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Indirect Inference: a Local Projection approach

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This paper uses Local Projection (LP) coefficients in an indirect inference exercise to estimate the structural parameters of a DSGE model. Monte Carlo analysis examines its small sample performance and compares the estimator to a more traditional approach that relies on VARs as an auxiliary model. Our approach produces consistent and computationally efficient estimates and outperforms the VAR approach in capturing the shape of true IRFs. This methodology is used to re-estimate the Smets and Wouters (2007) model to reconcile it with recent IRF evidence. We show that: (i) our parameter estimates are similar to those obtained under full information, (ii) the small differences in parameterizations are not enough to fully capture the effects of fiscal and monetary policy shocks, and (iii) the model does a good job in capturing the responses to technology shocks.

KEYWORDS. Indirect Inference, DSGE, Local Projection, Vector Autoregression, Monte Carlo, Estimation.

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1. INTRODUCTION

The Local Projection (LP) approach to understanding the dynamic effects of exogenous shocks, originating in Jordà (2005), has become a common tool for economic analyses. Though originally illustrated as an empirical model to study the impact of monetary shocks, this approach has become very widespread and used, as illustrated in Ramey (2016), to study the effects of fiscal and technology shocks as well. This paper, instead, uses the LP approach as an input into an indirect inference exercise to estimate the structural parameters of a large scale macroeconomic model.

In light of the theoretical result in Plagborg-Møller and Wolf (2021), i.e. VARs and LPs estimating the same impulse responses in population, one may think that using either VAR or LP should not matter for an indirect inference exercise. However, the finite sample properties of these two estimators differ. In particular, when p lags of the data are included in the VAR and as controls in the LP, IRFs approximately agree out to horizon p, but at longer horizons $h > p$ there is a bias-variance trade-off (Li et al., 2022).

This paper is complementary to Plagborg-Møller and Wolf (2021) and Li et al. (2022) in that our interest is as well on the performance of the LP approach. However, our focus is on the estimation of the structural parameters governing, for example, tastes and technology, rather than on the impulse response functions *per se*. Consequentially, the paper is organized around evaluating the use of LP as a basis for indirect inference.

There are two contributions in this paper. The first extends the methodology of Smith (1993) to the use of coefficients from LP estimation as moments. Through Monte Carlo we assess and compare the statistical properties of these two approaches. For these experiments, the data generating process is a parameterized version of the Smets and Wouters (2007) model, a leading representation of the aggregate economy. Consequently, we present a brief overview of the model in Section 2. This is followed by a comparison of the LP and VAR approaches to generating IRFs in Section 3. In particular, the model parameters are fixed at the Smets and Wouters (2007) mean estimated values reported in Tables 1A and 1B in their paper. The model is then simulated to create time series. Using these, we estimate bivariate VARs which are used as one auxiliary model, as in Smith (1993). We also use the simulated data to estimate the LP from a variety of shocks: (i) monetary (ii) technology and (iii) fiscal. In both approaches, we use the true values of the innovation to these shocks from the simulation of the Smets and Wouters (2007) model to identify the impulse responses. Finally, to evaluate the properties of these two econometric models as auxiliary ones in an indirect inference approach, in Section 4 we present Monte Carlo evidence. In particular, we focus on the structural parameters that characterize household preferences, capital adjustment costs and the determination of wage and price rigidities.

Monte Carlo results show that the LP approach to indirect inference produces consistent and computationally efficient estimates. Moreover, when compared to the traditional VAR approach, we also find that despite having mixed results based on RMSEs and J-statisitics, **the theoretical responses at estimated parameters are much closer to truth under the LP approach.** Overall, our findings seem to suggest that the LP approach is better at picking those parameters that have a bigger impact for the shape of the theoretical IRF at horizons $h > p$.

One potential concern relies on the observation of an innovation since in practice the constructed shocks are imperfect measures of the true innovation. Consequently, using simulated data, we also explore the impact of measurement error in the monetary innovation since this concern is raised frequently for that application. In particular, we distinguish two types of measurement error depending upon the noise being correlated or not with other shocks. If noise is purely random, the response of real variables to monetary innovations are attenuated and the estimated model does not match the true impulse responses as well. On the other hand, when the observed monetary innovations are correlated with the technology shock, the estimated model matches the impulse responses better because, as we shall see, variation coming from the technology shock is more useful when estimating the parameters of the model.

The second contribution is the re-estimation of the parameters of the Smets and Wouters (2007) model, which illustrates the power of this approach. Given the prominence of this model, it is relevant to evaluate how well it matches the new LP evidence on how the economy responds to aggregate shocks. To that end we first compare the impulse response functions (IRFs) from the Smets and Wouters (2007) at their mean parameter estimates to their empirical counterparts. In particular, we focus on the LP responses to technology and fiscal policy shocks as estimated in Ramey (2016) and to monetary policy shocks as estimated in Tenreyro and Thwaites (2016). Our first finding is that the Smets and Wouters (2007) IRFs do not match those from the data very well.

There are three possible explanations to this finding: (i) the new empirical evidence points towards a different set of structural parameters, (ii) the model misses certain key dimensions that makes impossible reconciling it with the data, and (iii) the empirical IRFs are not well identified. This paper thoroughly explores the first one, while also touches upon the second one by relating the Smets and Wouters (2007) model to other advancements in the literature and by proposing further changes based on the lessons drawn by our estimations.

Building on this, Section 5 presents our estimation results, using LP estimates from these three types of shocks from actual data, rather than from the model. The parameters can be estimated for each of these shocks independently or jointly. Compared to the estimates in Smets and Wouters (2007), our results show that there are some discrepancies, e.g. we obtain a lower intertemporal elasticity of substitution, higher frequencies of wage and price adjustment, and a lower degree of indexation to past wages and prices when we match either the responses to technology or fiscal shocks. These parameter values allow us to slightly improve the fit of the model when assessing the output, consumption, investment and hours worked responses to technology shocks, as well as the responses of output and hours worked to fiscal policy shocks. However, the model generates a negative consumption response and a large crowding out effect on investment when the economy is hit with a fiscal policy shock. These two features are at odds with the data. The poor performance of the model along these dimensions is not related to its parameterization, but rather to its structure. In fact, Galí et al. (2007) shows that one needs to introduce hand to mouth households into this type of models to be able to generate a rise in consumption in response to a government shock. Nevertheless, we cannot rule out that the disagreement between the model and the data is driven by the identifying assumptions used to empirically estimate the responses to a government spending shock.[1]

In regards to the monetary shocks, we find that the Smets and Wouters (2007) model is unable to match Tenreyro and Thwaites (2016) estimated responses of consumption, output and investment when we consider their linear LP model. Nevertheless, the disagreement between data and model doesn't seem to be related to the parameterization in general or our local projection approach in particular, but rather to the structure of the model and the empirical strategy, i.e. the linear LP model. As shown in Tenreyro and Thwaites (2016) paper, these responses are poor representations of reality since monetary policy is state dependent. It goes without saying that the log-linearized Smets and Wouters (2007) model is unable to generate such state-dependent responses to monetary shocks. In any case, we find interesting that the Smets and Wouters (2007) model is able to match the responses of consumption, output and investment to a contractionary shock during an expansion at both their estimates and ours; while it fails to do so during a recession. There is no economic model to date that is able to match the evidence in Tenreyro and Thwaites (2016). Based on our parameter estimates, which differ depending on which set of responses we are matching (boom vs. recession), we believe that an interesting avenue to reconcile the model with the data is through the firm's pricing decision. We left such extension of the model for future work.

We also estimate parameters from the joint response of the model to all three types of shocks, rather than individually. In this case, we focus on the response of the most informative variables to estimate the structural parameters.[2] Estimation results are slightly different to those obtained by matching responses individu-

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[1]As shown in Ramey (2016), if one relies on government spending being pre-determined within a quarter to identify the shock, as in Blanchard and Perotti (2002), then government spending rises consumption; while when one uses a narrative news approach, such as the one developed by Ramey (2011), the effect is often reversed and government spending lowers consumption.

[2]We find that the investment response to shocks is very informative about parameters and use that feature to structure our estimation.

ally, nevertheless, the re-estimated Smets and Wouters (2007) model still misses the same dimensions of the data.

Finally, Section 6 concludes and summarizes our insights of what we believe are two promising lines of research. First, the development of an economic model that is able to capture the state dependent responses of monetary policy. And second, the application of our local projection approach to estimating such state-dependent models.

2. Economic Model

The analysis builds on the model formulated and estimated in Smets and Wouters (2007). While other models may serve the same purpose, this structure captures many of the central channels of monetary and fiscal policy. For the first part of our analysis, we treat their estimated parameters as truth and see how close we come to them through the indirect inference approach. For the second part of the analysis, our estimated parameters are compared to those reported by Smets and Wouters (2007).

The Smets and Wouters (2007) model has become one, if not, the workhorse model in the DSGE literature. The model is based on Christiano et al. (2005) who added various frictions to a basic New Keynesian DSGE in order to capture the dynamic response to a monetary policy shock as measured by a structural vector autoregression (SVAR). In fact, price and wage stickiness paired with adjustment costs for investment, capacity utilization costs, habit formation in consumption, partial indexation of prices and wages as well as autocorrelated disturbance terms are able to generate a rich autocorrelation structure, which is key for capturing the joint dynamics of output, consumption, investment, hours worked, wages, inflation and the interest rate. These features of the model are crucial for our study since we are interested in matching the dynamic response of key macro aggregates to various shocks, as described in Section 3.

In what follows, we summarize the model components with an emphasis on key parameters. The summary builds upon Smets and Wouters (2003).

2.1 *Households*

Households are infinitely lived, working and consuming in each period of life. Their lifetime utility is given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, \bar{c}_{t-1}, n_t) \tag{1}$$

where c_t is the consumption of the representative agent, $\bar{c}_{t-1}$ represents an external habit and n_t is the amount of labor supplied. There are shocks associated with household impatience and the marginal rate of substitution between consumption and work. Household income comes from working, renting capital and dividends from the firms. Households save by holding bonds and also have access to state contingent securities which allow the household to smooth over taste shocks. Households also own the capital stock. This is rented to firms. Households incur an adjustment cost for changes in the capital stock. There is also a shock to this adjustment cost.

In the model, there are 4 parameters associated with the household problem: (i) the intertemporal elasticity of substitution σ_c, (ii) the strength of the habit h, (iii) the elasticity of labor supply σ_l, and (iv) the capital adjustment cost φ.

Further, each household acts as a wage setter, with a differentiated source of labor supply. There are associated parameters governing the likelihood of wage adjustment, $1 - \xi_w$ and the indexation of non-adjustment wages to past inflation, ι_w. These parameters are estimated as well.

2.2 *Firms*

The firm side of the economy entails a perfectly competitive final goods sector. Output is produced using differentiated intermediate goods. In the intermediate goods markets sellers have market power modelled as a monopolistic competition. The production of the intermediate goods requires capital and labor.

There are sticky prices in the market for intermediate goods. Price setting is not state dependent but rather firms are randomly granted an opportunity to adjust their prices, as in Calvo (1983). For those firms not adjusting their prices, there is partial indexation to past inflation.

There is an aggregate productivity shock associated with intermediate goods production. The TFP shock is assumed to follow an AR(1) process.

From this specification there are a couple of key parameters. One is the probability that a firm can adjust its price, denoted $1-\xi_p$ and the other one is the partial indexation parameter, denoted ι_p. These parameters are surely important for the effects of monetary policy. But, as we shall see, they also matter for the effects of other shocks and can be identified from impulse responses associated with non-monetary innovations.

2.3 *Government: Fiscal and Monetary Authorities*

Government spending is another source of stochastic variation in the model. Spending is specified as an AR(1) process with an iid normal error term and is also affected by the productivity shock.[3]

Monetary policy is modeled through a generalized Taylor rule that gradually adjusts the policy interest rate in response to changes in inflation and output gap. Innovations to this rule play a key role in the analysis.

3. Indirect Inference: LP vs. VAR impulse responses

This section fixes some basic ideas in order to clarify our approach and results. This includes the LP and VAR frameworks as well as the indirect inference approach.

Start by consider any economic model, M. Assume that the endogenous variables of this model, y_t, depend on its own lags y_{t-1} (endogenous states), some exogenous variables z_t (exogenous states) and some random errors u_t (shocks). Further assume that the model is parameterized by an ex-ante unknown vector of parameters Θ. That is, let

$$y_t = M(y_{t-1}, z_t, u_t; \Theta) \tag{2}$$

[3]There is no apparent mention of taxation in either Smets and Wouters (2003) or Smets and Wouters (2007). Taxes appear to be lump sum. Distortionary taxes can change the impact of fiscal policy shocks.

for $t = 1, 2, \dots, T$. Given an initial value for the endogenous variable y_{-1} and a sequence for the shocks $\{u_t\}_{t=1}^{T}$, it is possible to generate infinite data sequences $\{y_t\}_{t=1}^{T}$. This is a generic way to represent the Smets and Wouters (2007) model that provides the “sandbox” for our experiments.

3.1 *LP*

To understand the **LP** approach, consider the following regression:

$$\tilde{y}_{t+h} = \mu_h + \beta_h \tilde{x}_t + \sum_{\ell=1}^{p} \delta'_{h,\ell} w_{t-\ell} + \xi_{h,t}. \tag{3}$$

where $\tilde{y}_t$ is one of the variables of interest, $\tilde{x}_t$ denotes an innovation associated with a particular form of an aggregate shock. Finally, there are p lags of a vector of controls $w_t = \{\tilde{x}_t, \tilde{y}_t\}$.

The parameters in (3) are estimated at each horizon $h = 0, 1, 2, \dots, H$. This is simply an OLS regression of leads of $\tilde{y}_t$ on past innovations. For each horizon, $(\mu_h, \beta_h, \{\delta'_{h,\ell}\}_{\ell=1}^{p})$ are the projection coefficients.

DEFINITION 1. *The LP-IRFs of $\tilde{y}_t$ with respect to $\tilde{x}_t$ are given by $\{\beta_h\}_{h\geq 0}$ in* (3). Note that there are $H + 1$ coefficients generated for each of the variables of interest, $\tilde{y}_t$ for each type of innovation, $\tilde{x}_t$.

