# Indirect Inference: A Local Projection Approach *

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[FIRST DRAFT]

**Abstract**

This paper proposes an alternative way of generating moments to estimate the economic parameters of a Dynamic Stochastic General Equilibrium (DSGE) model using indirect inference methods. Until now, the most common auxiliary models that have been used to form a criterion function are Vector Autoregressions (VARs). However, given the increased popularity of Local Projections (LPs) to study the propagation of structural shocks, we ask: *how should one pick between VARs and LPs when choosing an auxilary model for indirect inference?* To answer this question, we make use of Monte Carlo methods to compare the performance of these two moment generating functions in a specific application involving the estimation of the parameters of the Smets and Wouters (2007) model. Then, we test our LP approach to indirect inference in a real world application, for which we borrow from the literature the empirically estimated responses of key macro variables to technology, fiscal and monetary shocks.

**Keywords:** *Indirect Inference, DSGE, Local Projections, VAR, Monte Carlo, Estimation*

**JEL classification:** C13, C15, E00

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# 1. Introduction

The Local Projection (LP) approach to understanding the dynamic effects of exogenous shocks, originating in Jordà (2005), has become a common tool for economic analyses. Though originally illustrated as an empirical model to study the impact of monetary shocks, this approach has become very widespread and used, as illustrated in Ramey (2016), to study the effects of fiscal and technology shocks as well.

We propose to use LPs as a way of summarizing key features of the data when estimating the structural parameters of a model in an indirect inference exercise. One issue with this approach is its comparison to the more common structural VAR approach. Plagborg-Møller and Wolf (2021) prove that VARs and LPs estimate the same impulse responses in population. Therefore, it may seem that using either VAR or LP should not matter for indirect inference. However, the finite sample properties of these two estimators differ. In particular, they show that when $p$ lags of the data are included in the VAR and as controls in the LP, impulse response functions (IRFs) approximately agree out to horizon $p$, but at longer horizons $h > p$ there is a bias-variance trade-off (Li et al., 2021). This paper is complementary to Plagborg-Møller and Wolf (2021) and Li et al. (2021) in that our interest is as well in the performance of the LP approach. But, our focus is on the estimation of structural parameters governing, for example, tastes and technology, rather than the impulse response functions *per se*. Smith (1993) illustrates the use of an autoregressive structure as an auxiliary model in a SMM estimation exercise.

Our goal is to study the comparable properties of parameter estimation using as moments the coefficients from an LP generated impulse response function. Given the findings of Plagborg-Møller and Wolf (2021) for finite samples, it is of interest to study the implications for estimation of underlying parameters using the LP rather than the VAR as the auxilary model for indirect inference.

Accordingly, we ask two questions. First, how do the properties of the parameter estimates depend on the use of a VAR as opposed to a LP as an auxiliary model? Second, how do the estimates based upon the LP auxiliary model from actual data compare to other methods?

The first question is answered through a Monte Carlo exercise, described in Section 4. For that experiment, the data generating mechanism is a parameterized version of the Smets and Wouters (2007) model. The structural parameters we focus on characterize household preferences, capital adjustment costs and the determination of wage and price rigidities. These parameters are fixed at the Smets and Wouters (2007) estimated values that Plagborg-Møller and Wolf (2021) use for their graphical illustration (section 2.4. of their paper).[1] The model is then simulated to create time series. Using these time series, we estimate a VAR and use this as one auxillary model, as in Smith (1993). We also use the simulated data to estimate the LP from a variety of shocks: (i) monetary (ii) technology and (iii) fiscal. In both approaches, we use the observed values of the innovation to these shocks from the simulation of the Smets and Wouters (2007) model.

From the Monte Carlo exercise, we find a tradeoff between these two approaches. The VAR approach generally has a lower RMSE and its average fit is better. However, the theoretical responses at estimated parameters are much closer to truth under the LP approach. Overall, our findings seem to suggest that the LP approach is better at picking those parameters that have a bigger impact for the shape of the theoretical IRF at long horizons.

The second question is answered by using LP estimates from these same three types of shocks from actual data, rather than from the Smets and Wouters (2007) model. The parameters can be estimated for each of these shocks independently or jointly. Compared to the estimates in Smets and Wouters (2007), our preliminary results show that there are some discrepancies, e.g. we obtain a higher intertemporal elasticity of substitution and lower frequencies of wage and price adjustments, which allow us to match better the response to a technology shock. The model's response to a fiscal shock is similar across estimation methodologies.

## 2. Economic Model

The analysis builds on the model formulated and estimated in Smets and Wouters (2007). While other models may serve the same purpose, this structure captures many of the central channels of monetary and fiscal policy. It was estimated using Bayesian methods. For the first part of our analysis, we take those estimated parameters as truth and see how close we

[1] These values correspond to those obtained using Johannes Pfeifer Dynare replication package: `https://github.com/JohannesPfeifer/DSGE_mod/tree/master/Smets_Wouters_2007`. Notice that these differ from the values reported in Smets and Wouters (2007) paper because a different sample is used.

come to them through the indirect inference approach. For the second part of the analysis, our estimated parameters are compared to those reported by Smets and Wouters (2007).

The Smets and Wouters (2007) model has become one, if not, the workhorse model in the DSGE literature. The model is based on Christiano et al. (2005) who added various frictions to a basic New Keynesian DSGE in order to capture the dynamic response to a monetary policy shock as measured by a structural vector autoregression (SVAR). In fact, price and wage stickiness paired with adjustment costs for investment, capacity utilization costs, habit formation in consumption, partial indexation of prices and wages as well as autocorrelated disturbance terms are able to generate a rich autocorrelation structure, which is key for capturing the joint dynamics of output, consumption, investment, hours worked, wages, inflation and the interest rate. These features of the model are crucial for our study since we are interested in matching the dynamic response of key macro aggregates to various shocks, as described in Section 3. Consequently, this explains why we use this model and not the traditional real business cycle as in the seminal paper by Smith (1993).

This section summarizes model components with an emphasis on key parameters. The summary builds upon Smets and Wouters (2003).

### 2.1. Households

Households are infinitely lived, working and consuming in each period of life. Their lifetime utility is given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, \bar{c}_{t-1}, n_t) \tag{1}$$

where $c_t$ is the consumption of the representative agent, $\bar{c}_{t-1}$ represents an external habit and $n_t$ is the amount of labor supplied. There are shocks associated with household impatience and the marginal rate of substitution between consumption and work. Household income comes from working, renting capital and dividends from the firms. Households save by holding bonds and also have access to state contingent securities which allow the household to smooth over taste shocks. Households also own the capital stock. This is rented to firms. Households incur an adjustment cost for changes in the capital stock. There is also a shock to this adjustment cost.

In the model, there are 4 parameters associated with the household problem: (i) the intertemporal elasticity of substitution $\sigma_c$, (ii) the strength of the habit $h$, (iii) the elasticity of labor supply $\sigma_l$, and (iv) the capital adjustment cost $\varphi$.

Further, each household acts as a wage setter, with a differentiated source of labor supply. There are associated parameters governing the likelihood of wage adjustment, $1 - \xi_w$ and the indexation of non-adjustment wages to past inflation, $\iota_w$. These parameters are estimated as well.

### 2.2. Firms

The firm side of the economy entails a perfectly competitive final goods sector. Output is produced using differentiated intermediate goods. In the intermediate goods markets sellers have market power modelled as a monopolistic competition. The production of the intermediate goods requires capital and labor.

There are sticky prices in the market for intermediate goods. Price setting is not state dependent but rather firms are randomly granted an opportunity to adjust their prices, as in Calvo (1983). For those firms not adjusting their prices, there is partial indexation to past inflation.

There is an aggregate productivity shock associated with intermediate goods production. The TFP shock is assumed to follow an AR(1) process.

From this specification there are a couple of key parameters. One is the probability that a firm can adjust its price, denoted $1 - \xi_p$ and the other one is the partial indexation parameter, denoted $\iota_p$. These parameters are surely important for the effects of monetary policy. But, as we shall see, they also matter for the effects of other shocks and can be identified from impulse responses associated with non-monetary innovations.

### 2.3. Government: Fiscal and Monetary Authorities

Government spending is another source of stochastic variation in the model. Spending is specified as an AR(1) process.[2]

Monetary policy is modelled through a generalized Taylor rule that gradually adjusts the

[2] There is no apparent mention of taxation in either Smets and Wouters (2003) or Smets and Wouters (2007). Taxes appear to be lump sum. Distortionary taxes can change the impact of fiscal policy shocks.

policy interest rate in response to changes in inflation and output gap. Innovations to this rule play a key role in the analysis.

## 3. Indirect Inference: LP vs. VAR impulse responses

This section fixes some basic ideas in order to clarify our approach and results. This includes the LP and VAR frameworks as well as the indirect inference approach.

