1 What's new with respect to Budianto et al. (2020)?

Let

$$\hat{\pi}_t = \pi_t + (1 - \omega)\hat{\pi}_{t-1}, \tag{1}$$

where π_t is the inflation rate between periods $t - 1$ and t. We want to consider two monetary policy rules. The first rule, identical to Budianto et al. (2020), is:

$$i_t = \max\left\{0, r_t^n + \phi_\pi \hat{\pi}_t\right\}, \tag{2}$$

where r_t^n is the natural real rate of interest. The second rule, new in the literature, is:

$$i_t = \max\left\{0, r_t^n + \phi_\pi \mathbb{E}_t \hat{\pi}_{t+1}\right\}. \tag{3}$$

For each rule, we solve the model for different values of ω ranging from zero to one. We set $\phi_\pi = 2$.

2 Baseline model: only natural rate shocks

Recall that the equilibrium conditions of the private sector are given by the following two equations:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \pi_{t+1} \tag{4}$$

$$y_t = \mathbb{E}_t y_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \tag{5}$$

We are interested in finding policy functions: $\pi_t(\cdot,\cdot), y_t(\cdot,\cdot), i_t(\cdot,\cdot)$.

2.1 State Variables

- Natural interest rate, r_t^n
- Moving average inflation rate, $\hat{\pi}_{t-1}$

2.2 The moving average inflation rate: an additional endogenous variable

Focus on the first monetary policy rule. In such case we would like to solve the system of equations given by equations (1), (2), (4) and (5). In matrix form this reads:

$$
\begin{bmatrix} 1 & -\kappa & 0 & 0 \\ 0 & 1 & 0 & \sigma \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -\phi_\pi & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ \hat{\pi}_t \\ i_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1-\omega \\ 0 \end{bmatrix} \hat{\pi}_{t-1} + \begin{bmatrix} \beta & 0 \\ \sigma & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t y_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma \\ 0 \\ 1 \end{bmatrix} r_t^n \tag{6}
$$

The same approach for the second monetary policy rule will lead us to the following system of equations:

$$
\begin{bmatrix} 1 & -\kappa & 0 & 0 \\ 0 & 1 & 0 & \sigma \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ \hat{\pi}_t \\ i_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1-\omega \\ 0 \end{bmatrix} \hat{\pi}_{t-1} + \begin{bmatrix} \beta & 0 & 0 \\ \sigma & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \phi_\pi \end{bmatrix} \begin{bmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t y_{t+1} \\ \mathbb{E}_t \hat{\pi}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma \\ 0 \\ 1 \end{bmatrix} r_t^n \tag{7}
$$

It goes without saying that the solution to the system of equations, i.e the solution of (6) or (7), is the *unconstrained* solution. Therefore, whenever $i_t < 0$ we recompute the value of inflation, output and the inflation target after imposing $i_t = 0$.

3 An extension: adding cost push shocks

The New Keynesian Phillips curve is now augmented with a cost-push shock:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \pi_{t+1} + u_t \tag{8}$$

where u_t follows a stationary autoregressive process of order one. That is, $u_t = \rho_u u_{t-1} + \sigma_u \varepsilon_t^u$ where $\varepsilon_t^u \sim \mathcal{N}(0,1)$. We set $\rho_u = 0.3$ and $\sigma_u = 0.1$.

3.1 State Variables

- Natural interest rate, r_t^n
- Moving average inflation rate, $\hat{\pi}_{t-1}$
- Cost-push shock, u_t

3.2 The new system of equations

Now the economy is described by the following system of equations:

$$\begin{bmatrix} 1 & -\kappa & 0 & 0 \\ 0 & 1 & 0 & \sigma \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -\phi_\pi & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ \hat{\pi}_t \\ i_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1-\omega \\ 0 \end{bmatrix} \hat{\pi}_{t-1} + \begin{bmatrix} \beta & 0 \\ \sigma & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t y_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ \sigma & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} r_t^n \\ u_t \end{bmatrix} \tag{9}$$

for the standard AIT rule. In a world in which the forecasting AIT rule prevails, then the economy behaves according to the following system:

$$\begin{bmatrix} 1 & -\kappa & 0 & 0 \\ 0 & 1 & 0 & \sigma \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ \hat{\pi}_t \\ i_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1-\omega \\ 0 \end{bmatrix} \hat{\pi}_{t-1} + \begin{bmatrix} \beta & 0 & 0 \\ \sigma & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \phi_\pi \end{bmatrix} \begin{bmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t y_{t+1} \\ \mathbb{E}_t \hat{\pi}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ \sigma & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} r_t^n \\ u_t \end{bmatrix} \tag{10}$$

4 A purely forward looking rule

Let

$$\tilde{\pi}_t = \pi_t + (1-\omega)\mathbb{E}_t\tilde{\pi}_{t+1} \tag{11}$$

where π_t is still the inflation rate between periods $t-1$ and t. Now we want to consider the following two monetary policy rules:

$$i_t = \max\{0, r_t^n + \phi_\pi\tilde{\pi}_t\} \tag{12}$$

$$i_t = \max\{0, r_t^n + \phi_\pi\mathbb{E}_t\tilde{\pi}_{t+1}\} \tag{13}$$

As a result, in the model with only demand shocks we have no endogenous state variable and we only need to keep track of the natural interest rate, r_t^n. Thus, the two system of equations we would like to solve are:

$$\begin{bmatrix} 1 & -\kappa & 0 & 0 \\ 0 & 1 & 0 & \sigma \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -\phi_\pi & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ \tilde{\pi}_t \\ i_t \end{bmatrix} = \begin{bmatrix} \beta & 0 & 0 \\ \sigma & 1 & 0 \\ 0 & 0 & 1-\omega \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbb{E}_t\pi_{t+1} \\ \mathbb{E}_t y_{t+1} \\ \mathbb{E}_t\tilde{\pi}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma \\ 0 \\ 1 \end{bmatrix} r_t^n \tag{14}$$

if (12) governs the nominal interest rate and

$$\begin{bmatrix} 1 & -\kappa & 0 & 0 \\ 0 & 1 & 0 & \sigma \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ \tilde{\pi}_t \\ i_t \end{bmatrix} = \begin{bmatrix} \beta & 0 & 0 \\ \sigma & 1 & 0 \\ 0 & 0 & 1-\omega \\ 0 & 0 & \phi_\pi \end{bmatrix} \begin{bmatrix} \mathbb{E}_t\pi_{t+1} \\ \mathbb{E}_t y_{t+1} \\ \mathbb{E}_t\tilde{\pi}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma \\ 0 \\ 1 \end{bmatrix} r_t^n \tag{15}$$

if instead is (13) which captures the behavior of the policy rate.