In our study, we focus on $\tilde{y}_t \in \{y_t, c_t, i_t, n_t\}$, being output, consumption, investment and hours worked respectively. Further $\tilde{x}_t \in \{\varepsilon_t^a, \varepsilon_t^g, \varepsilon_t^m\}$, so that we consider shocks to technology, government spending and monetary policy. Note that since we use the true innovations of the shock, the choice of p is irrelevant. Nevertheless, we use the same number of lags as in the VAR to be consistent with Plagborg-Møller and Wolf (2021).

3.2 *VAR*

The starting point for the multivariate linear **VAR(p) projection** is:

$$w_t = c + \sum_{\ell=1}^{p} A_\ell w_{t-\ell} + u_t \tag{4}$$

where u_t is the projection residual and $(c, \{A_\ell\}_{\ell=1}^{p})$ are the projection coefficients. Here p indicates the longest lag, matching the lag in the LP controls. Notice that given the definition of w_t, we are considering bivariate VAR(p) projections with the innovation ordered first Plagborg-Møller and Wolf (2021).

Let $\Sigma_u \equiv \mathbb{E}[u_t u_t']$ and define a *Cholesky decomposition* $\Sigma_u = BB'$ where B is lower triangular with positive diagonal entries. With this, consider the corresponding recursive *SVAR representation*:

$$A(L)w_t = c + B\eta_t \tag{5}$$

where $A(L) \equiv I - \sum_{\ell=1}^{p} A_\ell L^\ell$ and $\eta_t \equiv B^{-1}u_t$. Define the lag polynomial $\sum_{\ell=0}^{p} C_\ell L^\ell = C(L) \equiv A(L)^{-1}$.

DEFINITION 2. The SVAR-IRFs of $\tilde{y}_t$ with respect to an innovation in $\tilde{x}_t$ is given by $\{\theta_h\}_{h\geq 0}$ with $\theta_h \equiv C_{2,\bullet,h}B_{\bullet,1}$ where $\{C_\ell\}$ and B are defined in (5).

3.3 *Indirect Inference*

Smith (1993) refers to the indirect inference approach as an extended method of simulated moments (EMSM). In fact, indirect inference is very similar to the simulated method of moments (SMM) approach since it also constructs an estimate of the parameters by minimizing the distance between data and simulated moments. The subtle difference between the two is that SMM uses unconditional moments, while in an indirect inference exercise these come from an auxiliary econometric model. Thus, the indirect inference estimator of a $q \times 1$ vector of structural parameters Θ, $\hat{\Theta}$ solves

$$J = \min_{\Theta} \left(\beta - \beta(\Theta)\right)' W \left(\beta - \beta(\Theta)\right) \tag{6}$$

where β is a $m \times 1$ vector containing the estimates of the econometric model from the actual data, and $\beta(\Theta)$ is its synthetic counterpart from the artificial data generated by the economic model. In this quadratic form, W is a weighting matrix. The minimized value of the objective is the J-statistic.

In our application, Θ are the parameters characterizing household preferences, wage setting and price setting of firms in the Smets and Wouters (2007) model;

while β are the LP coefficients associated with the impulse responses, i.e. β_h from Definition 1. In our Monte Carlo study, we also consider the case in which β are the coefficients from a VAR (or to be more precise their associated impulse responses, denoted θ_h in Definition 2).[4]

4. A Monte Carlo Study

This section studies the small sample properties of the VAR and LP approaches to indirect inference under the hypothesis that the DGP and the estimated model are the same. Unlike other studies, we do not study such properties under the alternative hypothesis that the model is misspecified because moment-based methods tend to be robust to misspecification, as shown in Ruge-Murcia (2007).

4.1 *The Data Generating Process*

We solve the model in its log-linearized version, and then simulate it to generate an artificial database consisting of time series paths of four key macro aggregates: output, consumption, investment and hours worked $\{y_t, c_t, i_t, n_t\}$, as well as time paths for the innovations to technology, fiscal and monetary policy shocks $\{\varepsilon_t^a, \varepsilon_t^g, \varepsilon_t^g\}$.[5]

The Monte Carlo experiments are based on 100 replications using a sample size of 300 observations. Since LPs can be biased in smaller samples, as shown by Herbst and Johannsen (2021), we also repeat these experiments for a smaller sample of 100 observations. These results are shown in Appendix B.3.

We focus on the 8 structural parameters discussed above. The "true" values of these structural parameters are listed in Table 1, while the remaining ones are set and fixed at the estimated values from Smets and Wouters (2007). This reduces the computational burden in the Monte Carlo.

[4]We only use the coefficients associated to the impulse responses to guarantee the same number of moments across both econometric models.

[5]The log-linearized equilibrium conditions of the Smets and Wouters (2007) model are reproduced in Appendix A.

TABLE 1. True values of structural parameters

σ_c	h	σ_l	φ	ξ_w	ξ_p	ι_w	ι_p
1.26	0.80	2.52	6.31	0.70	0.66	0.58	0.24

Note: σ_c denotes the intertemporal elasticity of substitution, h, the habit parameter, σ_l the elasticity of labor supply, φ the investment adjustment cost parameter, ξ_w and ξ_p the probabilities of non adjustment for wage and prices, and ι_w and ι_p correspond to the degree of wage and price indexation to past inflation.

We consider eight scenarios which only differ in the set of coefficients we try to match. Six of these eight scenarios correspond to optimization routines that target either the LP or SVAR responses of output, consumption, investment and hours worked to technology, fiscal or monetary shocks. In the other two scenarios, we exploit the differences in responsiveness of these coefficients to changes in the structural parameters. Thus, based on the sensitivity analysis reported in Appendix B.6.2, we target the LP or SVAR responses of investment to the three aggregate shocks as well as that of consumption to a technology shock. We label this scenario "selected responses". In sum, in each scenario we target a total of 84 $(= 21 \times 4)$ moments to estimate the 8 parameters in Table 1.

The importance of each of these coefficients is weighted according to the inverse of the variance-covariance matrix of the moments, as suggested in Smith (1993). We compute it by estimating the IRF coefficients on simulated data at the true parameter vector and for 250 different draws of the innovation to the shocks.

Finally, we inflate the simulated sample size by a factor of 10 when computing $\beta(\Theta)$. In theory, we know that the asymptotic distribution of the estimates depends on this choice as simulation uncertainty decreases as the length of the simulated series to the sample size increases. However, in practice, having very long simulated series increases the computational cost and is not needed to obtain accurate estimates. Ruge-Murcia (2012) shows how this choice affects the parameter estimates in the context of DSGE models estimated by SMM.

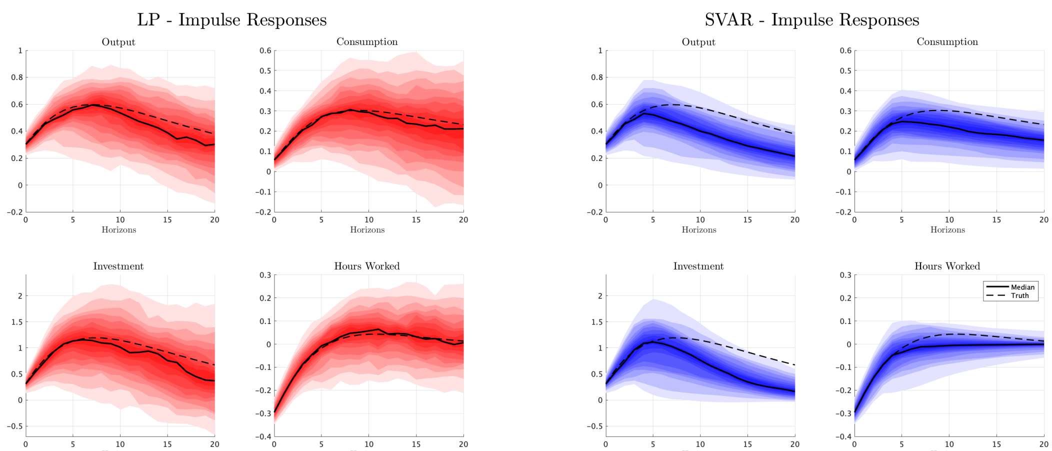
4.2 *The Moment Generating Functions*

We consider two auxiliary econometric models to summarize the key features of the data: the LP and the VAR. However, as anticipated in the previous section, we only use those coefficients that identify the IRFs, i.e. the β_h's and θ_h's from definitions 1 and 2 respectively. The motivation relies in the excessive parameterization of the LP, which although useful, results in an absurd number of moments. Moreover, matching impulse responses is more intuitive as they clearly summarize the dynamics of the model and the data.

Canova and Sala (2009) argue that identification problems may arise because impulse responses at long horizons are noisy and contain little information about the parameters. Their results are based on an estimation strategy that uses the theoretical, rather than the estimated, impulse responses. As we shall see, it is precisely the different behavior of LP- and SVAR-IRFs at long horizons that is going to drive the results in our Monte Carlo study. In any case, we set the horizon to 20 quarters and verify that local identification is achievable at the true vector of parameters. That is, we verify that the Jacobian matrix of the moments is full rank, as suggested in Ruge-Murcia (2012).

Figure 1 depicts the distribution of the LP-IRFs (left panel) and SVAR-IRFs (right panel) to a technology shock over the 100 draws of the DGP. The black solid line corresponds to the median response of output, consumption, invest-

FIGURE 1. Technology Shock



Note: the left panel depicts the distribution of the LP estimated responses to a technology shock, while the the right panel depicts the same estimated responses but using a SVAR. The solid black lines are the median of the distribution and the dashed black lines correspond to the structural IRFs.

ment and hours to the shock; while the black dashed line depicts the true/model generated IRFs.[6]

As demonstrated in Plagborg-Møller and Wolf (2021), the LP and VAR responses approximately agree up to horizon $h = 4$, which is the number of lags used in the VAR and the LP regression, $p = 4$. It is also clear from the figure that the variability of the estimated LP-IRFs is much larger than that of the SVAR-IRFs, specially at horizons $h > p$. Yet, from these figures it is also evident that the bias is much smaller for the LP-IRFs compared to the SVAR-IRFs. That is, the LP median response to the various shocks is closer to the true/model generated IRFs compared to the SVAR median response. In short, there is a bias-variance tradeoff at horizons $h > p$ in the IRFs estimated form the Smets and Wouters (2007) model.[7] These three results are also present if one looks at the responses to fiscal and monetary shocks.[8]

One may expect that the increased variability in the LP-IRFs is also going to lead to more variation in the estimated economic parameters. But, at the same time, it can be useful if the response coefficients also change by more when one of the parameters changes, which in turn will help to identify the true parameter vector.

4.3 *Results*

This section reports the results of our Monte Carlo experiments. Tables B.1 and B.2 in Appendixes B.1 and B.2 report bias, standard deviation and Root Mean Squared Error (RMSE) for each of the 8 parameters. These are computed as follows:

$$\text{Bias}_i \equiv \mathbb{E}\left[\hat{\Theta}_i\right] - \Theta_i^* \tag{7}$$

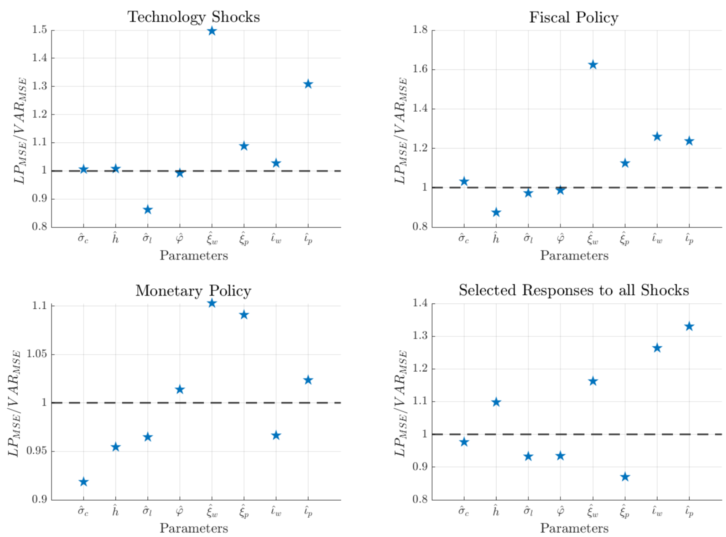
$$\text{Std dev}_i \equiv \sqrt{\text{Var}(\hat{\Theta}_i)} \tag{8}$$

[6]The true IRF are defined as the population LP-IRFs and SVAR-IRFs, i.e. those generated using infinite lag length. Note that these can be obtained directly from the model without simulation.

[7]This bias-variance trade off has been documented in Li et al. (2022) for other, more simple, DGP.

[8]The counterpart of Figure 1 for each of the two other shocks can be seen in Appendix B.6.1.

FIGURE 2. Relative performance across moment generating functions



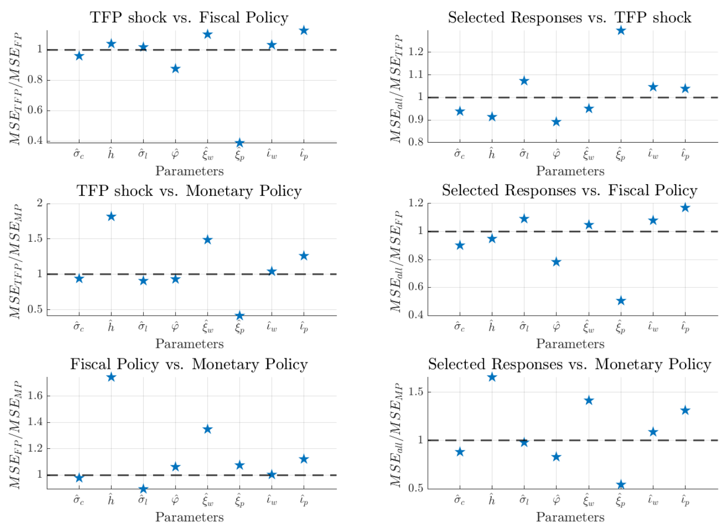
Note: this figure compares the RMSE measures for the SVAR and LP based parameter estimates by shock. A value smaller than 1 indicates that the RMSE coming from the estimation using SVAR-IRFs is larger, and consequently, that specific parameters is better identified through the LP approach.

$$RMSE_i \equiv \sqrt{\text{Bias}_i^2 + \text{Var}(\hat{\Theta}_i)} \tag{9}$$

where expectations are taken over the 100 Monte Carlo draws and i indicates a specific parameter in the vector of estimated parameters. The true value of the parameter is denoted Θ_i^* and its estimated value from a single Monte Carlo draw is given by $\hat{\Theta}_i$.