Start by consider any economic model, $M$. Assume that the endogenous variables of this model, $y_t$, depend on its own lags $y_{t-1}$ (endogenous states), some exogenous variables $z_t$ (exogenous states) and some random errors $u_t$ (shocks). Further assume that the model is parameterized by an ex-ante unknown vector of parameters $\Theta$. That is, let

$$y_t = M(y_{t-1}, z_t, u_t; \Theta) \tag{2}$$

for $t = 1, 2, \ldots, T$. Given an initial value for the endogenous variable $y_{-1}$ and a sequence for the shocks $\{u_t\}_{t=1}^{T}$, it is possible to generate infinite data sequences $\{y_t\}_{t=1}^{T}$. This is a generic way to represent the Smets and Wouters (2007) model that provides the "sandbox" for our experiments.

### 3.1. LP

To understand the **LP** approach, consider the following regression:

$$\tilde{y}_{t+h} = \mu_h + \beta_h \tilde{x}_t + \sum_{\ell=1}^{p} \delta'_{h,\ell} w_{t-\ell} + \xi_{h,t}. \tag{3}$$

where $\tilde{y}_t$ is one of the variables of interest, $\tilde{x}_t$ denotes an innovation associated with a particular form of an aggregate shock. Finally, there are $p$ lags of a vector of controls $w_t = \{\tilde{x}_t, \tilde{y}_t\}$.

The parameters in (3) are estimated at each horizon $h = 0, 1, 2, \ldots, H$. This is simply an OLS regression of leads of $\tilde{y}_t$ on past innovations. For each horizon, $(\mu_h, \beta_h, \{\delta'_{h,\ell}\}_{\ell=1}^{p})$ are the projection coefficients.

**Definition 1.** *The LP - IRFs of $\tilde{y}_t$ with respect to $\tilde{x}_t$ are given by $\{\beta_h\}_{h\geq 0}$ in (3). Note that there are $H$ coefficients generated for each of the variables of interest, $\tilde{y}_t$ for each type of innovation, $\tilde{x}_t$.*

In our study, we focus on $\tilde{y}_t \in \{y_t, c_t, i_t, n_t\}$, being output, consumption, investment and hours worked respectively. Further $\tilde{x}_t \in \{\varepsilon_t^a, \varepsilon_t^g, \varepsilon_t^m\}$, so that we consider shocks to technology, government spending and monetary policy.

### 3.2. VAR

The starting point for the multivariate linear **VAR($p$) projection** is:

$$w_t = c + \sum_{\ell=1}^{p} A_\ell w_{t-\ell} + u_t \tag{4}$$

where $u_t$ is the projection residual and $(c, \{A_\ell\}_{\ell=1}^p)$ are the projection coefficients. Here $p$ indicates the longest lag, matching the lag in the LP controls. Notice that given the definition of $w_t$, we are considering bivariate VAR(p) projections with the innovation ordered first.

Let $\Sigma_u \equiv \mathbb{E}[u_t u_t']$ and define a *Cholesky decomposition* $\Sigma_u = BB'$ where $B$ is lower triangular with positive diagonal entries. With this, consider the corresponding recursive *SVAR representation*:

$$A(L) w_t = c + B\eta_t \tag{5}$$

where $A(L) \equiv I - \sum_{\ell=1}^{p} A_\ell L^\ell$ and $\eta_t \equiv B^{-1} u_t$. Define the lag polynomial $\sum_{\ell=0}^{p} C_\ell L^\ell = C(L) \equiv A(L)^{-1}$.

**Definition 2.** *The SVAR - IRFs of $\tilde{y}_t$ with respect to an innovation in $\tilde{x}_t$ is given by $\{\theta_h\}_{h\geq 0}$ with $\theta_h \equiv C_{2,\bullet,h} B_{\bullet,1}$ where $\{C_\ell\}$ and $B$ are defined in (5).*

### 3.3. Indirect Inference

The indirect inference approach is a form of minimum distance estimation. The parameter estimates are given by:

$$\hat{\Theta} = \arg\min_{\Theta} \left(M^d - M^s(\Theta)\right)' W \left(M^d - M^s(\Theta)\right) \tag{6}$$

In (6), $M^d$ are moments from the data and $M^s(\Theta)$ are the counterparts from simulated data, where $\Theta$ is the vector of structural parameters. In our application, these are the parameters characterizing household preferences, wage setting and price setting of firms in the Smets and Wouters (2007) model. In this quadratic form, $W$ is a weighting matrix.

In the indirect inference approach, the moments taken from the data are regression coefficients of an auxiliary model. As in Smith (1993), the auxiliary model could be the coefficients from a VAR (or the associated impulse response, denoted $\theta_h$ in Definition 2). An alternative, explored here, is to use the LP as the source of moments, matching the $\beta_h$ coefficients from Definition 1.

# 4. A Monte Carlo Study

This section uses the VAR and LP approaches to indirect inference in order to estimate a subset of the parameters of the Smets and Wouters (2007) model using repeated samples from the data generating process associated with their DSGE model. The goal of this Monte Carlo study is to compare the performance of these two moment generating functions in a controlled setting where we know the true parameter vector, $\Theta = \Theta^*$.

## 4.1. Setting up the Monte Carlo: The Data Generating Process

For the Monte Carlo study, we solve the model in its log-linearized version, and then simulate it to generate an artificial database consisting of time series paths of four key macro aggregates: output, consumption, investment and hours worked $\{y_t, c_t, i_t, n_t\}$, as well as time paths for the innovations to technology, fiscal and monetary policy shocks $\{\varepsilon_t^a, \varepsilon_t^g, \varepsilon_t^g\}$. The log-linearized equilibrium conditions of the Smets and Wouters (2007) model are reproduced in Appendix A.

The "observed" sample size is set at $T = 300$, which coincides with the sample size chosen by Òscar Jordà in the Monte-Carlo study of his seminal paper "Estimation and Inference of Impulse Responses by Local Projections" (Jordà, 2005). We are aware that sample size in the time dimension is typically much smaller in empirical applications and more importantly that LPs can be biased for small samples as shown by Herbst and Johannsen (2021). Moreover, we will also show that estimated impulse responses have particularly large variability in small samples.[3] Nevertheless, we haven't yet explored how this choice influences our parameter

[3] Many applied macroeconomists are aware of the erratic behavior of the impulse responses estimated by LPs, which has even lead to Barnichon and Brownlees (2019) to propose a new estimation method that smooth

estimates, but we conjecture that the LP approach to indirect inference may also be very sensible to changes in $T$.

We focus on the 8 structural parameters discussed above. The "true" values of these structural parameters are listed in Table 1, while the remaining ones are set and fixed at the estimated values from Smets and Wouters (2007).[4]

Table 1: True values of structural parameters

| $\sigma_c$ | $h$ | $\sigma_l$ | $\varphi$ | $\xi_w$ | $\xi_p$ | $\iota_w$ | $\iota_p$ |
|---|---|---|---|---|---|---|---|
| 1.2679 | 0.8056 | 2.5201 | 6.3144 | 0.7668 | 0.5304 | 0.5345 | 0.1779 |

Here: $\sigma_c$ : intertemporal elasticity of substitution, $h$ : habit parameter, $\sigma_l$ : elasticity of labor supply, $\varphi$ : investment adjustment cost parameter, $\xi_w$, $\xi_p$ : Probabilities of no adjustment for wage and prices, $\iota_w$, $\iota_p$ : Degree of wage and price indexation to past inflation.

### 4.2. The Moment Generating Functions

Indirect inference applications typically use an auxiliary econometric model that help researchers summarize key features of the data that they wish to match with their economic model. In most cases, and specially for representative agent models (like most DSGEs), a VAR is chosen. This approach was popularized by Smith (1993) in his paper "Estimating Nonlinear Time-Series Models Using Simulated Vector Autoregressions", in which he proposes to match all the coefficients of the VAR as well as the associated variance-covariance matrix of the error term. Alternatively, as we propose in this paper, one could also use LP coefficients.