We summarize this information through Figure 2 which analyses how the two moment generating functions, LPs and SVARs, pick up the different parameters when matching the same responses to the various shocks. In each of the four subplots a value of the ratio greater than one indicates that a specific parameter is better identified using the SVAR as auxiliary model since its $RMSE$ is smaller. This is typically true for the Calvo adjustment and indexation parameters; while for the curvature, habit, elasticity of labor supply and capital adjustment parameters the relationship is reversed and identification is better via the LP approach, i.e. the $RMSE$ ratio is smaller than one.

FIGURE 3. Relative performance across shocks



Note: this figure shows the RMSE measures for the LP based parameter estimates comparing one source of variation to another. For example a value smaller than 1 in the first subfigure indicates that a parameter is better identified using variation coming from TFP shocks rather than from variation in government spending (fiscal policy shock).

The nature of the shock is itself relevant for assessing the estimation results. To see this, Figure 3 compares the RMSE for each estimated parameter depending on the type of shock used to estimate the IRF coefficients. Here we restrict our attention to the LP approach. A few interesting patterns arise. On the one hand, there are some parameters that are better pinned down by a single source of variation. For example, the habit formation parameter h or the Calvo wage adjustment probability $(1-\xi_w)$ are much better identified by monetary shocks alone (see subplot in the second column, third row). On the other hand, other parameters, such as the curvature in utility σ_c or the capital adjustment parameter φ are better pinned down when targeting the responses to all shocks. Finally, we also find surprising that the Calvo adjustment probability of prices $(1-\xi_p)$ is better identified by a technology shock given its importance for inflation dynamics and therefore monetary policy. We do not interpret this result as problem but instead as an opportunity. Why? Because one can exploit the variability coming from a non-monetary shock to estimate some of the key parameters for monetary policy, such as $1-\xi_p$, and then evaluate the validity of the model "out of sample" with respect to an untargeted response to a monetary shock.

These parameter-by-parameter comparisons are a good way to assess the relative performance of these two moment generating functions for each of the estimated parameters in a case-by-case basis. However, one would also like to know how these two distinct approaches to indirect inference perform when all the parameters involved in the estimation are considered as a whole. Thus, we evaluate the value of the criterion function, equation (6), at the optimal/estimated parameter values. This metric, usually referred to as the J-statistic, gives an overall measure of how well we match our targeted moments.

Table 2 reports the average and the maximum value of this statistic across the 100 Monte Carlo draws as well as the time needed to complete the estimation for each of the eight different scenarios. When comparing across rows, we observe that the "selected responses" scenario, in which we match the most responsive IRF coefficients, delivers the lowest average J statistic. A result that is probably expected. If one instead compares across moment generating functions, one finds that the LP approach does worse on average, but for very bad draws of the shocks it performs much better, i.e. lower maximum J. Moreover, the computational time needed to reach a minimum is significantly lower.[9]

Finally, we consider a third measure to evaluate the performance of the LP and SVAR-IRFs approaches to indirect inference since neither the RMSE or the J-statistic inform us about how close we are from the true/model-implied impulse response functions, the ultimate object of interest. Therefore, we look at the weighted distance between the theoretical IRFs coming from the model at the estimated parameter values $\hat{\Theta}$ and at the true values Θ^*. Table 3 summarizes our findings.

The results are striking. According to this metric, the LP-IRFs approach is able to match the theoretical IRFs much better than its SVAR counterpart. That is the

[9]Notice that the lower computing comes from the lower number of iterations needed to reach the minimum since per iteration computing the LP-IRFs takes longer as one has to estimate more coefficients.

TABLE 2. Overall performance: Estimated Impulse Responses

	Local Projections			Vector Autoregression		
	Avg. J	Max. J	Time (in min.)	Avg. J	Max. J	Time (in min.)
Technology Shock	87.23	117.00	28.62	82.80	247.94	67.98
Fiscal Policy	87.72	129.96	24.68	86.28	251.90	52.49
Monetary Policy	88.58	121.48	23.38	82.77	221.87	53.04
Selected Responses	86.56	128.65	20.03	82.63	240.07	72.42

Note: J is the value of the J-statistic defined as the minimized value of (6). The econometric parameters $(\beta, \beta(\Theta))$ used as moments for the indirect inference exercise come from the LP or SVAR response of output, consumption, investment and hours worked to each of the three shocks. For the "selected responses" scenario we use the LP or SVAR response of investment to the three shocks and that of consumption to the technology shock.

case because the LP approach does a significantly better job in picking those parameters that are relevant for capturing the shape of the true impulse responses. In light of this evidence, we argue that using the LP-IRFs approach to indirect inference is the better alternative despite the mixed findings regarding the J-statistic and the RMSE.

An insight into the lower J^* in the LP approach comes from the choice of p, the lag length. From Appendix Table B.6, we know that as p gets large, the J^* from the SVAR approach gets closer and closer to the J^* from the LP approach which in turn is unaffected by p. Longer lags help the SVAR approach to reduce the misspecification in the IRFs and make these moments more similar to those estimated by LP which are essentially independent of p since shocks are purely exogenous.

4.4 *Measurement Error*

Until now we have assumed that the econometrician observes the true shock. This is a useful but at the same time a reasonable concern, which directly relates to the choice of p, the lag length on controls. Note that a branch of the literature, specially in the context of monetary shocks, tries to construct proxy variables that resemble the shock, and then use lagged controls in an LP regression to wipe out serial correlation in the measured shock. But if the innovation is accurately measured, as we have assumed so far, then the LP regression coefficients on the inno-

TABLE 3. Overall performance: Model Impulse Responses

	Local Projections		Vector Autoregression	
	Avg. J^*	Max. J^*	Avg. J^*	Max. J^*
Technology Shock	2.57	9.43	34.67	228.41
Fiscal Policy	3.05	13.88	58.12	692.14
Monetary Policy	2.71	16.89	178.17	853.72
Selected Responses	8.37	44.69	230.46	1130.58

Note: J^* measures the fit of the model relative to the theoretical IRFs produced by the model at the true parameter vector, Θ^*. In particular, J^* is computed as follows: 1) compute the true/model IRFs using the true parameter vector, Θ^*; 2) use the estimated parameters under the LP and SVAR approaches, $\hat{\Theta}$, to recover the true/model IRFs; 3) compute the weighted distance between the two.

vation are independent of p. This is not true for the SVAR and variations in p lead to different IRFs. As a result, indirect inference results are robust to the choice of p in our LP approach while they are not for the SVAR, as shown in Appendix B.4. Anyhow, what happens if the constructed shocks are contaminated?

We focus on the monetary shock and assume that the econometrician does not observe the true shock, i.e. $\varepsilon_t^{m,obs} \neq \varepsilon_t^m$, and in particular, we distinguish two scenarios: (i) the observed shock contains some error that is uncorrelated with other shocks that hit the economy (classical measurement error case), and (ii) the observed shock is correlated with some other shocks in the economy, e.g. the technology shock. Note that the presence of this noise will bias the impulse responses used as data moments, however, our object of interest are the structural parameters of the economic model.

4.4.1 *Classical measurement error* Formally, here we work under the assumption that the observed monetary innovation in the data generating process is given by: $\varepsilon_t^{m,obs} = \varepsilon_t^m + \sigma_\nu \nu_t$ where $\nu_t \sim \mathcal{N}(0,1)$. That is, the econometrician estimates the impulse responses to monetary shocks using $\varepsilon_t^{m,obs}$, while the macroeconomist uses ε_t^m when constructing the simulated moments for the indirect inference exercise. Here we target the responses of output, consumption, investment and hours worked to the noisy monetary policy shocks.

Appendix Table B.7 reports the bias, standard deviation and the root mean squared error for the estimation of the model with classical measurement error

TABLE 4. Classical Measurement Error in the Monetary Shock

	Avg. J	Max. J	Avg. J^*	Max. J^*
Monetary Policy	88.58	121.48	2.71	16.89
Measurement Error ($\sigma_\nu = 0.25$)	87.23	118.58	3.61	9.07
Measurement Error ($\sigma_\nu = 0.5$)	80.56	113.54	4.98	29.46

Note: this table reports the previously defined J and J^* measures for the Monte Carlo that includes measurement error in the data moments. Measurement error enters additively in the monetary innovation and it is uncorrelated with other shocks.

in the monetary innovation. Here we see that the point estimates are not that far from the model without measurement error, and for all parameters the differences are statistically insignificant. Given the muted responses of real variables to the (noisy) monetary innovation, the estimated model has more price and wage flexibility than the baseline, though the difference is not statistically significant. The capital adjustment cost is also biased upwards, presumably to limit the response of investment to the monetary shock.

Table 4 summarizes the measures of model fit for the LP approach in this experiment. The average value of the J-statistic is lower, as is the maximum, the larger is the variance of the noise. This can be interpreted as a consequence of the measurement error biasing all responses towards zero and thus creating the illusion of a better fit. However, the fit is worse relative to the true model IRFs, reflecting the actual effects of the measurement error.

4.4.2 *Correlated measurement error* In this section, we work under the assumption that the observed monetary innovation is correlated with the technology shock. That is, $\varepsilon_t^{m,obs} = \varepsilon_t^m + \rho_{m,a}\varepsilon_t^a$ where $\rho_{m,a}$ is the correlation coefficient. As before, the econometrician computes the impulse responses to $\varepsilon_t^{m,obs}$, while the macroeconomist still uses ε_t^m when generating the simulated moments. Unlike the previous case, here the macroeconomist targets the responses of output and consumption to *both* technology and monetary shocks.

Appendix Table B.8 reports the bias, standard deviation and the root mean squared error for the estimation of the model with correlated measurement error in the monetary innovation. Similarly to the classical measurement error case,

TABLE 5. Overall Performance – Correlated Measurement Error in the Monetary Shock

	Avg. J	Max. J	Avg. J^*	Max. J^*
Technology & Monetary Policy	91.15	124.33	5.01	89.67
Measurement Error ($\rho_{a,m} = 0.25$)	90.62	124.81	4.74	90.44
Measurement Error ($\rho_{a,m} = 0.5$)	90.74	127.86	4.25	21.00

Note: this table reports the previously defined J and J^* measures for the Monte Carlo that includes measurement error in the data moments. Measurement error enters additively in the monetary innovation and it is correlated with the technology shocks. We only target output and consumption responses in this exercise

we see that these estimates are within the range of those coming from the true innovation with the exception of the capital adjustment costs and the price flexibility which are upward biased. In any case, note that targeting the responses to the true technology innovation is key since as we seen in Table 3 we are able to exploit the variation coming from these responses alone to identify the parameters.

Table 5 reports the two measures of overall fit, the value of the loss function (J) and the distance to the true IRFs (J^*), for various values of the correlation coefficient, $\rho_{m,a} \in \{0, 0.25, 0.5\}$. The average and the maximum J is similar across scenarios, while the J^* is smaller the higher is the correlation between the monetary and the technology shock. Since parameters are better identified using variation coming from responses to technology shocks (see Table 3), it is not a surprise that the fit increases as we rely more on the technology shock to draw inferences on the structural parameters.

There is a final point to make. Based upon the results from the Monte Carlo experiments, there is another way to deal with the concern over measurement error in, say, the monetary innovation. If there is another innovation, say the technology shock that is measured without error, then parameters can be estimated from LP coefficients based upon those innovations rather than the monetary innovations. Using these parameter estimates, the response to a monetary innovation can be simulated through the model.

5. An Empirical Application: Re-Estimating the Model

This last section re-estimates the Smets and Wouters (2007) model using our local projection approach to indirect inference. We target the empirical LP impulse responses to technology and fiscal shocks from Ramey (2016), and to monetary policy shocks from Tenreyro and Thwaites (2016). We estimate the same 8 parameters from the Monte Carlo analysis by exploiting the variation from each shock individually since our Monte Carlo results revealed that some of these parameters were better identified this way. Nevertheless, it wasn't true for all them. Therefore, we also re-estimate the model using the response coefficients from all three shocks jointly. For these estimation exercises, we weight the importance of each coefficient via the inverse of a diagonal matrix comprised of the standard errors of the LP coefficients. Finally, we compute the standard errors of the estimated parameters through bootstrapping since Ruge-Murcia (2012) showed that there are discrepancies between the asymptotic and finite sample distributions of the estimates obtained via SMM.

5.1 *Technology Shocks*

Technology shocks are the most important type of non-policy shocks. In fact, there is a vast literature on identification of these shocks on time series models. From all these, we use Francis et al. (2014) approach and identify an unanticipated TFP shock through medium-run restrictions.[10]

The identified shock is then used as dependent variable in a local projection regression to estimate the impulse response of the variable of interest. Following Ramey (2016), we estimate the following series of regressions:

$$\tilde{z}_{t+h} = \alpha_h + \beta_h \cdot \text{shock}_t + \varphi_h(L) \cdot \text{control}_{t-1} + \text{ quadratic trend } + \varepsilon_{t+h} \qquad (10)$$

where $\tilde{z}_{t+h}$ are the variables of interest (real GDP, consumption, non-residential investment, and hours worked), $shock_t$ is the innovation to the growth rate of TFP, $control_{t-1}$ includes two lags each of the shock, real GDP, stock prices, labor pro-

[10] A review of the literature on TFP shock identification can be found in Section 5 of Ramey (2016) handbook chapter.