When comparing these two approaches in our indirect inference exercise, we depart from Smith (1993) in that we simply use the estimated impulse responses instead of all the estimated coefficients. We prefer this approach because IRFs are excessively parameterized in the LP framework, resulting in an absurd number of moments. However, one should not confuse this with IRFs matching, another popular approach to estimate the parameters of DSGE models, see for example Christiano et al. (2005). The subtle, but important difference, between IRF matching and our approach to indirect inference is that we are estimating the IRF using

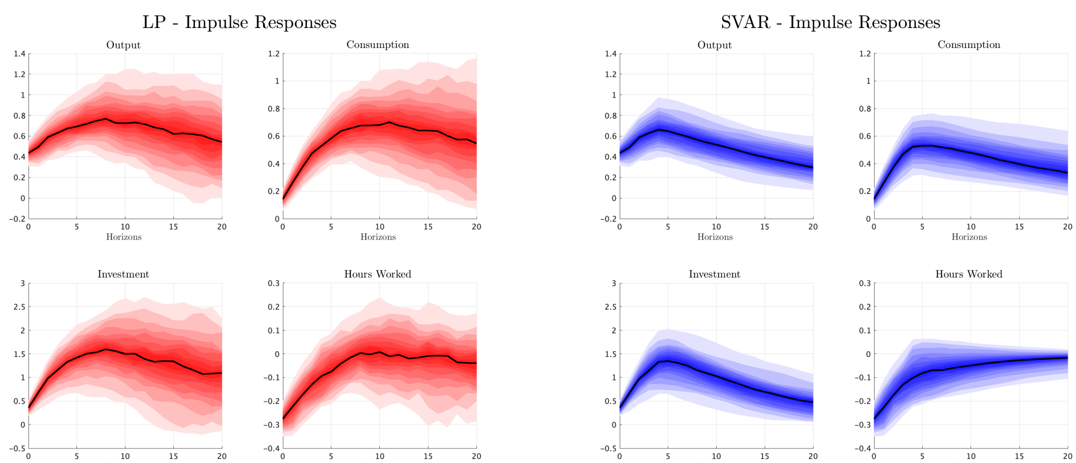
out the response. We'll show that in our set-up, in which we have access to an infinite amount of data, the variability disappears as one increases T.

[4] The main reason why we estimate a rather limited subset of the parameters is to reduce the computational burden. Our LP approach can be very easily adapted to estimate a large number of parameters as it is very easy to generate moments, e.g. by matching the response to a different variable, shock or over a longer horizon.

the model's simulated data either via LPs or via a VAR regression followed by a Cholesky decomposition; while in an IRF matching exercise the responses are directly recovered from the model. In other words, these two approaches are only equivalent when the econometric model used to estimate the IRF is not misspecified, and therefore the estimated IRF coincides with the true one.[5]

We construct data moments as follows. For each of the 100 simulated datasets at $\Theta = \Theta^*$, which are of length $T = 300$, we estimate either the LP or the SVAR-IRFs to technology, fiscal, or monetary shocks as described in Section 3. We set $p = 4$ and $H = 20$. Figure 1 depicts the distribution of these LP-IRFs (left panel) and SVAR-IRFs (right panel) to a technology shock. The black solid line corresponds to the median response of output, consumption, investment and hours to the shock. The counterparts of Figure 1 for each of these two other shocks can be seen in Appendix C.1.

Figure 1: Technology Shock




(a) Local Projections IFRs

(b) Structural Vector Autoregression IRFs

Notice from Figure 1 that the variability of the estimated LP-IRFs is much larger than that of the SVAR-IRFs, specially at horizons $h > p$. As a result, one may expect that the variance of the estimated economic parameters is also going to be larger when using the LP approach to indirect inference. And indeed, this is true for the majority of the estimated parameters as

[5] In our specific application, model misspecification will depend upon the choices of $T^s$ and $p$ as shown in Appendix B.

we will discuss in Section 4.3. This result, i.e. the wider distribution around the LP-IRFs, also holds for the fiscal and monetary shocks.

### 4.3. Results

The structural model is estimated $S = 100$ times, that is once for each of the 100 vectors of data moments corresponding to one of our eight scenarios.[6] In particular, six of these eight scenarios corresponds to estimation routines that match LP or SVAR responses to only one of the three shocks. While the remaining two correspond to a selection of the LP or SVAR responses to all of the three shocks. We base this selection on the sensibility analysis of the moments (see figures in Appendix C.2) and try to match the response of investment to the three aggregate shocks as well as the response of consumption to a technology shock.

To construct the counterpart of the data moments, we simulate the model at $\Theta = \Theta^{guess}$ for $T^s = 3,000$ periods, so that the simulated database is 10 times larger that the "observed" one as in Smith (1993). To minimize the effects of initial conditions, we simulate $0.1 \times T^s$ periods which are then discarded.[7] Then, we estimate (3) and (5) on this simulated dataset using the same auxiliary model as for the data moments.

Thus, in each scenario we target a total of 84 ($= 21 \times 4$) moments to estimate 8 parameters. As we have an over-identified model we use the optimal weighting matrix, that is the inverse of the variance covariance matrix of the moments, to solve the minimization problem stated in equation (6).

The solution to that problem is a vector of estimated parameters for each of the 100 vectors of data moments, i.e $\hat{\Theta} = \{\hat{\Theta}^s\}_{s=1}^{S}$. Following Smith (1993), we compute for each of the estimated parameters $\hat{\Theta}_i \in \hat{\Theta}$, the following statistics:

$$\text{Bias}_i \equiv \mathbb{E}\left[\hat{\Theta}_i\right] - \Theta_i^* \tag{7}$$

$$\text{Std dev}_i \equiv \sqrt{\text{Var}(\hat{\Theta}_i)} \tag{8}$$

$$RMSE_i \equiv \sqrt{\text{Bias}_i^2 + \text{Var}(\hat{\Theta}_i)} \tag{9}$$

where expectations are taken over the $S$ Monte Carlo draws. Tables 5 and 6 in Appendix

[6] To be clear, the same auxiliary model is used for each of these 100 vector of moments. The only source of variability comes from the different draws of the shocks.

[7] This is also done for the "observed" sample size.

D report these statistics for all the considered scenarios. These two tables contain a lot of information, thus, we will mainly focus on the Root Mean Squared Error (RMSE) going forward. Notice that the RMSE combines the information of both bias and variance. Therefore, it seems natural to start the discussion about the relative performance of the two moment generation functions as well as the performance across shocks by looking at this metric parameter by parameter.

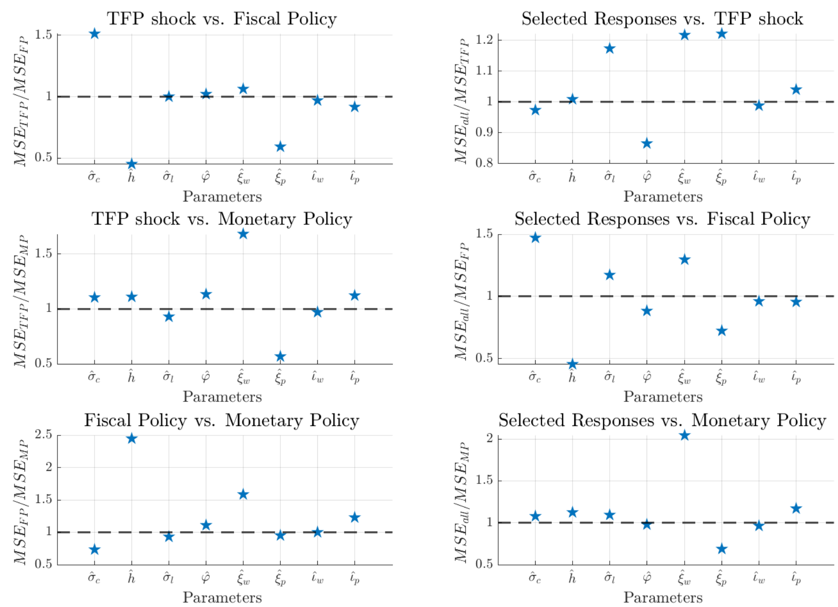
Figure 2 tries to summarize this comparison. Panel (a) assesses how well responses to different shocks pick up different parameters when the LP IRFs have been used as the source of moments. For example, focus on the comparison between the estimates obtained after matching the responses to a technology shock versus those coming from targeting the responses to a monetary shock (subplot in the first column, second row). We can see that most parameters are better estimated through the responses to a monetary shock. That is the ratio of RMSEs is greater than 1 in most cases. There are though a few exceptions: the elasticity of labor supply ($\sigma_l$) and the Calvo price adjustment probability ($1 - \xi_p$) have a ratio of RMSEs below 1. It seems surprising that the Calvo price adjustment probability is better picked up by a non-monetary shock given its importance for inflation dynamics and therefore for monetary policy. In any case, this can be seen as an opportunity, since one can exploit the variability coming from a non-monetary shock to estimate some of the key parameters for monetary policy, e.g. $1 - \xi_p$, and then evaluate the validity of the model "out of sample" with respect to an untargeted response to a monetary shock.

Panel (b) focuses on the comparison across moment generating functions. The message is a lot more clear in this case. The SVAR - IRF approach to indirect inference seems to be superior for most parameter values. This result could be explained by the well-known fact that model misspecification is not an issue for indirect inference. Therefore, the relatively less variance of the SVAR - IRFs across different shock draws leads also to better estimates in terms of RMSE.[8]

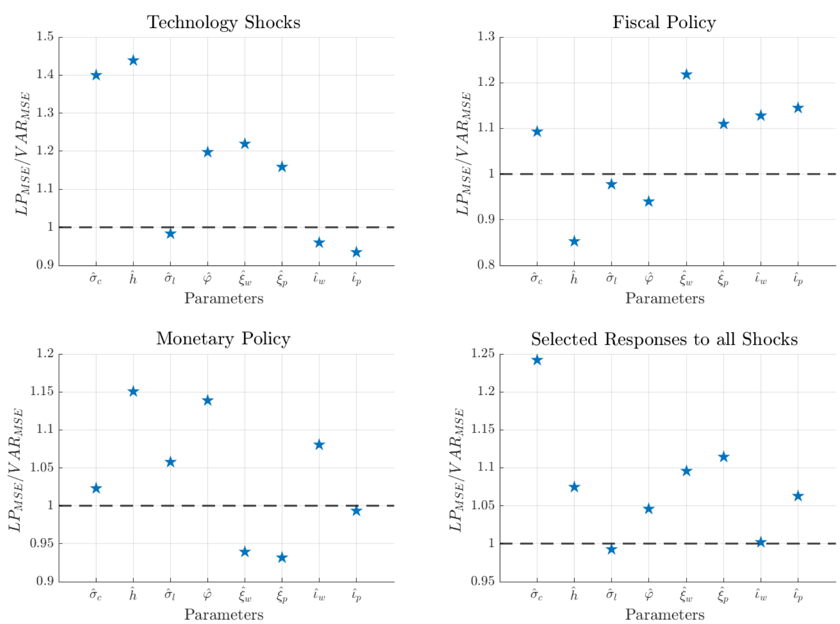
These parameter-by-parameter comparisons are a good way to assess the relative performance of these two moment generating functions for each of the estimated parameters in a case-by-case basis. However, one would also like to know how these two distinct approaches to

[8] It is still needs to be studied how much of this result is explained by the bias and how much by the variance since RMSE gives them equal weights.

Figure 2: Relative performance in terms of RMSE




(a) Across Different Shocks (LP - IRFs approach)




(b) LP - IRFs vs. SVAR - IRFs

indirect inference perform when all the parameters involved in the estimation are considered as a whole. To do so we evaluate the value of the criterion function, i.e. the RHS of equation (6), at the optimal/estimated parameter values $\hat{\Theta}^s$. This metric, usually refer to as the $J$-statistic, gives an overall measure of how well we match our targeted moments. Thus, the closer $J$ is to zero, the better the match. However, also notice that even at $\Theta^*$ these metric won't be zero because the simulated moments are obtained in a different sample ($T \neq T^s$) with different draws for the shock.

Table 2 reports the average and the maximum value of this statistic across the S Monte Carlo draws for each of the eight different scenarios, i.e. four different sets of targeted IRFs estimated under the two alternative moment generating functions. We start by fixing the moment generating function and comparing across targeted IRFs. Notice how the tailored scenario, for which we picked the responses that were more sensitive to small changes in the parameters, gives the lowest average $J$. In any case, values are similar to each other and there is still some room for improvement as the $J$-statistic is far away from zero.

If we now fix the targeted responses and compare across moment generating functions, then, we see how the LP - IRFs approach does worse on average, which is consistent with the discussion about the RMSE. However, the maximum value of $J$ is always larger, and by a significant margin, for the SVAR - IRFs scenario. Moreover, it is also important to notice the time needed to estimate the parameters over the S repeated samples is much lower for the LP - IRFs, something that it's worth taking into account for models in which the solution is time-intensive.[9] Thus, these last two observations favor our proposed approach of using the LP - IRFs as moments.

Table 2: Overall performance: Estimated Impulse Responses

| | Local Projections | | | Vector Autoregression | | |
|---|---|---|---|---|---|---|
| | Avg. $J$ | Max. $J$ | Time (in min.) | Avg. $J$ | Max. $J$ | Time (in min.) |
| *Technology Shock* | 86.93 | 117.61 | 17.38 | 83.71 | 245.40 | 69.90 |
| *Fiscal Policy* | 88.02 | 127.94 | 21.63 | 83.62 | 250.59 | 68.03 |
| *Monetary Policy* | 89.68 | 123.26 | 20.31 | 82.69 | 210.79 | 68.42 |
| *Selected Responses* | 86.28 | 123.03 | 20.11 | 81.97 | 237.03 | 62.49 |

[9] Notice that the lower computing time need to estimate the parameters comes from the lower number of iterations needed to reach the minimum. The number of iterations is significantly smaller since per iteration computing the LP-IRFs takes longer since one has to estimate more coefficients.

Finally, we consider a third measure to evaluate the performance of the LP and SVAR - IRFs approaches to indirect inference since neither RMSE or J-statistics informs us about how close we are from the true/theoretical impulse response functions, the ultimate object of interest. Therefore, we look at the weighted distance between the theoretical IRFs coming from the model at the estimated parameter values $\hat{\Theta}$ and at the true values $\Theta^*$.

Table 3 shows the average and the maximum value of this metric across all the $\hat{\Theta}^s$. Results differ substantially. According to this metric, the LP - IRFs approach is able to match the theoretical IRFs much better than its SVAR counterpart, which may seem inconsistent with our previous findings. However, this measure is simply telling us that the LP - IRFs approach does a significantly a better job in picking those parameters that are relevant for capturing the shape of the true impulse responses. Consequently, it may still be desirable to use the LP - IRFs approach to indirect inference despite it underperformance in terms of the J-statistic and the RMSE for most parameters.

Table 3: Overall performance: Model Impulse Responses

| | Local Projections | | Vector Autoregression | |
|---|---|---|---|---|
| | Avg. $J^*$ | Max. $J^*$ | Avg. $J^*$ | Max. $J^*$ |
| *Technology Shock* | 2.90 | 7.84 | 226.54 | 1014.14 |
| *Fiscal Policy* | 3.36 | 14.94 | 289.26 | 1893.02 |
| *Monetary Policy* | 3.14 | 12.63 | 821.66 | 3380.27 |
| *Selected Responses* | 9.37 | 75.05 | 686.72 | 9307.71 |

## 5. An Empirical Application

Finally, we put to test our local projection approach to indirect inference by estimating a subset of the parameters of the Smets and Wouters (2007) model using empirically estimated responses to technology, fiscal and monetary shocks as our data moments. It goes without saying that all these responses have been estimated within the Jordà (2005) local projection framework.

### 5.1. Technology Shocks

Technology shocks are the most important type of non-policy shocks. In fact, there is a vast literature on identification of these shocks on time series models. A review of the literature on this topic can be found in Valerie A. Ramey's Chapter in the Handbook of Macroeconomics (Ramey, 2016). We actually borrow from her the LP estimated responses to an unanticipated TFP shock, measured as in Francis et al. (2014).[10]

She reports the responses of various variables among which we select the response of real GDP, consumption, non-residential investment, and hours. Figure 3 depicts the median responses of these four variables (solid blue line) during 21 periods. Thus, we target a total of 84 ($21 \times 4$) data moments during the estimation.

We estimate the same eight parameters from the Monte Carlo study, as well as the persistence and the variance of the TFP shock. That is, we look over the following parameter space $\Theta_{TFP} = \{\sigma_c, h, \sigma_l, \varphi, \xi_p, \xi_w, \iota_p, \iota_w, \rho_a, \sigma_a\}$ to minimize the quadratic distance between data and simulated moments.

Simulated moments are computed using equation (3), that is regressing the variable of interest on the TFP innovation over $H$ horizons while controlling for $p$ lags of itself and the shock. In particular, we estimate this LP regression on model simulated data of length $T^s = 300,000$, which is sufficiently long to have well behaved responses.[11] In Appendix B, we discuss how these responses depend upon the choice of $T^s$ under this specification and why is important to simulate the economic model long enough. In any case, this way of computing the moments may be controversial since we use the actual innovation as our independent variable of interest, which the econometrician would never be able to observe. However, it is common these days that researchers come up with smart strategies to recover some approximation to the shock. And in fact, most LP specifications in the literature, as well as Ramey (2016)'s, have the following form:

$$\tilde{z}_{t+h} = \alpha_h + \theta_h \cdot \text{shock}_t + \varphi_h(L) \cdot \text{control}_{t-1} + \varepsilon_{t+h} \tag{10}$$

[10] They identify the shock through medium-run restrictions.

[11] Notice that for ergodic processes, like the Smets and Wouters (2007) model, it is equivalent to estimate the median response on $S$ samples, each length for $T$; than use a single sample of length $T^s = T \times S$.

where $\tilde{z}_{t+h}$ is the variable of interest, and $\varepsilon_{t+h}$ the error term. Equation (10) looks very similar to (3) with the important nuance that the shock may have some associated measurement error which is potentially correlated with the error term.[12] Thus, we argue that since the true model shock is *iid*, then there is no need to add extra controls to recover the correct IRFs. This simplifies things, since many times we don't have access to the model counterpart of those control variables. For example, Ramey (2016) includes the log real stock prices per capita as a control, a variable that is not available within the Smets and Wouters (2007) model. We do, however, control for the same number of lags of the shock and the dependent variable.