TABLE 6. Estimates using LP coefficients

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
S&W 07	1.26	0.80	2.52	6.31	0.70	0.66	0.58	0.24
	Technology Shocks							
Median	0.85	0.69	3.28	8.20	0.44	0.59	0.47	0.14
10th pctl.	0.76	0.48	1.51	3.79	0.42	0.40	0.35	0.14
90th pctl.	1.36	0.89	3.28	8.20	0.84	0.86	0.75	0.31
	Fiscal Policy							
Median	1.01	0.85	1.51	3.90	0.42	0.40	0.36	0.14
10th pctl.	0.81	0.48	1.51	3.79	0.42	0.40	0.35	0.14
90th pctl.	1.57	0.96	3.00	7.89	0.81	0.82	0.72	0.30

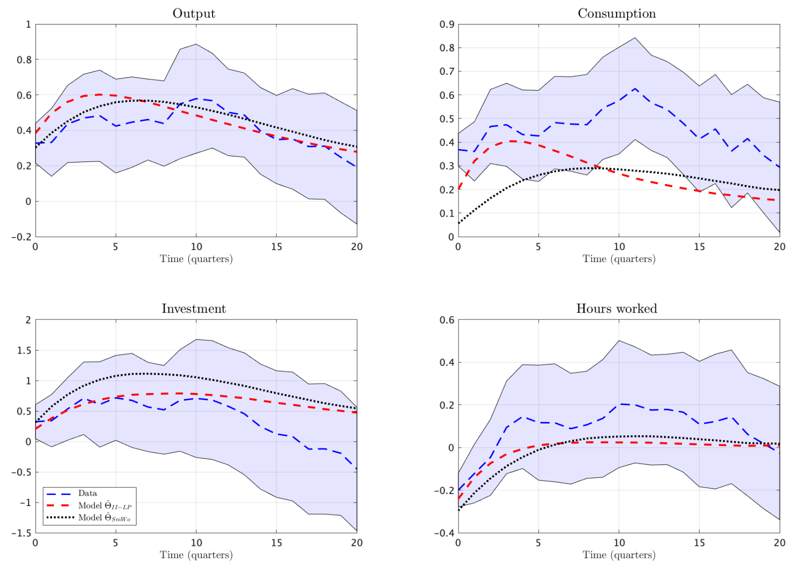
Note: this table reports parameter estimates using the estimated LP coefficients for technology or fiscal policy shocks as targeted moments.

ductivity and the dependent variable, and ε_{t+h} is the error term. As in equation (3), β_h gives the response of $\tilde{z}$ at time $t+h$ to a shock at time t. However, the regressions in (3), which we use in the simulated data, differ from the regressions in (10) in that (i) we do no control for the trend in the regression on model simulated data because variables are already in deviations from steady state, and (ii) we only control for lags of the dependent variable and the shock. Since we use the actual innovation of the shock in the simulated data, we think that these differences won't affect our results.[11]

Thus, we target the β_h coefficients associated with the responses of real GDP per capita, consumption, non-residential investment and hours in our indirect inference estimation. Results are reported in the second subtable of Table 6. Our median estimates are reasonably close to the mean estimates in Smets and Wouters (2007), which are reproduced again at the top of the table for convenience. In fact, their point estimates fall within our confidence intervals. Nevertheless, we estimate a lower intertemporal elasticity of substitution, habit parameter, and degrees of indexation to past wage and price inflation. Moreover,

[11]The identified shock form the data may still suffer from measurement error and be correlated with the error term. Thus, researchers typically use additional controls, which sometimes are not observable in the model. On the other hand, the innovation of shock in the model is purely exogenous, thus there is no need for additional controls.

FIGURE 4. TFP shocks – Empirical vs. Model IRFs



Note: this figure depicts the empirical IRFs to a technology shock (dashed blue lines) as well as those coming from the model at our parameter estimates (dash red lines) and at Smets and Wouters (2007) parameters (dotted black line). Confidence intervals for the empirical responses are in shaded blue.

we obtain higher labor supply elasticity, adjustment costs and Calvo adjustment probabilities.

To assess the performance of our estimation, we compute the estimated responses on simulated data at our median estimates. To reduce simulation error we report the mean β_h coefficients across 1,000 draws. These responses (red dashed line) are compared to the targeted ones (blue dashed line) in Figure 4.

Overall, the Smets and Wouters (2007) model at our parameter estimates does a good job in capturing these responses. The match is particularly close for output, investment and hours. However, the model is not able to produce the large effect on consumption upon impact nor its delayed peak. These discrepancies with the data are partially solved through the parameterization of the model. In fact, the same impulse response at the Smets and Wouters (2007) parameters, the black dotted line, generates a smaller effect on consumption upon impact and a much smoother hump.

5.2 *Government Spending Shocks*

Now we turn to fiscal policy shocks. As for the technology shock there are many identification strategies.[12] We identify this shock as in Blanchard and Perotti (2002), that is we order government spending first in a Cholesky decomposition. This is our preferred strategy because it does not rely on the news of a fiscal intervention, an aspect that is not present in the Smets and Wouters (2007) model.[13]

We target the responses of GDP, non-durables and services consumption, non-residential investment and hours worked to the Blanchard and Perotti (2002) shock. These LP coefficients are estimated by means of regressions (10) but now with the shock being the identified government spending shock, and the controls being two lags each of the shock, real GDP, real government purchases, and the tax rate. These responses are depicted in blue in Figure 5, where we also plot the estimated impulse responses at our median parameter estimates (red dashed line) as well as at Smets and Wouters (2007) mean estimates (black dotted line). The parameters used to generate these responses are reported in the third subtable of Table 6.

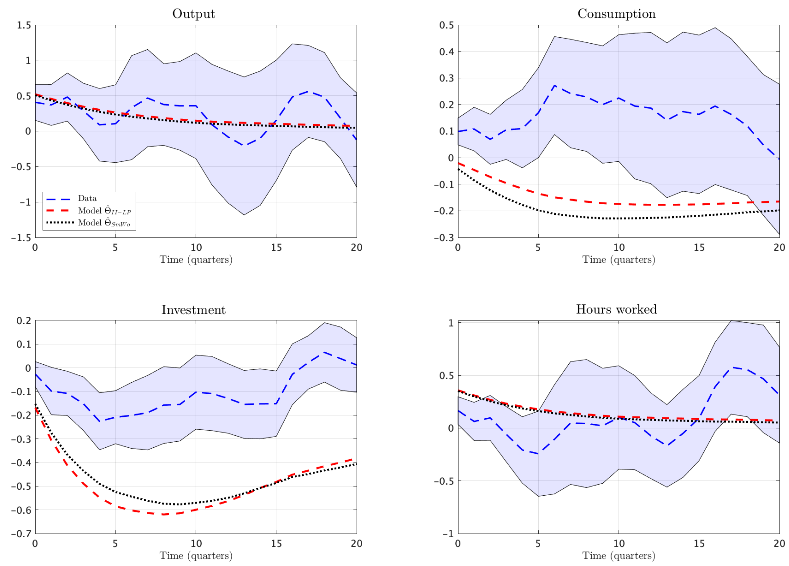
As with the technology shock, we also obtain lower curvature in utility, higher wage and price adjustment probabilities and lower degrees of indexation to past inflation than Smets and Wouters (2007). Moreover, trying to match only the response to government spending leads to higher consumption habit and lower elasticity of labor supply and adjustment costs.

These new estimated parameters only marginally change the impulse responses. Thus, the model at either our or their estimated parameters is only able to match the response of output. As for hours worked, the model has a bigger response than in the data in the first five quarters, it predicts a much larger crowding out effect on investment, and predicts a drop in consumption. Thus,

[12] Ramey (2016) summarizes some of the most prominent identification strategies for government spending shocks in Section 4 of her handbook chapter.

[13] Ramey (2011) or Ben Zeev and Pappa (2017) are leading examples that rely on the news to identify government spending shocks.

FIGURE 5. Fiscal Policy – Empirical vs. Model Estimated IRFs



Note: this figure depicts the empirical IRFs to a fiscal policy shock (dashed blue lines) as well as those coming from the model at our parameter estimates (dash red lines) and at Smets and Wouters (2007) parameters (dotted black line). Confidence intervals for the empirical responses are in shaded blue.

the model is unable to match many dimensions of the dynamic response of the economy to a government spending shock.

The most worrisome is the response of consumption given its importance for the calculation of fiscal multipliers. It is known that in these class of models households anticipate future increases in (lump-sum) taxes which makes them increase their labor supply to compensate for the negative wealth effect which in turn brings consumption down. Galí et al. (2007) demonstrate that the inclusion of hand to mouth consumers breaks the Ricardian equivalence. Thus, introducing household heterogeneity into the model helps in generating a rise of aggregate consumption in response to an unexpected increase in government spending.[14]

[14]Another important limitation of the Smets and Wouters (2007) model for the study of fiscal policies and the propagation of shocks is the absence of distortionary taxation.

Nevertheless, there is no consensus in the empirical literature on the response of consumption. As illustrated in Ramey (2016), the identification strategies that rely on the assumption of government spending being predetermined within a quarter, as done for example in Blanchard and Perotti (2002), find that government purchases rise consumption; while those that rely on the narrative news approach estimate the opposite effect, i.e. a reduction in consumption as a result of an increasing in government spending.

5.3 *Monetary Policy Shocks*

The estimation of parameters based upon monetary policy shocks rests upon the local projection estimates in Tenreyro and Thwaites (2016). They identify the monetary policy shock using a non-linear Romer and Romer (2004) regression on 40 years of quarterly data. Their specification is of particular interest as it allows a state dependent response to monetary innovations. In particular, they find that a monetary contraction during a boom creates responses in key macroeconomic variables, such as output, (nondurable) consumption and investment, that are quite different from the responses to a monetary contraction during a recession.[15] Their local projection estimation is based upon:

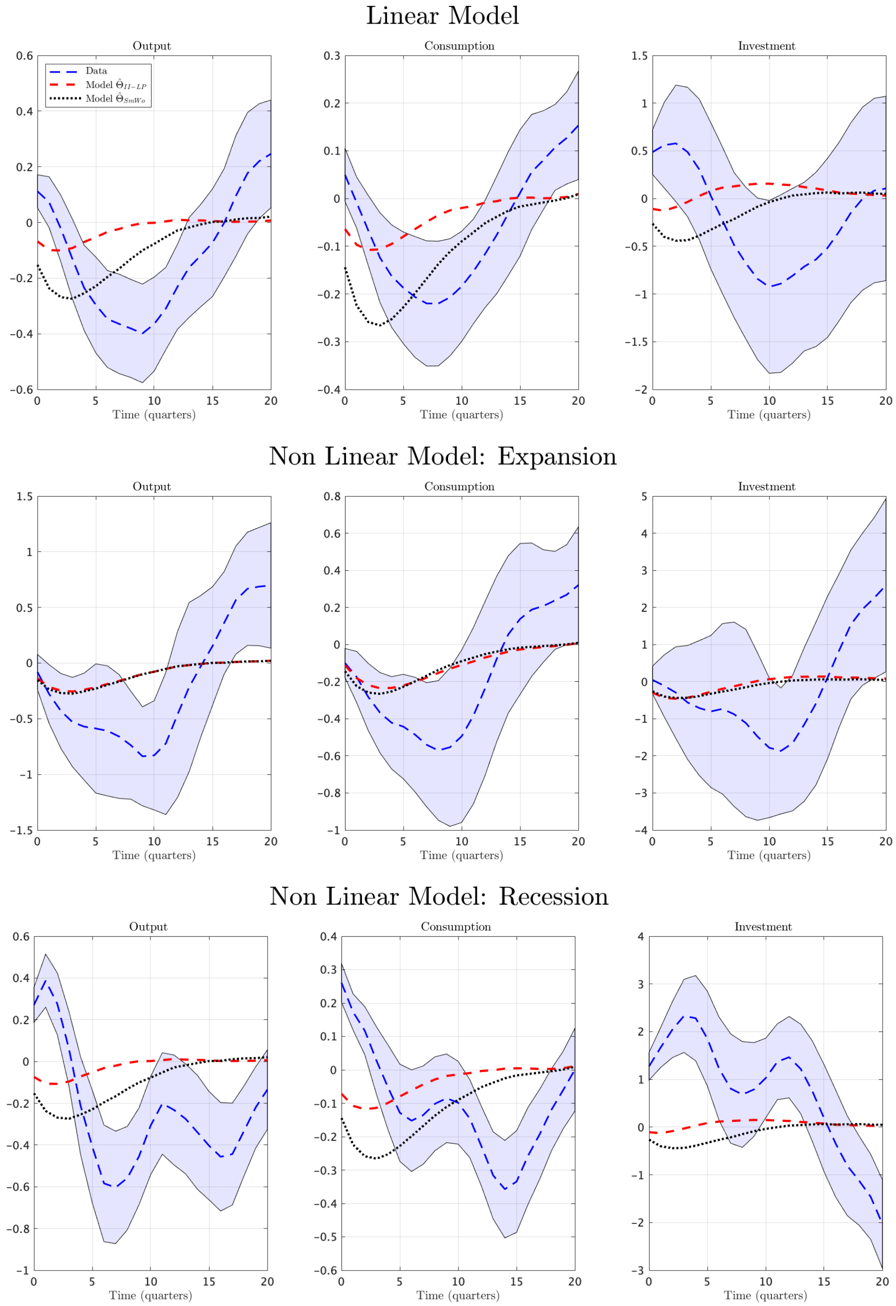
$$y_{t+h} = \tau t + F(z_t)\left(\alpha_h^b + \beta_h^b \varepsilon_t + \gamma^{\mathbf{b}'}\mathbf{x_t}\right) + (1 - F(z_t))\left(\alpha_h^r + \beta_h^r \varepsilon_t + \gamma^{\mathbf{r}'}\mathbf{x_t}\right) + u_t \quad (11)$$

where τ is a time trend, α_h^j is a constant and $\mathbf{x_t}$ are controls. $F(z_t)$ is a smooth increasing function of an indicator of the state of the economy. This is the way in which the state dependence of monetary policy is captured.