After setting the weighting matrix to the identity, we have all the ingredients to solve (6) and estimate the parameters of the model using our LP - IRFs approach. We repeat this estimation 100 times to get a distribution around our parameter estimates, however, the data moments are now fixed and the distribution comes only from the different draws of the shock. Results are summarized in the first subtable of Table 4.

Table 4: SMM using the LP - IRFs approach

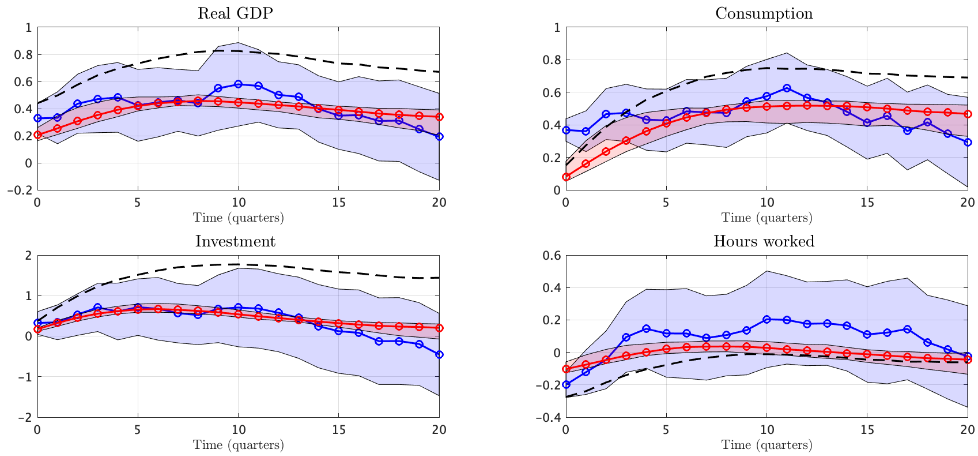
| | Technology Shocks | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{\sigma}_c$ | $\hat{h}$ | $\hat{\sigma}_l$ | $\hat{\varphi}$ | $\hat{\xi}_w$ | $\hat{\xi}_p$ | $\hat{\iota}_w$ | $\hat{\iota}_p$ | $\rho_a$ | $\sigma_a$ |
| *P-M&W – S&W* | 1.2679 | 0.8056 | 2.5201 | 6.3144 | 0.7668 | 0.5304 | 0.5345 | 0.1779 | 0.9826 | 0.5017 |
| *Median* | 1.57 | 0.80 | 3.15 | 3.79 | 0.87 | 0.32 | 0.66 | 0.21 | 1.00 | 0.20 |
| *5th pctl.* | 0.77 | 0.57 | 1.54 | 3.79 | 0.82 | 0.32 | 0.56 | 0.12 | 1.00 | 0.17 |
| *95th pctl.* | 1.57 | 0.85 | 3.15 | 4.56 | 0.95 | 0.32 | 0.66 | 0.21 | 1.00 | 0.24 |
| | Fiscal Policy | | | | | | | | | |
| | $\hat{\sigma}_c$ | $\hat{h}$ | $\hat{\sigma}_l$ | $\hat{\varphi}$ | $\hat{\xi}_w$ | $\hat{\xi}_p$ | $\hat{\iota}_w$ | $\hat{\iota}_p$ | $\rho_g$ | $\sigma_g$ |
| *P-M&W – S&W* | 1.2679 | 0.8056 | 2.5201 | 6.3144 | 0.7668 | 0.5304 | 0.5345 | 0.1779 | 0.70 | 0.6752 |
| *Median* | 1.57 | 0.90 | 1.51 | 7.89 | 0.93 | 0.66 | 0.53 | 0.20 | 0.75 | 0.33 |
| *5th pctl.* | 1.01 | 0.48 | 1.51 | 3.79 | 0.46 | 0.32 | 0.32 | 0.10 | 0.52 | 0.20 |
| *95th pctl.* | 1.57 | 1.00 | 1.53 | 7.89 | 0.95 | 0.66 | 0.66 | 0.21 | 1.00 | 0.39 |

Here : *P-M&W – S&W* refers to the parameters used in Plagborg-Møller and Wolf (2021) graphical illustration. They are different from those reported in Smets and Wouters (2007) paper.

We also compute the estimated responses on simulated data using the entire distribution of parameter estimates. The resulting impulse responses are depicted in red in Figure 3. The red dash line is the median response, while the lower and upper bounds of the shaded area correspond to the 5th and 95th percentiles, respectively.

[12] Stock and Watson (2018) explain very neatly all these issues in empirical applications and how to tackle them by means of external instruments.

Figure 3: TFP shocks – Empirical vs. Model Estimated IRFs




Overall, the Smets and Wouters (2007) model does a good job in capturing these responses at the estimated parameters by our LP - IRFs approach. However, the very tight confidence intervals come from the fact that the estimation leads us to the pre-specified upper or lower bounds for certain parameter values during the optimization stage. When looking over a wider parameter space, the optimization algorithm bring us to parameter combinations where the Blanchard-Kahn conditions are not satisfied.

Finally, we also report the estimated impulse responses at the Smets and Wouters (2007) median estimates, the black dash line in Figure 3. It is important to note that these parameters were obtained using Bayesian estimation techniques on a different sample. Nevertheless, we think the comparison is still interesting. Notice, for example, how at the Smets and Wouters (2007) parameters the estimated responses from the model overshoot the response of output, consumption and investment, and undershoot the response of hours; while at our parameter estimates we are undershooting consumption and overshooting hours but only upon impact.

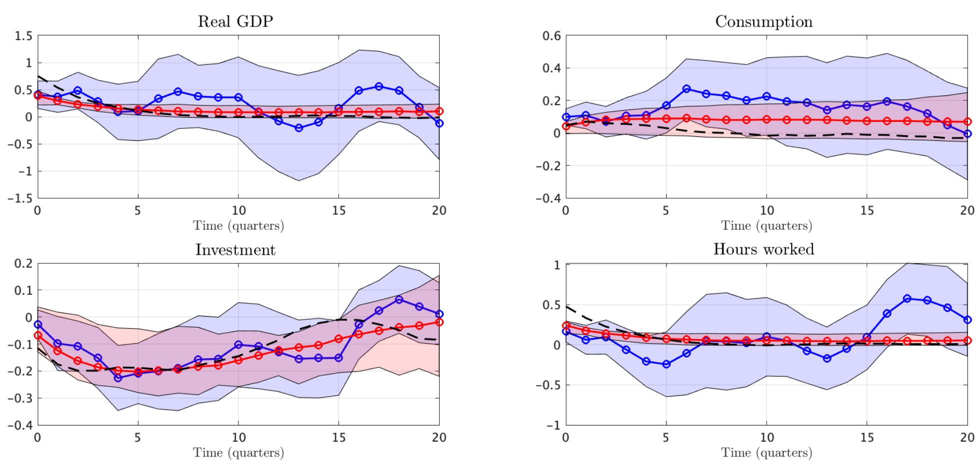
### 5.2. Government Spending Shocks

Now we turn to fiscal policy shocks, for which we also borrow the responses from Ramey (2016). In particular, we target the responses of GDP, non-durables + services consumption, non-residential investment and hours worked to an unanticipated government spending shock. As for the technology shock, Ramey (2016) reports these responses to shocks identified in various forms. We focus on those that were generated using Blanchard and Perotti (2002) strategy. That is, the shock is identified by assuming that government purchases were predetermined within the quarter. In other words, the shock comes from a standard Cholesky decomposition with government spending ordered first. These responses are depicted in blue in Figure 4.

As for the technology shock we use these moments to estimate the same eight parameters from the Monte Carlo study, as well as the persistence and the variance of the government spending shock. That is, we look over $\Theta_{FP} = \{\sigma_c, h, \sigma_l, \varphi, \xi_p, \xi_w, \iota_p, \iota_w, \rho_g, \sigma_g\}$ to minimize the quadratic distance between data and simulated moments. Simulated moments are computed in a similar fashion but now using the innovation of the government spending shock (instead of that of the TFP shock) in equation (3).

The distribution of the parameters estimates that we obtain using our LP - IRFs indirect inference approach are summarized in the second subtable in Table 4. In Figure 4, we also plot (in red) the estimated impulse responses at these optimal parameter values. As for the technology shock, the Smets and Wouters (2007) model at our estimated parameters does a good job in capturing the dynamic responses of output, consumption, investment and hours worked.

Figure 4: Fiscal Policy – Empirical vs. Model Estimated IRFs




Finally, we compare our results to those obtained using Bayesian techniques. The black dash line in Figure 4 corresponds to the estimated responses at Smets and Wouters (2007) parameters, which are also able to capture the responses quite well. In fact, they are within our confidence intervals and those of the empirical responses.