Figure 6 illustrates the state dependent estimated effects of monetary policy shocks on output, consumption and investment. For the linear model (thus imposing no state dependence), output, investment and consumption show a slight increase on impact of the contractionary shock. By the second quarter, output and consumption fall, while investment does not fall until 5 quarters. The response to the shock is prolonged, including an overshooting after approximately 15 quarters.

[15] They derive economic expansions and contractions in terms of output growth, not levels.

FIGURE 6. Monetary Policy – Empirical vs. Model IRFs



Note: the top panel depicts the empirical IRFs to a monetary policy shock (dashed blue lines) using a linear LP specification. As in previous figures the responses coming from the model at our parameter estimates are shown in dash red lines while those at Smets and Wouters (2007) parameters appear in dotted black lines. Confidence intervals for the empirical responses are in shaded blue. The remaining two panels show the responses coming from the state dependent LP model during a boom (middle) and during a recession (bottom).

These impulse responses combine the effects of monetary contractions in booms and recessions. Focusing on boom times, the second row of the figure shows more "conventional" effects of a contraction, including an immediate and prolonged fall in output, consumption and investment combined with a slow return. From the third row, the effects of a monetary contraction during a recession are quite different. One natural interpretation is that the contractionary policy during a recession is occurring during a period of stagflation, with monetary policy focusing on combating inflation.

The Smets and Wouters (2007) model clearly does not contain the necessary non-linearities that might give rise to the state dependent effects of monetary policy. Nonetheless it is instructive to see how well we can match the moments from the state dependent estimation as well as the linear model. Note, however, that when matching the moments form the state dependent estimation we still use a linear local projection on the model simulated data. That is, we allow the parameters to vary with the treatment to provide a sense of how the state dependent responses translate into different parameter estimates. Clearly if we had a model with nonlinearities, we would fix the parameters and try to match the state dependent responses. Table 7 reports our parameter estimates.

Looking first at the linear model, our median parameter estimates are generally close to the mean estimates in Smets and Wouters (2007) and the reported confidence intervals contain their parameter estimates. Our estimated elasticity of substitution is about 25% larger than theirs. The habit is stronger in our model and labor is more elastic. Our point estimates of the price adjustment parameter, $\hat{\xi}_p$, is much lower than theirs, indicating that in our estimated model prices adjust more frequently.

Focusing on the effects of monetary policy contractions during booms, our estimate of price flexibility match theirs, though the wage flexibility and the elasticity of labor supply is a bit lower than theirs. In contrast, the estimated habit is stronger as is the curvature in utility.

The impulse response function at our median estimates in each of these three cases are shown (in red) in Figure 6. The estimated model can match the basic pattern of the responses to monetary contractions during an expansion. Out-

TABLE 7. Estimates using LP coefficients

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
Sm&Wo 2007	1.26	0.80	2.52	6.31	0.70	0.66	0.58	0.24
	Linear Model							
Median	1.57	0.88	3.15	7.89	0.46	0.32	0.63	0.11
10th pctl.	0.92	0.71	1.51	3.79	0.46	0.32	0.32	0.10
90th pctl.	1.57	0.95	3.15	7.89	0.80	0.66	0.66	0.21
	Non-Linear Model: Expansion							
Median	1.46	0.83	2.20	4.98	0.64	0.66	0.33	0.21
10th pctl.	0.81	0.51	1.51	3.79	0.46	0.32	0.32	0.10
90th pctl.	1.57	0.97	3.15	7.89	0.84	0.66	0.66	0.21
	Non-Linear Model: Recession							
Median	1.57	0.86	3.15	7.89	0.46	0.32	0.66	0.21
10th pctl.	0.87	0.60	1.51	3.79	0.46	0.32	0.32	0.10
90th pctl.	1.57	0.98	3.15	7.89	0.77	0.66	0.66	0.21

Note: this table reports parameter estimates using the LP coefficients for linear and non-linear specifications.

put, consumption and investment fall on impact and recover slowly, with a slight hump shape. The impulse responses from Tenreyro and Thwaites (2016) show a more pronounced hump-shape: it is both larger and more delayed. We also do not capture their overshooting.

The estimated model is unable to capture the effects of monetary contractions during recessions. The underlying Smets and Wouters (2007) model is evidently unable to reproduce the initial positive output response to a contractionary shock. This same point applies to the linear model, thought to a lesser degree.

The impulse responses at Smets and Wouters (2007) mean parameters are similar in the linear model except that at their parameter estimates the model predicts a more pronounced drop in output, consumption and investment, which is more similar to the data. However, the hump is more delayed in the data, around Q10, than in the model, around Q3. The model at the two set of parameter estimates are quite close when studying the effects of contractionary policy during an expansion. Neither of the models can match the effects of monetary contractions during recessions.

TABLE 8. SMM estimates combining information

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
Sm&Wo 2007	1.26	0.80	2.52	6.31	0.70	0.66	0.58	0.24
	Jointly							
Median	1.00	0.62	3.15	5.89	0.42	0.40	0.43	0.14
10th pctl.	0.76	0.48	1.51	3.79	0.42	0.40	0.35	0.14
90th pctl.	1.57	0.90	3.15	7.89	0.86	0.82	0.72	0.30
	Independently							
Technology	0.85	0.69	3.28	8.20	0.44	0.59	0.47	0.14
Fiscal Policy	1.01	0.85	1.51	3.90	0.42	0.40	0.36	0.14
Monetary Policy	1.26	0.91	3.15	7.89	0.46	0.32	0.32	0.10

Note: the top block recalls the estimates of Smets and Wouters (2007), the middle block reports parameter estimates using the LP coefficients from the response of investment to all three shocks, and the bottom block recalls the median estimates from the previous estimations targeting only one source of variation.

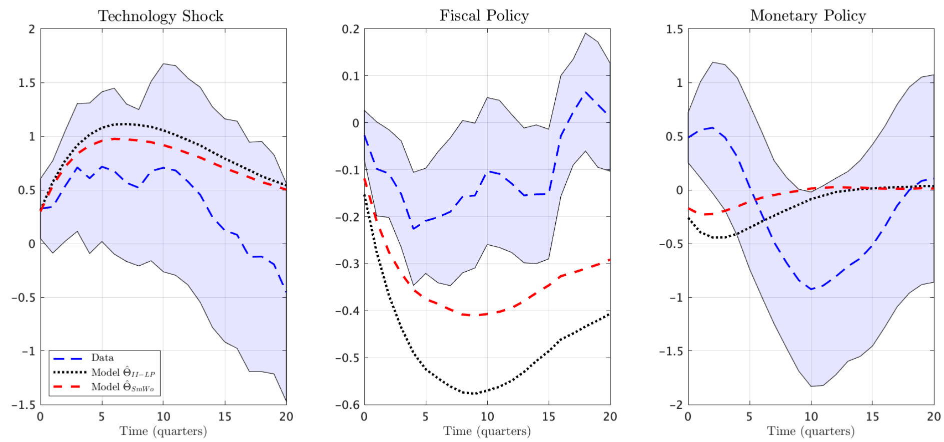
5.4 *All Shocks: The Response of Investment*

As a final exercise, we consider all three sources of variation simultaneously, rather than individually. In doing so, we are uncovering the structural parameters that best match the responses jointly. For the previous exercises, this restriction to a single vector of parameters across estimation exercises was not imposed. So, for example, the median estimate of σ_l was more that 2 times larger when matching the response to technology shocks compared to fiscal policy. The current exercise restricts these parameter to be the same across sources of variation.

As noted earlier, the response of investment is very sensitive to a change in the parameters. Accordingly, this estimation uses the LP investment response to the three types of shocks as targeted moments.[16] The results are shown in Table 8. The first block reports the estimates of Smets and Wouters (2007) which allowed multiple shocks, the middle block shows the parameter estimates from the joint shock case, and the last block recalls the estimates by type of shock.

[16] An alternative to pursue would be to include other variables but only at short/medium term horizons such that the moments are even more responsive to parameters. Note that the IRFs tend to die out at long horizons.

FIGURE 7. Targeted Investment Response to All Three Shocks



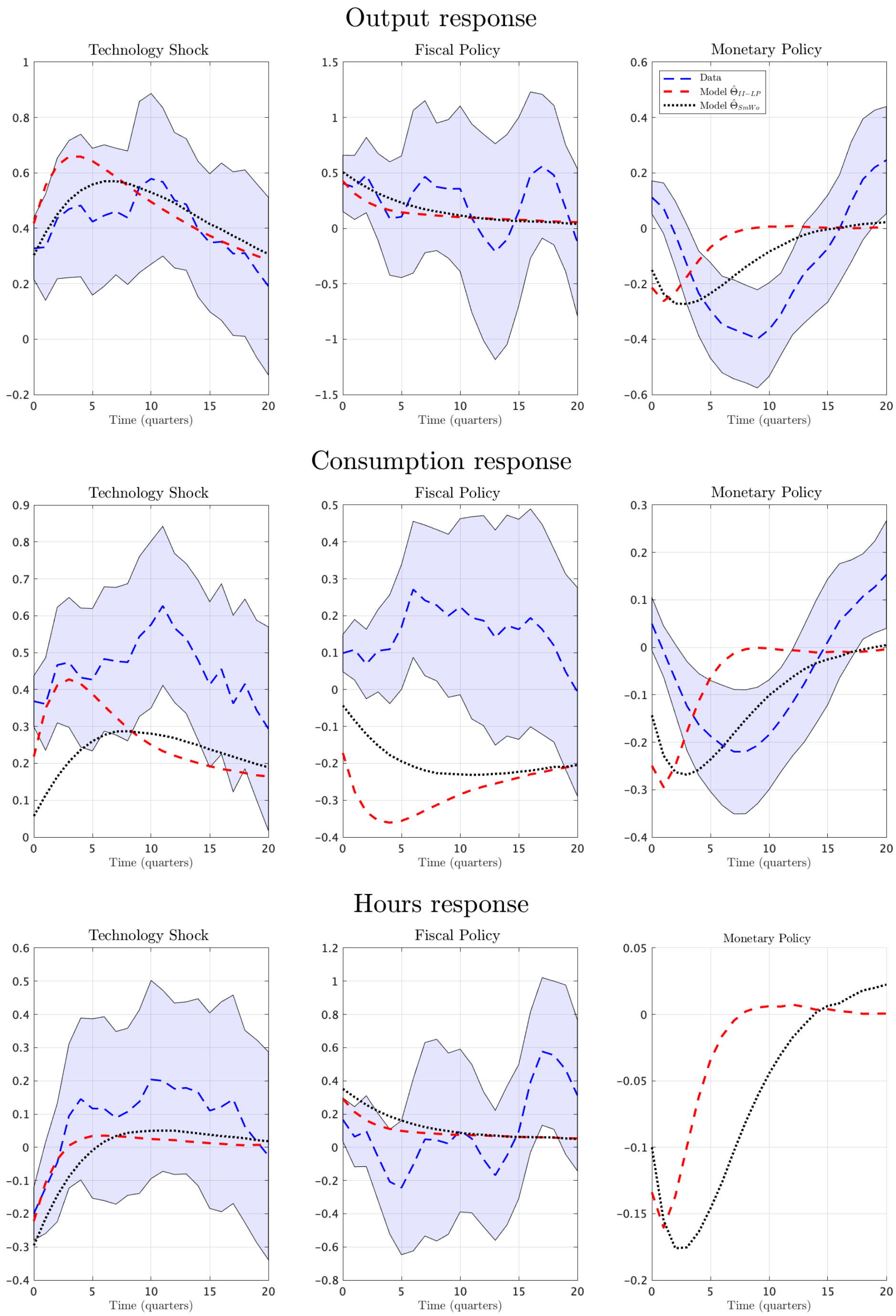
Note: this figure depicts the empirical investment IRFs to a technology, fiscal policy and monetary policy shock (dashed blue lines) as well as those coming from the model at our parameter estimates (dash red lines) and at Smets and Wouters (2007) parameters (dotted black line). Confidence intervals for the empirical responses are in shaded blue

Comparing the last two blocks, for the most part, the estimates for the cases of the individual shocks lie within the intervals created by the joint estimation. The estimated curvature of 0.91 lies between the other estimates. The habit parameter, $\hat{h}$ is smaller than any of the other point estimates. The frequency of wage and price adjustment is similar to the estimates based upon the response to fiscal shocks. It is in this sense the joint estimation exercise combines the responses to the individual shocks.

Compared to the estimates of Smets and Wouters (2007), the median estimate of the utility function is lower and the frequency of wage and price adjustment is considerably higher. That said, the point estimates of Smets and Wouters (2007) do lie in the interval between the 10th and 90th percentile.

It is important to note, however, that all these parameters have been estimated using information from different samples. For example, the responses to technology and fiscal shocks are recovered from data spanning from the late 1940s to the early 2010s, while the monetary shocks are only available from 1969 to 2007. Therefore, it is possible that some of the discrepancies we found, specially those related to the Calvo parameters, arise due to sample selection. As

FIGURE 8. Untargeted Response to All Three Shocks



Note: this figure depicts the untargeted empirical output, consumption and hours worked IRFs to a technology, fiscal policy and monetary policy shock (dashed blue lines) as well as those coming from the model at our parameter estimates (dash red lines) and at Smets and Wouters (2007) parameters (dotted black line). Confidence intervals for the empirical responses are in shaded blue noted by Fernández-Villaverde et al. (2007), there is some evidence that certain DSGE parameters, such as those characterizing the pricing behavior of firms and households, change depending on the sample used for estimation. In fact, Smets and Wouters (2007) find a higher degree of price and wage stickiness when their model is estimated only using data from the "Great Moderation" period (1984Q1 - 2004Q4).

Do the different sources of variation help in reconciling the model with the data? Figure 7 shows the targeted response of investment to the three types of shocks while Figure 8 shows the untargeted output, consumption, and hours response.