### 5.3. Monetary Policy Shocks

We haven't yet tested our approach for monetary policy shocks. Nevertheless, we plan to match the impulse response functions estimated in Tenreyro and Thwaites (2016). They show state-dependent effects of monetary policy, which suggest that it will be also interesting to consider one additional data generating process for the Monte Carlo study that features some form of non-linearities, specially given the recent findings of Gonçalves et al. (2021) about the construction of non-linear impulse responses within the LP framework.

## 6. Conclusion

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# A. Log-Linearized Equilibrium Conditions

- The aggregate resource constraint:

$$\hat{y}_t = c_y \hat{c}_t + i_y \hat{i}_t + z_y \hat{z}_t + \varepsilon_t^g \tag{A.1}$$

- The consumption Euler equation:

$$\begin{aligned} \hat{c}_t = & \frac{h/\gamma}{1+h/\gamma}\hat{c}_{t-1} + \frac{1}{1+h/\gamma}\mathbb{E}_t\hat{c}_{t+1} + \frac{wl_c\,(\sigma_c-1)}{\sigma_c(1+h/\gamma)}\left(\hat{l}_t - \mathbb{E}_t\hat{l}_{t+1}\right) + \\ & - \frac{1-h/\gamma}{(1+h/\gamma)\sigma_c}\left(\hat{r}_t - \mathbb{E}_t\hat{\pi}_{t+1}\right) - \frac{1-h/\gamma}{(1+h/\gamma)\sigma_c}\varepsilon_t^b \end{aligned} \tag{A.2}$$

- The investment Euler equation:

$$\hat{i}_t = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\hat{i}_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}\mathbb{E}_t\hat{i}_{t+1} + \frac{1}{\varphi\gamma^2\left(1+\beta\gamma^{(1-\sigma_c)}\right)}\hat{q}_t + \varepsilon_t^i \tag{A.3}$$

- The arbitrage equation for the value of capital:

$$\hat{q}_t = \beta(1-\delta)\gamma^{-\sigma_c}\mathbb{E}_t\hat{q}_{t+1} - \hat{r}_t + \mathbb{E}_t\hat{\pi}_{t+1} + \left(1-\beta(1-\delta)\gamma^{-\sigma_c}\right)\mathbb{E}_t\hat{r}_{t+1}^k - \varepsilon_t^b \tag{A.4}$$

- The aggregate production function:

$$\hat{y}_t = \Phi\left(\alpha\hat{k}_t^s + (1-\alpha)\hat{l}_t + \varepsilon_t^a\right) \tag{A.5}$$

- Capital services:

$$\hat{k}_t^s = \hat{k}_{t-1} + \hat{z}_t \tag{A.6}$$

- Capital utilization:

$$\hat{z}_t = \frac{1-\psi}{\psi}\hat{r}_t^k \tag{A.7}$$

- The accumulation of installed capital:

$$\hat{k}_t = \frac{(1-\delta)}{\gamma}\hat{k}_{t-1} + (1-(1-\delta)/\gamma)\hat{i}_t + (1-(1-\delta)/\gamma)\varphi\gamma^2\left(1+\beta\gamma^{(1-\sigma_c)}\right)\varepsilon_t^i \tag{A.8}$$

- Cost minimization by firms implies that the price mark up:

$$\hat{\mu}_t^p = \alpha\left(\hat{k}_t^s - \hat{l}_t\right) - \hat{w}_t + \varepsilon_t^a \tag{A.9}$$

- New Keynesian Phillips curve:

$$\begin{aligned}\hat{\pi}_t =& \frac{\beta\gamma^{(1-\sigma_c)}}{1+\iota_p\beta\gamma^{(1-\sigma_c)}}\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\iota_p}{1+\beta\gamma^{(1-\sigma_c)}}\hat{\pi}_{t-1} + \\ & - \frac{\left(1-\beta\gamma^{(1-\sigma_c)}\xi_p\right)(1-\xi_p)}{\left(1+\iota_p\beta\gamma^{(1-\sigma_c)}\right)(1+(\Phi-1)\varepsilon_p)\,\xi_p}\hat{\mu}_t^p + \varepsilon_t^p\end{aligned} \tag{A.10}$$

- Cost minimization by firms implies that the rental rate of capital:

$$\hat{r}_t^k = \hat{l}_t + \hat{w}_t - \hat{k}_t^s \tag{A.11}$$

- In the monopolistically competitive labor market, the wage mark-up

$$\hat{\mu}_t^w = \hat{w}_t - \sigma_l\hat{l}_t - \frac{1}{1-h/\gamma}\left(\hat{c}_t - h/\gamma\hat{c}_{t-1}\right) \tag{A.12}$$

- Wage adjustment:

$$\begin{aligned}\hat{w}_t =& \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}\left(\mathbb{E}_t\hat{w}_{t+1} + \mathbb{E}_t\hat{\pi}_{t+1}\right) + \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\left(\hat{w}_{t-1} - \iota_w\hat{\pi}_{t-1}\right) + \\ & - \frac{1+\beta\gamma^{(1-\sigma_c)}\iota_w}{1+\beta\gamma^{(1-\sigma_c)}}\hat{\pi}_t - \frac{\left(1-\beta\gamma^{(1-\sigma_c)}\xi_w\right)(1-\xi_w)}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)(1+(\lambda_w-1)\,\epsilon_w)\,\xi_w}\hat{\mu}_t^w + \varepsilon_t^u\end{aligned} \tag{A.13}$$

- Monetary policy reaction function:

$$\hat{r}_t = \rho\hat{r}_{t-1} + (1-\rho)\left(r_\pi\hat{\pi}_t + r_y\left(\hat{y}_t - \hat{y}_t^*\right)\right) + r_{\Delta y}\left(\left(\hat{y}_t - \hat{y}_t^*\right) - \left(\hat{y}_{t-1} - \hat{y}_{t-1}^*\right)\right) + \varepsilon_t^r \tag{A.14}$$

# B. The Role of the Sample Size

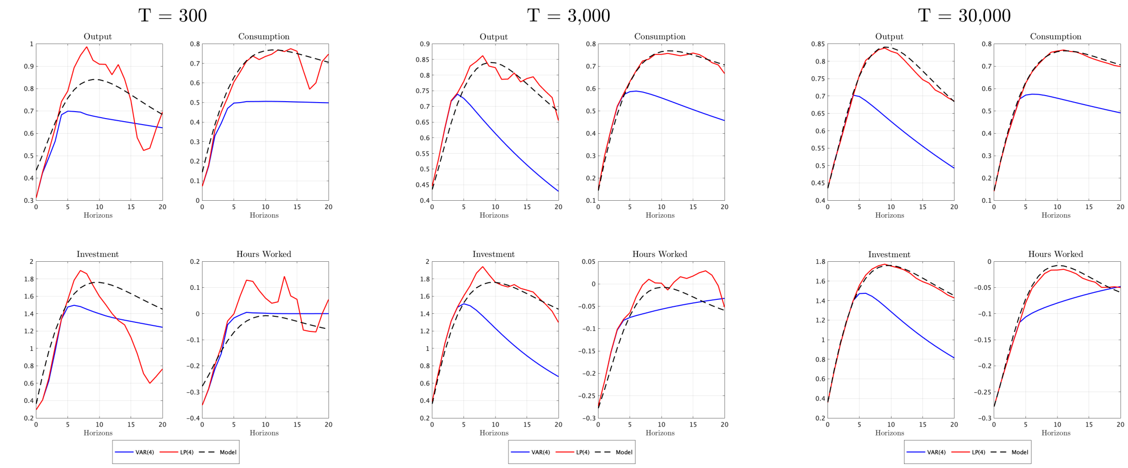
We have already stressed the importance of the sample size. In this section, however, we want to be more specific about what we actually mean. Start from looking at panel (a) of Figure B.0.1, where we plot the estimated LP - IRFs (red line) and SVAR - IRFs (blue line) on a sample with $T = 300$ observations, as well as the true impulse response function coming from the model (black dash line).

First, notice that LP - IRFs and SVAR - IRFs approximately agree until horizon $p = 4$, and after that, their behavior diverges. This is consistent with the results in Plagborg-Møller and Wolf (2021) and Li et al. (2021). The result is explained by the different extrapolation at horizons $h > p$: a VAR with p lags extrapolates its long-run responses from the first $p$ sample autocovariances, and therefore it yields lower variance but a high bias. On the other hand, LPs uses the sample covariances more flexibly by directly projecting at future horizons $h$ on current covariates (see (3) for more clarity). The similar behavior of the two estimators up to horizon $p$, and their disagreement beyond it, is very nicely seen in the response of output. Notice how the LP - IRF follows more closely the shape of the true response (low bias), but with a lot more ups and downs around it (high variance). On the other hand, the SVAR - IRF follows the true response only up to horizon $p$ and then misses consistently the dynamics (high bias, low variance).

Second, as we move towards panels (b) and (c), which depict the same estimated impulse responses but on databases with a much longer time dimension ($T = 3,000$ and $T = 30,000$, respectively), we see how the LP - IRF matches more and more closely the true IRF and the wiggles at further horizons disappear. On the other hand, sample size doesn't seem to matter for the SVAR - IRFs. The response is still similar to that obtained through LPs until horizon $p$ but then it diverges. To get closer to the true IRF with the SVAR approach one most likely needs to increase $p$. Recall that we picked $p$ in a heuristic way, i.e. we haven't rely on the AIC or any other statistical criteria to make that choice.

What are then the possible implications of these two results for estimation? First, one may think that using LP - IRFs as moments may result in estimates of the economic parameters that are closer to the truth but with larger confidence intervals than if one uses SVAR - IRFs as moments. And second, if one simulates its model long enough, the estimates obtained through

Figure B.0.1: The Role of T for the Estimated Impulse Responses to a Technology Shock



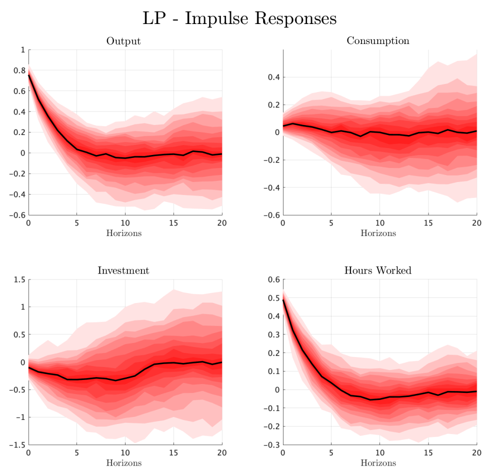

(a) T = 300 (b) T = 3,000 (c) T = 30,000

LP - IRFs will coincide with those obtained by a maximum likelihood or IRFs matching approach since the auxiliary model is well-specified (estimated IRF = true IRF); while for the SVAR - IRFs approach to indirect inference it will require the researcher to fine-tune the optimal lag length and/or the inclusion of more variables in the system to avoid model misspecification.

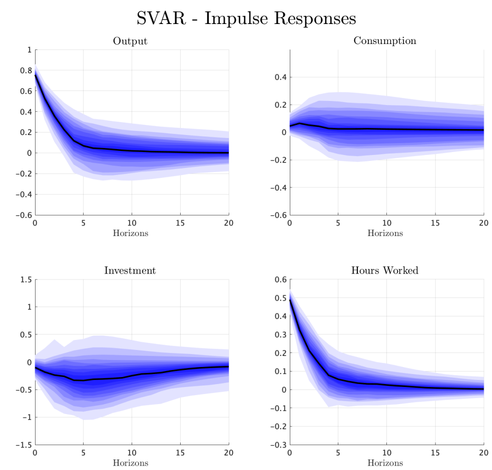
# C. Additional Figures

## C.1. Fan Charts: Fiscal and Monetary Policy Shocks

Figure C.1.1: Fiscal Policy Shock

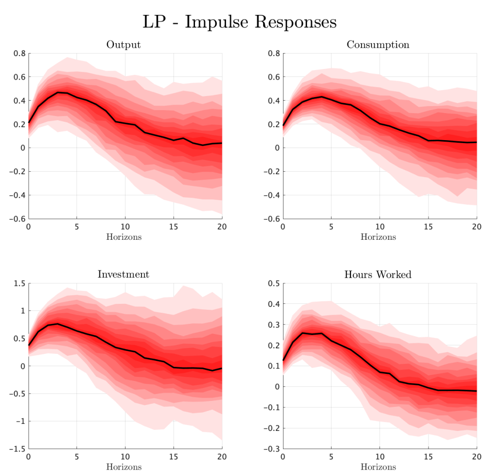



(a) Local Projections

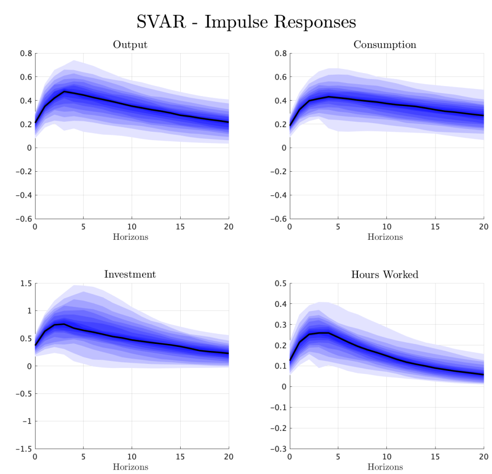



(b) Vector Autoregression

Figure C.1.2: Monetary Policy Shock



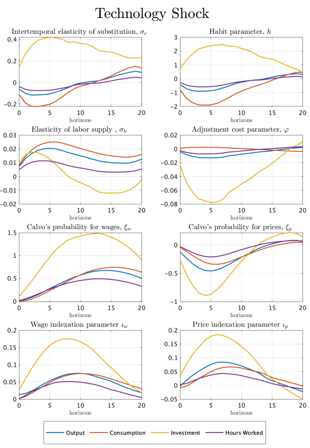

(a) Local Projections



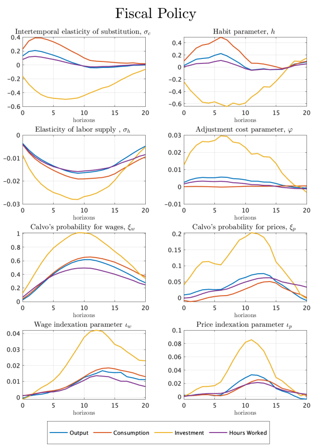

(b) Vector Autoregression

## C.2. Sensitivity of the Moments to Changes in the Parameters

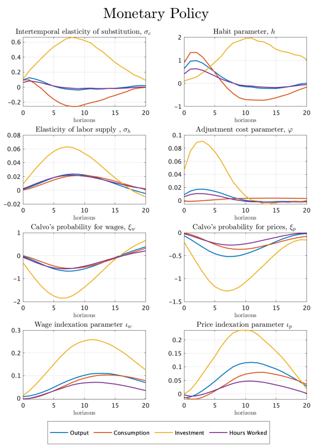
Figure C.2.1: LP-IRFs elasticities




(a) Technology shock

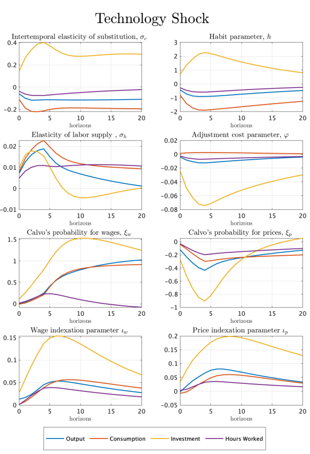



(b) Fiscal Policy

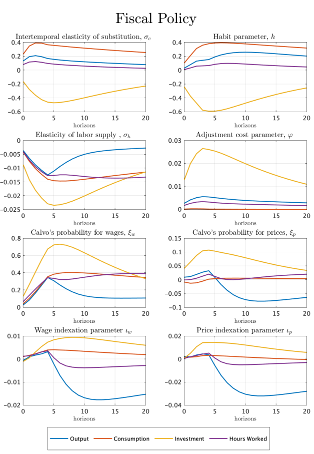



(c) Monetary Policy

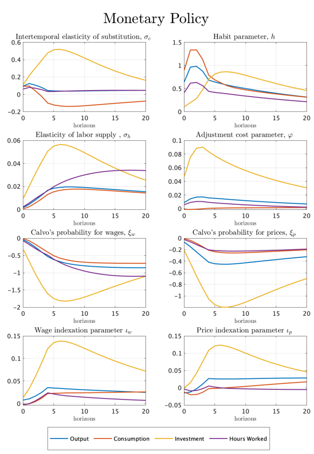
Figure C.2.2: SVAR-IRFs elasticities




(a) Technology shock



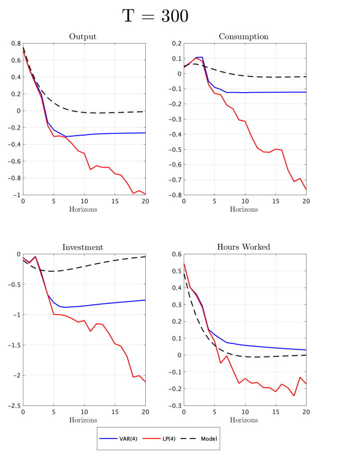

(b) Fiscal Policy



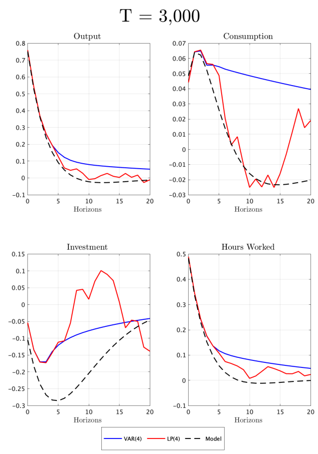

(c) Monetary Policy

## C.3. The Role of Sample Size for Fiscal and Monetary Policy Shocks

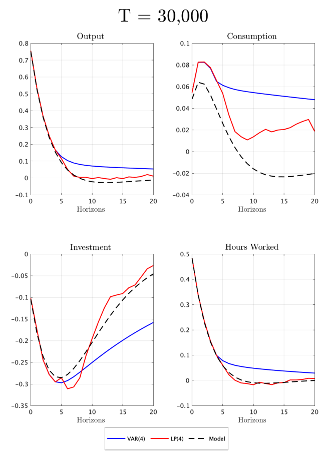
Figure C.3.1: The Role of T for the Estimated Impulse Responses to Fiscal Policy