As noted earlier, the estimated model does well in matching the investment response to technology shocks but not so well to fiscal and monetary innovations. These findings remain in the case of the joint estimation. However, through exploiting the variation of the three shocks, the estimated model is able to improve the match of the investment response to a fiscal policy shock, although it is not sufficient to reconcile it completely with the data. Moreover, the investment does not longer increase in the medium run as a result of a monetary contraction.

Looking at the untargeted output response, the Smets and Wouters (2007) parameterization again produces a deeper and longer reduction in output in response to a monetary contraction compared to these estimates based upon the shocks together. A key difference is in the frequency of wage and price adjustment. The responses to technology and fiscal policy shocks are similar to those obtained in the shock-by-shock estimations.

Regarding consumption, the model at our parameter estimates is still able to capture its response to a technology shock better than at Smets and Wouters (2007) parameters, even when we do not target it directly. However, its response to the government spending shock is better captured when estimated individually. This is probably reflected in the lower intertemporal elasticity of substitution obtained at the joint estimation.

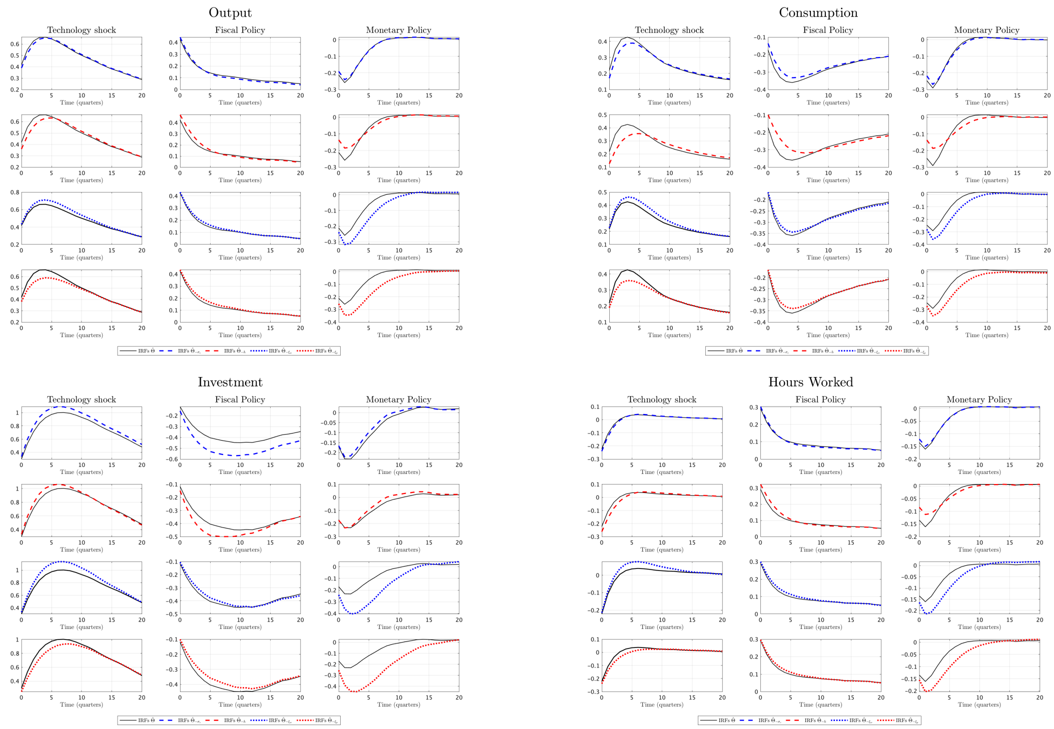
Finally, the untargeted response of hours worked is similar to those obtained when targeting its response to either the technology or the fiscal shock. For the monetary policy shock, we do not have an empirical counterpart. In any case, it is reassuring that the response is similar to that obtained at Smets and Wouters (2007) parameters. Nevertheless, the initial drop is less prolonged and the rate at which the effect dies out is bigger at our parameters than at theirs.

5.4.1 *What do the parameter differences imply for the responses?* To dig a bit deeper into differences in estimates compared to Smets and Wouters (2007), Figure 9 shows the impulse responses to the three shocks (columns) for alternative parametrizations (rows). We construct each of these four alternatives by changing one at a time the curvature of utility, the habit parameter and the Calvo's probabilities to be equal to the Smets and Wouters (2007) estimates instead of ours. So, for example, in the first row the IRFs depicted by the blue dash line are labeled $\hat{\Theta}_{-\sigma_c}$ indicating that these are the median estimated parameters from the joint estimation except for the treatment of $\hat{\sigma}_c$, which corresponds to the elasticity estimated by Smets and Wouters (2007) instead of ours. The model is simulated (not re-estimated) and the resulting IRFs are plotted along with the baseline IRFs, i.e. those generated at our median estimates from the joint estimation (solid black lines).

Looking across these combinations, we see some cases where differences in the estimated parameters matter for the economic responses. Looking first at the estimated wage and price flexibility, the output and investment responses to monetary policy are different in the two parameterizations because of the increased frequency of adjustment in our estimated model. The investment response is about half that of Smets and Wouters (2007) in our model. Both wage and price flexibility impact the magnitude of the response. Interestingly, the magnitude of the effects of technology shocks on hours is not very sensitive, at least for the first 5 quarters, to differences in price flexibility in the two models.

The differences in preference parameters, both the elasticity of substitution and the habit, matter most for the response of consumption to all three shocks as well as the response of investment to a government spending shock. Recall that our estimates are much closer to log utility so that consumption smoothing is less important to the household. Accordingly, the consumption responses produced by our model are generally larger except for the response of consumption

FIGURE 9. Parameter Decomposition



Note: this figure plots with solid black lines the IRFs of output (top left), consumption (top right), investment (bottom left) and hours worked (bottom right) to each of the three aggregate shocks (technology, fiscal and monetary policy) at our parameter estimates. These estimated parameters correspond to those obtained by matching the response of investment to all three shocks. We report the same impulse responses using instead the intertemporal elasticity of substitution (blue dashed lines), habit formation (red dashed lines), Calvo adjustment probability for wages (blue dotted lines) and prices (red dotted lines) from Smets and Wouters (2007).

to a monetary innovation, which seems insensitive to this elasticity. Perhaps this reflects that channels of monetary policy other than intertemporal substitution are operative. Both models produce hump-shaped responses. Our predicted response has a peak that is earlier and more pronounced mostly due to differences in the estimate of h.

6. Conclusion

This paper studies the use of LP coefficients in an indirect inference approach to structural estimation. Monte Carlo analysis shows that the theoretical responses at the estimated parameters are much closer to the truth than if one relies in the more traditional approach that uses VAR coefficients. Moreover, the time spent in the estimation is significantly reduced, an important consideration for large scale DSGE models.

The application of this approach to the estimation of the Smets and Wouters (2007) model, despite successful, has revealed some shortcomings of the model. For example, the model is not able to replicate the large initial response and the more delayed hump in consumption in response to a technology shock, even if we target it. The responses of investment and consumption to a government spending shock are also at odds with the data whenever the empirical estimated shock is identified recursively. Nonetheless, its most relevant and obvious flaw is its inability to capture the state dependent effects of monetary innovations, specially during a recession. Our parameter estimates suggest that Calvo (1983) pricing may be behind it since we obtain a much lower degree of wage and price stickiness in that case.

Based on these findings, we believe that state-dependent rather than time-dependent pricing will help in reconciling the model with the data. Thus, one fruitfully line of research is the extension of Smets and Wouters (2007) model to allow for state-dependent pricing. And in fact, if such model is truly state-dependent one should also be able to infer its parameters by jointly targeting the state-dependent LP coefficients, e.g. those coming from the state-dependent responses to monetary policy documented in Tenreyro and Thwaites (2016) or Ascari and Haber (2022). Consequentially, the application of our indirect inference LP approach in a non-linear environment is another interesting avenue for future research.

APPENDIX A: LOG-LINEARIZED EQUILIBRIUM CONDITIONS

- The aggregate resource constraint:

$$\hat{y}_t = c_y \hat{c}_t + i_y \hat{i}_t + z_y \hat{z}_t + \varepsilon_t^g \tag{12}$$

- The consumption Euler equation:

$$\begin{aligned}\hat{c}_t = &\frac{h/\gamma}{1+h/\gamma}\hat{c}_{t-1} + \frac{1}{1+h/\gamma}\mathbb{E}_t\hat{c}_{t+1} + \frac{wl_c(\sigma_c-1)}{\sigma_c(1+h/\gamma)}\left(\hat{l}_t - \mathbb{E}_t\hat{l}_{t+1}\right) + \\ &- \frac{1-h/\gamma}{(1+h/\gamma)\sigma_c}\left(\hat{r}_t - \mathbb{E}_t\hat{\pi}_{t+1}\right) - \frac{1-h/\gamma}{(1+h/\gamma)\sigma_c}\varepsilon_t^b\end{aligned} \tag{13}$$

- The investment Euler equation:

$$\hat{i}_t = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\hat{i}_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}\mathbb{E}_t\hat{i}_{t+1} + \frac{1}{\varphi\gamma^2\left(1+\beta\gamma^{(1-\sigma_c)}\right)}\hat{q}_t + \varepsilon_t^i \tag{14}$$

- The arbitrage equation for the value of capital:

$$\hat{q}_t = \beta(1-\delta)\gamma^{-\sigma_c}\mathbb{E}_t\hat{q}_{t+1} - \hat{r}_t + \mathbb{E}_t\hat{\pi}_{t+1} + \left(1-\beta(1-\delta)\gamma^{-\sigma_c}\right)\mathbb{E}_t\hat{r}_{t+1}^k - \varepsilon_t^b \tag{15}$$

- The aggregate production function:

$$\hat{y}_t = \Phi\left(\alpha\hat{k}_t^s + (1-\alpha)\hat{l}_t + \varepsilon_t^a\right) \tag{16}$$

- Capital services:

$$\hat{k}_t^s = \hat{k}_{t-1} + \hat{z}_t \tag{17}$$

- Capital utilization:

$$\hat{z}_t = \frac{1-\psi}{\psi}\hat{r}_t^k \tag{18}$$

- The accumulation of installed capital:

$$\hat{k}_t = \frac{(1-\delta)}{\gamma}\hat{k}_{t-1} + (1-(1-\delta)/\gamma)\hat{i}_t + (1-(1-\delta)/\gamma)\varphi\gamma^2\left(1+\beta\gamma^{(1-\sigma_c)}\right)\varepsilon_t^i \tag{19}$$

- Cost minimization by firms implies that the price mark up:

$$\hat{\mu}_t^p = \alpha\left(\hat{k}_t^s - \hat{l}_t\right) - \hat{w}_t + \varepsilon_t^a \tag{20}$$

- New Keynesian Phillips curve:

$$\begin{aligned}\hat{\pi}_t =&\frac{\beta\gamma^{(1-\sigma_c)}}{1+\iota_p\beta\gamma^{(1-\sigma_c)}}\mathbb{E}_t\hat{\pi}_{t+1} + \frac{\iota_p}{1+\beta\gamma^{(1-\sigma_c)}}\hat{\pi}_{t-1}+ \\ &- \frac{\left(1-\beta\gamma^{(1-\sigma_c)}\xi_p\right)(1-\xi_p)}{\left(1+\iota_p\beta\gamma^{(1-\sigma_c)}\right)(1+(\Phi-1)\varepsilon_p)\,\xi_p}\hat{\mu}_t^p + \varepsilon_t^p\end{aligned} \tag{21}$$

- Cost minimization by firms implies that the rental rate of capital:

$$\hat{r}_t^k = \hat{l}_t + \hat{w}_t - \hat{k}_t^s \tag{22}$$

- In the monopolistically competitive labor market, the wage mark-up

$$\hat{\mu}_t^w = \hat{w}_t - \sigma_l\hat{l}_t - \frac{1}{1-h/\gamma}\left(\hat{c}_t - h/\gamma\hat{c}_{t-1}\right) \tag{23}$$

- Wage adjustment:

$$\begin{aligned}\hat{w}_t =&\frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}\left(\mathbb{E}_t\hat{w}_{t+1} + \mathbb{E}_t\hat{\pi}_{t+1}\right) + \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\left(\hat{w}_{t-1} - \iota_w\hat{\pi}_{t-1}\right) + \\ &- \frac{1+\beta\gamma^{(1-\sigma_c)}\iota_w}{1+\beta\gamma^{(1-\sigma_c)}}\hat{\pi}_t - \frac{\left(1-\beta\gamma^{(1-\sigma_c)}\xi_w\right)(1-\xi_w)}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)\left(1+(\lambda_w-1)\,\epsilon_w\right)\xi_w}\hat{\mu}_t^w + \varepsilon_t^u\end{aligned} \tag{24}$$

- Monetary policy reaction function:

$$\hat{r}_t = \rho\hat{r}_{t-1} + (1-\rho)\left(r_\pi\hat{\pi}_t + r_y\left(\hat{y}_t - \hat{y}_t^*\right)\right) + r_{\Delta y}\left(\left(\hat{y}_t - \hat{y}_t^*\right) - \left(\hat{y}_{t-1} - \hat{y}_{t-1}^*\right)\right) + \varepsilon_t^r \tag{25}$$

Appendix B: The Monte Carlo Study. Results in Detail

B.1 *The Local Projection Approach to Indirect Inference*

Table B.1 report the mean, bias, standard deviation and root mean squared error for each of the 8 structural parameters considered in the indirect inference exercise that uses LP coefficients as moments.