(a) T = 300

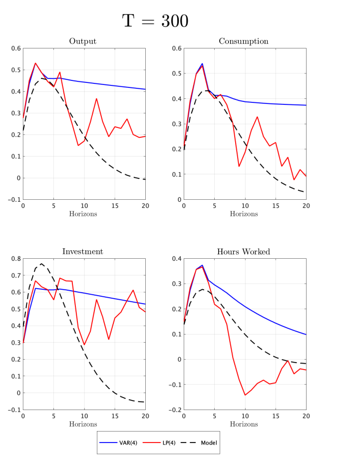



(b) T = 3,000

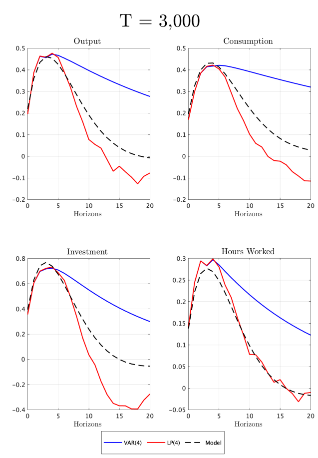



(c) T = 30,000

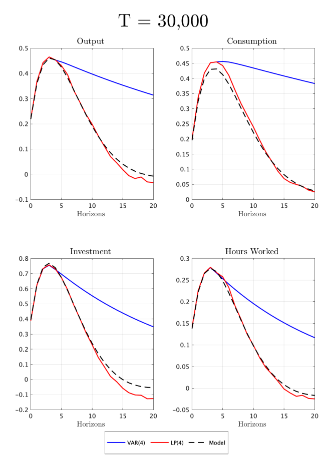
Figure C.3.2: The Role of T for the Estimated Impulse Responses to Monetary Policy




(a) T = 300




(b) T = 3,000




(c) T = 30,000

# D. The Monte Carlo Study: Results in Detail

## D.1. The Local Projection Approach to Indirect Inference

Table 5: SMM estimates using LP - IRFs

| | $\hat{\sigma}_c$ | $\hat{h}$ | $\hat{\sigma}_l$ | $\hat{\varphi}$ | $\hat{\xi}_w$ | $\hat{\xi}_p$ | $\hat{\iota}_w$ | $\hat{\iota}_p$ |
|---|---|---|---|---|---|---|---|---|
| | Technology shock, $\varepsilon_t^a$ | | | | | | | |
| *Mean* | 1.35 | 0.79 | 2.66 | 5.73 | 0.67 | 0.59 | 0.48 | 0.16 |
| *Bias* | 0.08 | -0.02 | 0.14 | -0.58 | -0.10 | 0.06 | -0.05 | -0.02 |
| *Std dev.* | 0.28 | 0.06 | 0.70 | 1.79 | 0.17 | 0.08 | 0.16 | 0.05 |
| *RMSE* | 0.29 | 0.06 | 0.71 | 1.89 | 0.19 | 0.10 | 0.17 | 0.06 |
| | Fiscal Policy, $\varepsilon_t^g$ | | | | | | | |
| *Mean* | 1.40 | 0.80 | 2.74 | 6.41 | 0.63 | 0.43 | 0.50 | 0.15 |
| *Bias* | 0.13 | -0.01 | 0.22 | 0.10 | -0.14 | -0.10 | -0.03 | -0.03 |
| *Std dev.* | 0.14 | 0.13 | 0.68 | 1.84 | 0.12 | 0.12 | 0.17 | 0.05 |
| *RMSE* | 0.19 | 0.13 | 0.71 | 1.84 | 0.18 | 0.16 | 0.18 | 0.06 |
| | Monetary Policy, $\varepsilon_t^m$ | | | | | | | |
| *Mean* | 1.44 | 0.79 | 2.57 | 5.68 | 0.70 | 0.43 | 0.48 | 0.18 |
| *Bias* | 0.17 | -0.02 | 0.05 | -0.63 | -0.06 | -0.10 | -0.06 | -0.00 |
| *Std dev.* | 0.20 | 0.05 | 0.77 | 1.54 | 0.10 | 0.14 | 0.17 | 0.05 |
| *RMSE* | 0.26 | 0.05 | 0.77 | 1.66 | 0.12 | 0.17 | 0.18 | 0.05 |
| | Selected Responses to All Shocks | | | | | | | |
| *Mean* | 1.36 | 0.80 | 2.11 | 5.45 | 0.60 | 0.51 | 0.49 | 0.15 |
| *Bias* | 0.10 | -0.01 | -0.41 | -0.86 | -0.17 | -0.02 | -0.04 | -0.02 |
| *Std dev.* | 0.26 | 0.06 | 0.73 | 1.38 | 0.17 | 0.12 | 0.16 | 0.05 |
| *RMSE* | 0.28 | 0.06 | 0.84 | 1.63 | 0.24 | 0.12 | 0.17 | 0.06 |

## D.2. The Structural Vector Autoregression Approach to Indirect Inference

Table 6: SMM estimates using SVAR - IRFs

| | $\hat{\sigma}_c$ | $\hat{h}$ | $\hat{\sigma}_l$ | $\hat{\varphi}$ | $\hat{\xi}_w$ | $\hat{\xi}_p$ | $\hat{\iota}_w$ | $\hat{\iota}_p$ |
|---|---|---|---|---|---|---|---|---|
| | Technology shock, $\varepsilon_t^a$ | | | | | | | |
| *Mean* | 1.24 | 0.81 | 2.59 | 6.78 | 0.67 | 0.52 | 0.45 | 0.15 |
| *Bias* | -0.03 | 0.01 | 0.07 | 0.47 | -0.10 | -0.01 | -0.09 | -0.03 |
| *Std dev.* | 0.20 | 0.04 | 0.72 | 1.50 | 0.12 | 0.08 | 0.16 | 0.05 |
| *RMSE* | 0.21 | 0.04 | 0.73 | 1.57 | 0.16 | 0.08 | 0.18 | 0.06 |
| | Fiscal Policy, $\varepsilon_t^g$ | | | | | | | |
| *Mean* | 1.35 | 0.78 | 2.50 | 5.90 | 0.67 | 0.48 | 0.56 | 0.17 |
| *Bias* | 0.08 | -0.03 | -0.02 | -0.41 | -0.09 | -0.05 | 0.02 | -0.01 |
| *Std dev.* | 0.16 | 0.15 | 0.73 | 1.92 | 0.12 | 0.14 | 0.15 | 0.05 |
| *RMSE* | 0.18 | 0.16 | 0.73 | 1.96 | 0.15 | 0.15 | 0.16 | 0.05 |
| | Monetary Policy, $\varepsilon_t^m$ | | | | | | | |
| *Mean* | 1.29 | 0.80 | 2.57 | 5.70 | 0.69 | 0.37 | 0.53 | 0.17 |
| *Bias* | 0.02 | -0.01 | 0.05 | -0.62 | -0.08 | -0.16 | -0.00 | -0.00 |
| *Std dev.* | 0.25 | 0.05 | 0.72 | 1.32 | 0.09 | 0.08 | 0.16 | 0.05 |
| *RMSE* | 0.26 | 0.05 | 0.73 | 1.46 | 0.12 | 0.18 | 0.16 | 0.05 |
| | Selected Responses to All Shocks | | | | | | | |
| *Mean* | 1.25 | 0.81 | 2.14 | 5.59 | 0.60 | 0.51 | 0.49 | 0.16 |
| *Bias* | -0.02 | 0.00 | -0.38 | -0.72 | -0.17 | -0.02 | -0.04 | -0.02 |
| *Std dev.* | 0.23 | 0.06 | 0.75 | 1.38 | 0.14 | 0.10 | 0.16 | 0.05 |
| *RMSE* | 0.23 | 0.06 | 0.84 | 1.56 | 0.22 | 0.10 | 0.17 | 0.06 |