Table B.1. SMM estimates using LP-IRFs

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
	Technology shock, ε_t^a							
Mean	1.23	0.82	2.81	5.91	0.57	0.62	0.48	0.15
Bias	-0.03	0.02	0.29	-0.40	-0.13	-0.04	-0.10	-0.09
Std dev.	0.26	0.10	0.61	1.74	0.15	0.07	0.17	0.05
RMSE	0.26	0.10	0.67	1.78	0.20	0.08	0.19	0.10
	Fiscal Policy, ε_t^g							
Mean	1.40	0.80	2.60	5.90	0.54	0.46	0.52	0.17
Bias	0.14	0.00	0.08	-0.41	-0.16	-0.20	-0.06	-0.07
Std dev.	0.23	0.09	0.70	1.85	0.11	0.14	0.17	0.05
RMSE	0.27	0.09	0.70	1.89	0.19	0.25	0.18	0.09
	Monetary Policy, ε_t^m							
Mean	1.38	0.79	2.36	5.52	0.62	0.53	0.47	0.16
Bias	0.12	-0.01	-0.16	-0.79	-0.08	-0.13	-0.11	-0.08
Std dev.	0.26	0.06	0.77	1.60	0.14	0.14	0.17	0.05
RMSE	0.28	0.06	0.79	1.78	0.15	0.19	0.20	0.09
	Selected Responses to All Shocks							
Mean	1.29	0.81	2.56	5.75	0.56	0.59	0.47	0.15
Bias	0.03	0.01	0.04	-0.56	-0.14	-0.07	-0.11	-0.09
Std dev.	0.25	0.09	0.74	1.40	0.14	0.10	0.17	0.05
RMSE	0.25	0.10	0.74	1.50	0.19	0.12	0.20	0.10

B.2 *The Structural Vector Autoregression Approach to Indirect Inference*

Table B.2 report the mean, bias, standard deviation and root mean squared error for each of the 8 structural parameters considered in the indirect inference exercise that uses SVAR coefficients as moments.

TABLE B.2. SMM estimates using SVAR-IRFs

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
	Technology shock, ε_t^a							
Mean	1.28	0.82	2.36	6.81	0.64	0.62	0.43	0.14
Bias	0.02	0.02	-0.16	0.50	-0.06	-0.04	-0.15	-0.10
Std dev.	0.20	0.08	0.77	1.46	0.10	0.07	0.15	0.05
RMSE	0.20	0.08	0.78	1.54	0.12	0.08	0.21	0.11
	Fiscal Policy, ε_t^g							
Mean	1.32	0.82	2.65	5.44	0.68	0.47	0.52	0.18
Bias	0.06	0.02	0.13	-0.87	-0.02	-0.19	-0.06	-0.06
Std dev.	0.25	0.11	0.68	1.82	0.07	0.16	0.16	0.05
RMSE	0.25	0.11	0.70	2.02	0.08	0.25	0.17	0.07
	Monetary Policy, ε_t^m							
Mean	1.32	0.79	2.39	5.54	0.66	0.48	0.50	0.17
Bias	0.06	-0.01	-0.13	-0.77	-0.04	-0.18	-0.08	-0.07
Std dev.	0.27	0.06	0.80	1.63	0.09	0.14	0.17	0.05
RMSE	0.27	0.06	0.81	1.80	0.10	0.23	0.18	0.08
	Selected Responses to All Shocks							
Mean	1.21	0.85	2.57	6.45	0.58	0.58	0.50	0.15
Bias	-0.05	0.05	0.05	0.14	-0.12	-0.08	-0.08	-0.09
Std dev.	0.24	0.08	0.75	1.39	0.12	0.10	0.16	0.05
RMSE	0.24	0.09	0.75	1.40	0.17	0.13	0.18	0.10

B.3 *The Role of the Sample Size*

In our baseline results we have set $T = 300$ because that is the sample size chosen by Jordà (2005) in the Monte Carlo study of his seminal paper. However, most empirical applications that used identified shocks as regressors within the LP framework employ fewer observations. For such sample sizes, LPs suffer from small sample bias as documented in Herbst and Johannsen (2021). Thus, to check if our Monte Carlo results still hold in smaller samples we repeat all the analyses using $T = 100$.

In particular, we generate a new repeated dataset consisting of time series of length 100, which corresponds to 25 years of quarterly data. The new simulated dataset is used to generate S vectors of the true data moments. Simulated mo-

TABLE B.3. SMM estimates using LP-IRFs

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
	Technology shock, ε_t^a							
Mean	1.28	0.85	2.79	5.67	0.57	0.53	0.44	0.15
Bias	0.02	0.05	0.27	-0.64	-0.13	-0.13	-0.14	-0.09
Std dev.	0.28	0.09	0.65	1.74	0.15	0.13	0.16	0.05
RMSE	0.28	0.11	0.71	1.85	0.19	0.19	0.21	0.11
	Fiscal Policy, ε_t^g							
Mean	1.19	0.83	2.34	4.47	0.48	0.42	0.56	0.13
Bias	-0.07	0.03	-0.18	-1.83	-0.22	-0.24	-0.02	-0.11
Std dev.	0.25	0.15	0.73	1.35	0.06	0.14	0.16	0.05
RMSE	0.26	0.15	0.75	2.28	0.22	0.28	0.16	0.12
	Monetary Policy, ε_t^m							
Mean	1.33	0.79	2.74	5.10	0.69	0.43	0.55	0.19
Bias	0.07	-0.01	0.22	-1.21	-0.01	-0.23	-0.03	-0.05
Std dev.	0.33	0.08	0.68	1.55	0.16	0.15	0.16	0.05
RMSE	0.34	0.08	0.72	1.97	0.16	0.27	0.16	0.07
	Selected Responses to All Shocks							
Mean	1.48	0.79	2.35	5.00	0.54	0.51	0.45	0.15
Bias	0.22	-0.01	-0.17	-1.31	-0.16	-0.15	-0.13	-0.09
Std dev.	0.18	0.08	0.78	1.61	0.14	0.15	0.17	0.05
RMSE	0.29	0.08	0.80	2.07	0.21	0.21	0.21	0.11

ments are computed on a sample that is 10 times as large, i.e. $T^s = 1000$. All remaining hyper-parameters used in the optimization stage are unchanged. Notice, however, that the optimal weighting matrix is now computed over repeated samples of length 100.

Results are summarized in tables B.3, B.4 and B.5 as well as figure B.1. Based on all this information, we can confirm that the small sample bias associated with LPs is not an issue for indirect inference. In fact, all our results from the Monte Carlo hold when we reduce the sample size to $T = 100$. That is, the LP approach produces consistent estimates despite biases being higher for certain parameters such as for example the investment adjustment cost parameter (see table B.3). Compared to the SVAR approach, the RMSE is larger for more than half of the parameters (figure B.1) and the overall fit in terms of the J-statistic is worse (table

FIGURE B.1. Relative performance in terms of RMSE with T = 100

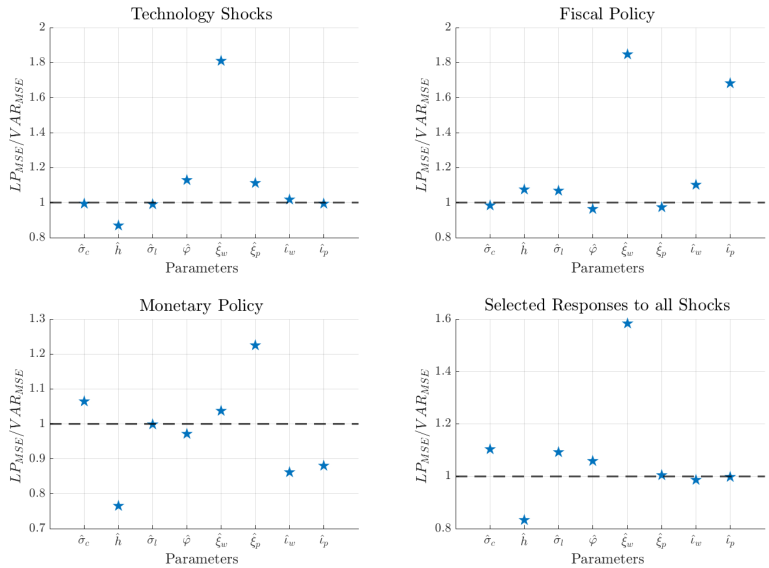


TABLE B.4. Overall performance: Estimated Impulse Responses

	Local Projections			Vector Autoregression		
	Avg. J	Max. J	Time (in min.)	Avg. J	Max. J	Time (in min.)
	Observed Sample Size, T = 100					
Technology Shock	86.44	134.40	8.31	83.09	233.56	16.88
Fiscal Policy	90.74	135.14	8.73	83.96	230.66	13.72
Monetary Policy	89.43	139.47	8.37	83.64	222.38	11.80
Selected Responses	87.66	151.53	8.25	82.79	198.70	14.48
	Observed Sample Size, T = 300					
Technology Shock	87.23	117.00	28.62	82.80	247.94	67.98
Fiscal Policy	87.72	129.96	24.68	86.28	251.90	52.49
Monetary Policy	88.58	121.48	23.38	82.77	221.87	53.04
Selected Responses	86.56	128.65	20.03	82.63	240.07	72.42

B.4). However, when looking at the distance of the true/model generated IRFs at the true parameter vector versus at the estimated parameters using the LP or SVAR approach, we still find that the LP structural estimates generate IRFs that are closer to the truth (see table B.5).

TABLE B.5. Overall performance: Model Impulse Responses

	Local Projections		Vector Autoregression	
	Avg. J^*	Max. J^*	Avg. J^*	Max. J^*
	Observed Sample Size, T = 100			
Technology Shock	2.04	6.14	6.81	103.25
Fiscal Policy	1.31	5.99	5.01	57.40
Monetary Policy	2.10	10.90	7.80	47.45
Selected Responses	14.18	49.06	29.81	218.14
	Observed Sample size, T = 300			
Technology Shock	2.57	9.43	34.67	228.41
Fiscal Policy	3.05	13.88	58.12	692.14
Monetary Policy	2.71	16.89	178.17	853.72
Selected Responses	8.37	44.69	230.46	1130.58

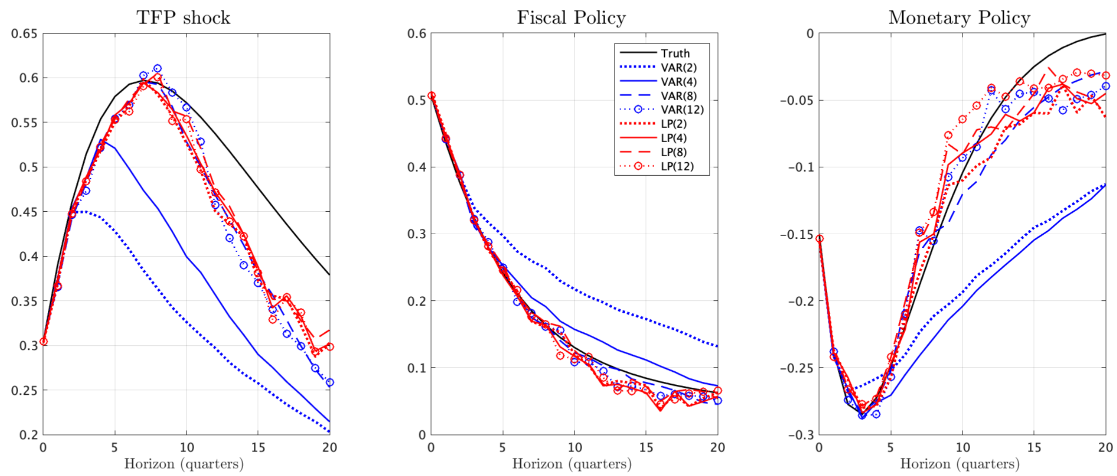
B.4 *The Role of Lag Length*

The choice of lag-length is not relevant for the estimated LP-IRFs since we use the true innovation as our dependent variable of interest. However, it matters quite substantially for the SVAR-IRFs. In fact, increasing the number of lags in the SVAR approach reduces the distance of the SVAR-IRFs from the true ones. This can be seen in figure B.2 below, where we plot the mean IRFs estimated by LPs or VARs with different choices for the lag length along with the true IRFs coming from the model.

In light of this evidence, we try to answer the following question: what are the implications for the structural parameters of choosing different lag lengths when constructing the moments used in the estimation? To answer this question we repeat the Monte Carlo experiments presented in the main text but under different choices of p.

Table B.6 shows the average and mean value of our second and most preferred measure of overall performance, i.e. the weighted distance of the structural IRFs at the estimated parameters with respect to those at the true parameters, for various lag lengths, $p = \{2, 4, 8, 12\}$, and the two moment generating functions. Three clear messages arise from this table.

FIGURE B.2. Output response to the three aggregate shocks



First, as expected, the difference between the LP and SVAR approach to indirect inference, measured as the difference in J^*'s, becomes smaller and smaller as we increase the lag length used in the moment generation stage. Since the IRFs coming form these two econometric models approximately agree out to horizon p, then, the larger p, the more similar are the set of moments used in the estimation stage. Hence, J^*'s for LP and SVAR are more similar with $p = 12$ than for $p = 2$.

Second, the overall performance barely changes when looking at the LP approach under different lag lengths. Since LP-IRFs do not change with the choice of p, the estimated structural parameters are also not affected by this choice. Hence, the value of the J^*'s are not affected either.

And third, the overall performance of the SVAR approach improves as we increasing p. Longer lags help the SVAR approach in reducing the misspecification of the IRFs and makes this responses be more similar to those estimated by LP. As a result, the set of moments are more similar to the LP approach and the values of J^* fall and get closer to those obtain with the LP approach.

B.5 *Estimates with Measurement Error*

B.5.1 *Classical Measurement Error* Table B.7 reports the mean, bias, standard deviation and root mean squared error for each of the 8 structural parameters

TABLE B.6. Overall performance – Sensitivity to lag length

	Local Projections		Vector Autoregression	
	Avg. J^*	Max. J^*	Avg. J^*	Max. J^*
		p=2		
Technology Shock	2.60	8.69	284021.78	2587907.99
Fiscal Policy	3.14	13.07	86762.92	1003678.62
Monetary Policy	2.77	13.44	-1090391.15	361979.19
Selected Responses	9.10	47.94	2617507.84	16254584.36
		p=4		
Technology Shock	2.57	9.43	34.67	228.41
Fiscal Policy	3.05	13.88	58.12	692.14
Monetary Policy	2.71	16.89	178.17	853.72
Selected Responses	8.37	44.69	230.46	1130.58
		p=8		
Technology Shock	2.47	13.33	2.87	19.93
Fiscal Policy	2.93	15.79	3.21	20.77
Monetary Policy	2.71	15.82	5.26	20.57
Selected Responses	9.66	50.95	11.84	79.72
		p=12		
Technology Shock	2.33	8.67	2.43	14.75
Fiscal Policy	2.74	16.47	2.43	14.68
Monetary Policy	2.63	15.34	3.20	23.26
Selected Responses	8.58	52.95	10.79	53.69

considered in the indirect inference exercise that uses LP responses to a noisy monetary innovation as targeted moments. Here the noise comes from some random variable that is uncorrelated with other shocks.

B.5.2 *Correlated Measurement Error* Table B.8 reports the mean, bias, standard deviation and root mean squared error for each of the 8 structural parameters considered in the indirect inference exercise that uses LP responses of output and consumption to a noisy monetary innovation as targeted moments. Here the noise comes from different degrees of correlation with the technology shock.

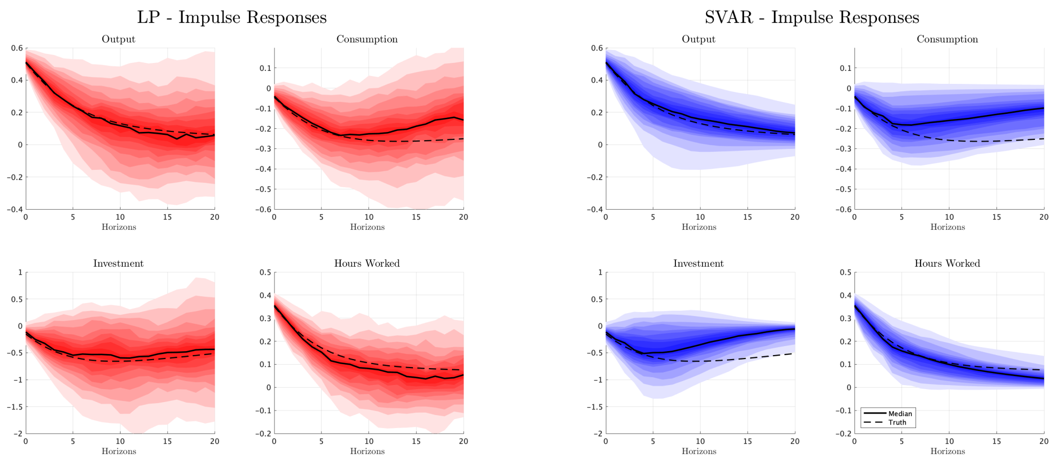
TABLE B.7. SMM Estimates: Classical Measurement Error

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
	True innovation, ε_t^m							
Mean	1.38	0.79	2.36	5.52	0.62	0.53	0.47	0.16
Bias	0.12	-0.01	-0.16	-0.79	-0.08	-0.13	-0.11	-0.08
Std dev.	0.26	0.06	0.77	1.60	0.14	0.14	0.17	0.05
RMSE	0.28	0.06	0.79	1.78	0.15	0.19	0.20	0.09
	Small measurement error, $\varepsilon_t^{m,obs}$ with $\sigma_\nu = 0.25$							
Mean	1.25	0.81	2.60	6.13	0.56	0.49	0.53	0.17
Bias	-0.01	0.01	0.08	-0.18	-0.14	-0.17	-0.05	-0.07
Std dev.	0.31	0.05	0.73	1.68	0.12	0.15	0.11	0.03
RMSE	0.31	0.05	0.74	1.69	0.19	0.23	0.12	0.08
	Large measurement error, $\varepsilon_t^{m,obs}$ with $\sigma_\nu = 0.5$							
Mean	1.20	0.84	2.65	6.55	0.54	0.45	0.55	0.17
Bias	-0.06	0.04	0.13	0.24	-0.16	-0.21	-0.03	-0.07
Std dev.	0.31	0.06	0.71	1.59	0.12	0.15	0.10	0.04
RMSE	0.31	0.07	0.73	1.61	0.20	0.26	0.11	0.08

TABLE B.8. SMM Estimates: Correlated Measurement Error

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
	True innovations, ε_t^a and ε_t^m							
Mean	1.42	0.79	2.37	6.34	0.56	0.58	0.50	0.16
Bias	0.16	-0.01	-0.15	0.03	-0.14	-0.08	-0.08	-0.08
Std dev.	0.20	0.05	0.77	1.72	0.14	0.10	0.11	0.03
RMSE	0.26	0.05	0.79	1.72	0.19	0.13	0.14	0.09
	Small correlation, ε_t^a and $\varepsilon_t^{m,obs}$ with $\rho_{m,a} = 0.25$							
Mean	1.42	0.79	2.37	6.91	0.56	0.55	0.50	0.16
Bias	0.16	-0.01	-0.15	0.60	-0.14	-0.11	-0.08	-0.08
Std dev.	0.20	0.05	0.77	1.42	0.13	0.11	0.11	0.04
RMSE	0.25	0.05	0.78	1.54	0.19	0.16	0.14	0.09
	Large correlation, ε_t^a and $\varepsilon_t^{m,obs}$ with $\rho_{m,a} = 0.5$							
Mean	1.40	0.80	2.39	7.31	0.56	0.52	0.50	0.17
Bias	0.14	0.00	-0.13	1.00	-0.14	-0.14	-0.08	-0.07
Std dev.	0.21	0.05	0.78	1.15	0.12	0.12	0.12	0.04
RMSE	0.25	0.05	0.79	1.53	0.19	0.19	0.14	0.08

FIGURE B.3. Fiscal Policy Shock



B.6 *Additional Figures*

B.6.1 *Fan Charts: Fiscal and Monetary Policy Shocks* Here we report the distribution of IRFs to fiscal policy (Figure B.3) and monetary policy (Figure B.4). These are used in the Monte Carlo experiment. As for Figure 1, it is clear that the bias-variance trade off between the LP-IRFs and the SVAR-IRFs still persists in the IRFs to fiscal and monetary shocks. In fact, we see that in both figures LP-IRFs are less biased since the median (black solid) is closer to the truth (black dashed) and that it also suffers from more variability because the distribution is wider specially at long horizons.

B.6.2 *Elasticity and Sensitivity of the Moments to Changes in the Parameters* We report the elasticity of the moments with respect to small changes in the parameters. The elasticity is computed using numerical differentiation (forward difference). That is,

$$\text{elasticity}(\Theta_i) = \frac{\dfrac{M(\Theta_i+\Delta,\Theta_{-i})-M(\Theta)}{M(\Theta)}}{\dfrac{\Theta_i+\Delta-\Theta_i}{\Theta_i}} = \frac{\dfrac{M(\Theta_i+\Delta,\Theta_{-i})-M(\Theta)}{M(\Theta)}}{\Delta/\Theta_i} \tag{26}$$

where Δ is a small change in parameter Θ_i. Results are shown in Figures B.5 and B.6. A quick inspection of these figures reveal that these metric is not well suited

FIGURE B.4. Monetary Policy Shock

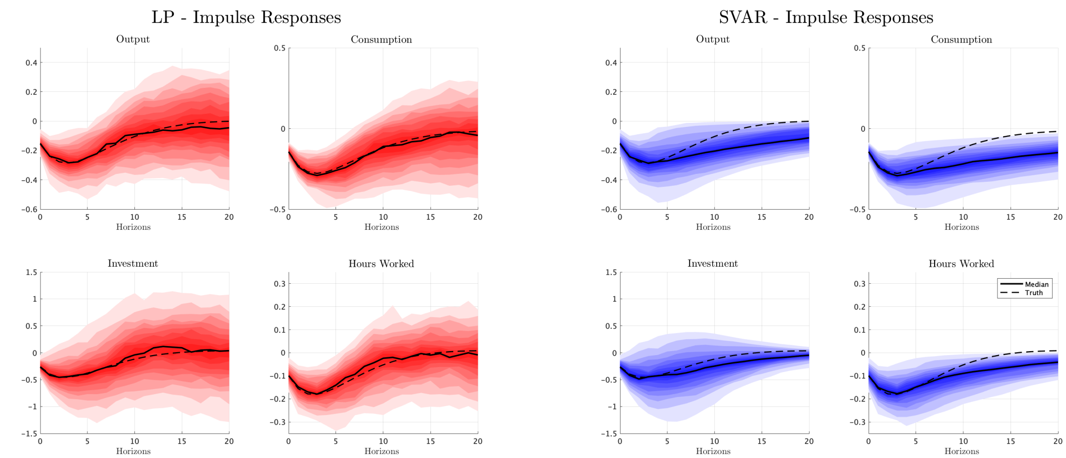


FIGURE B.5. LP-IRFs elasticities

to deal with IRFs converging towards zero or changing sign. Therefore, we also compute the sensitivity of the moments in absolute terms as shown in equation (27):

$$\text{sensitivity}(\theta) = \frac{M(\theta + \Delta) - M(\theta)}{\theta + \Delta - \theta} = \frac{M(\theta + \Delta) - M(\theta)}{\Delta} \tag{27}$$

Results are depicted in Figures B.7 and B.8. Here we see that investment (yellow line) is the most responsive to small changes in the estimated parameters. This observation motivates our focus on investment for the "selected responses"

FIGURE B.6. SVAR-IRFs elasticities

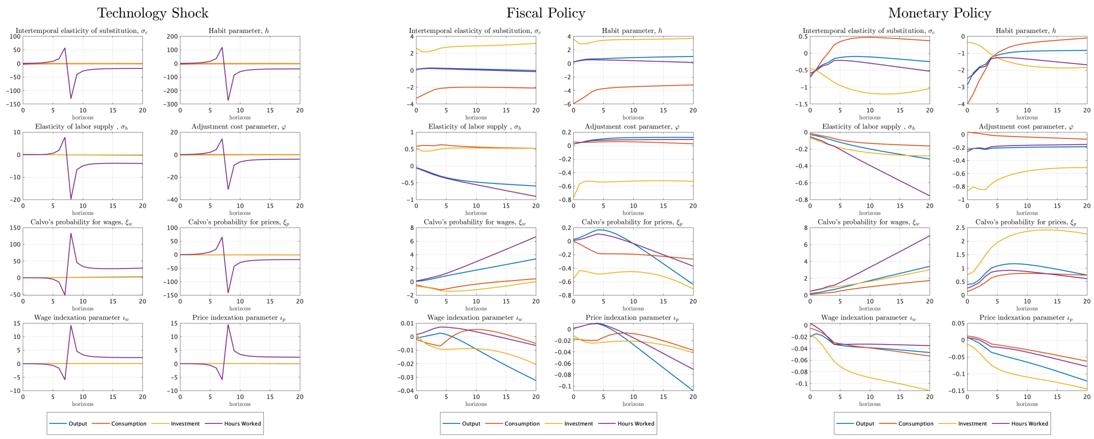


FIGURE B.7. LP-IRFs sensitivity

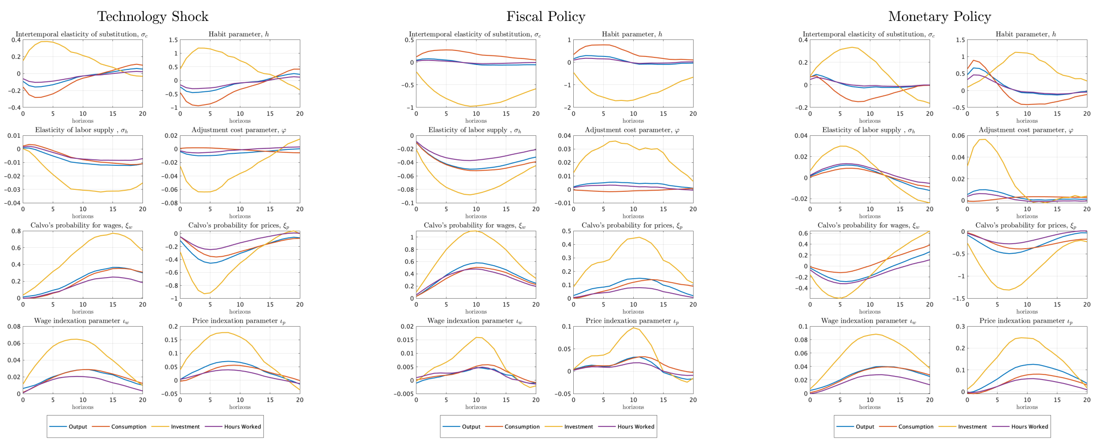
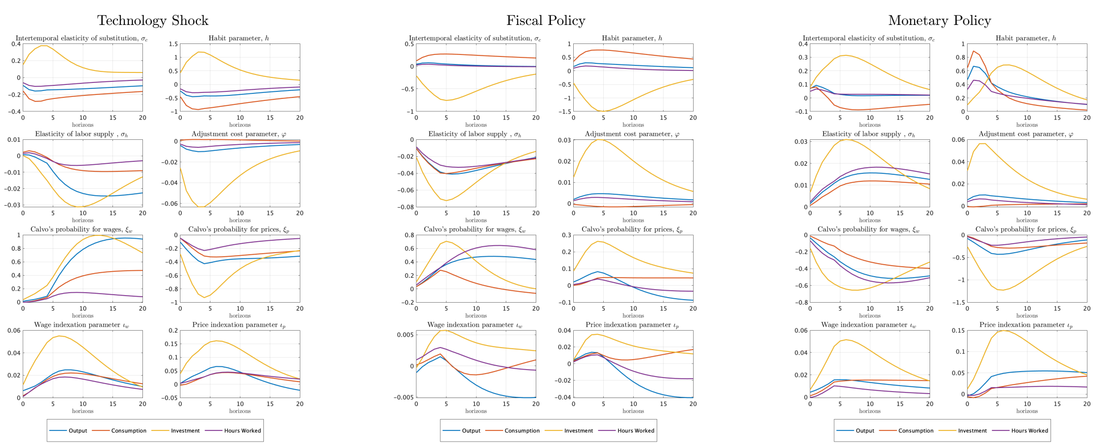


FIGURE B.8. SVAR-IRFs sensitivity



scenario in the Monte Carlo as well as for the re-estimation of the model using the responses to all three shocks jointly.

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