Indirect Inference: A Local Projection Approach

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Motivation

- **Estimating** the structural **parameters** of economic models can be quite **challenging**:
  - ∗ *Frequentist approach* $\Longrightarrow$ intractability or very difficult to evaluate likelihoods.
  - ∗ *Bayesian approach* $\Longrightarrow$ no analytical posterior distribution for DSGEs.
- **Indirect Inference** is an alternative to *maximum likelihood* or *bayesian* estimation that does not require analytical tractability.
- A key feature of indirect indirect inference is the use of an **auxiliary (econometric) model** to form a criterion function.
  - ∗ The most popular approach is a *vector autoregression* (VAR) for the variables of interest.
- Starting with Jordà (2005), **local projections** (LP) have become an increasing widespread alternative to study the propagation of structural shocks.
- ***How should we choose between VAR and LP*** *when picking the window through which we view the actual data and the simulated data generated by the economic model?*

What We Do

- We use **Monte Carlo** methods to compare the performance of the two moment generating functions in a specific application involving the estimation of the parameters of a dynamic general equilibrium model (DSGE).
  * We use the log-linearized version of *Smets and Wouters (2007) model* as our data generating process.
  * Given the complexity of the SW model we can consider the propagation of different shocks: technology, fiscal and monetary policy, and study their ability in picking up different parameter values.
  * We use the Root Mean Squared Error (RMSE) as a measure to assess the performance parameter by parameter and the J-statistic for the overall performance.
- We use our Local Projection approach to Indirect Inference in a real world **empirical application**.
  * We study how well the Smets and Wouters model is able to capture the response of key macro aggregates to technology, fiscal and monetary shocks.
  * We compare how the resulting parameter estimates differ from those in their paper.

Where We Stand in the Literature

- There are two papers that stand out and that we use as our starting point:
  - **Smith (1993)** as the seminal contribution that proposed the use of VAR coefficients as moments in a simulated method of moments exercise.
  - **Plagborg-Møller and Wolf (2021)** because they show that LP with p lags as controls and VAR(p) estimators approximately agree at impulse response horizons $h \leq p$.
- However, we **differ from these two papers** in a few important matters:
  - In contrast with Smith (1993), we propose a different way of summarizing the actual and the simulated data by means of impulse responses estimated using local projections.
  - Our ultimate object of interest is different than that of Plagborg-Møller and Wolf (2021). They are interested in the impulse responses, while for us they are simply a mean of getting an estimate of our model parameters.
- The paper is also related to the literature on:
  - *Estimation of DSGE models* $\Longrightarrow$ Christiano et al. (2005), Smets and Wouters (2007) or Scalone (2018)
  - *Simulation studies of LP & VAR methods* $\Longrightarrow$ Li, Plagborg-Møller and Wolf (2021), Herbst and Johannsen (2020)

A Quick Refresher on SMM

A General Indirect Inference Exercise

- Suppose we have an ***economic model*** that takes the form:

$$y_t = M(y_{t-1}, x_t, u_t; \Theta) \quad \text{for} \quad t = 1, 2, \dots, T$$

  where $\{x_t\}_{t=1}^T$ is a sequence of observed exogenous variables, $\{y_t\}_{t=1}^T$ is a sequence of endogenous variables and $\{u_t\}_{t=1}^T$ is a sequence of random errors.

  * Requirement: being able to numerically simulate the model.

- Assume that we have another ***auxiliary model*** that captures, or summarizes, certain features of the data by means of some estimated parameter vector $\hat{\beta}$.

  * Requirement: the number of parameters in the auxiliary model must be at least as large as the number of parameters in the economic model ($m \geq k$).

- The **goal** is to choose the parameters of the economic model so that the observed data and the simulated data look the same from the point of view of the auxiliary econometric model.

- The most common way to achieve this goal is by **minimizing the quadratic distance** between the two vectors of estimated parameters, i.e.

$$\hat{\Theta} = \arg\min_{\Theta} \left(\hat{\beta} - \tilde{\beta}(\Theta)\right)' W \left(\hat{\beta} - \tilde{\beta}(\Theta)\right)$$

A Monte-Carlo Study

Setting up the Monte Carlo Study

The Smets and Wouters Model

- **The Supply Side = Production**: capital + labor $\Longrightarrow$ int. good $\Longrightarrow$ final good
  - A *perfectly competitive* firm combines intermediate goods to produce the **final good** in the economy.
  - There is *monopolistic competition* in the **intermediate goods** market: each intermediate good is produced by a single firm j.
  - Each of these firms combine **capital** and **labor** to produce the differentiated goods. And **total factor productivity** follows an AR(1).
- **The Demand Side = Expenditures**: cons. + investment + exogenous spending
  - Consumption growth follows an Euler Equation which takes into account the existence of **external habit formation**.
  - Households rent capital services to firms and decide how much capital to accumulate given certain **adjustment costs** associated to the degree of capital utilization.
  - Exogenous spending has two components: **government spending** and an element related to productivity.

The Smets and Wouters Model II

- **Sticky Prices**:
  - Firms are not allowed to change their prices unless they receive a random signal as in **Calvo (1983)**.
  - Firms that do not receive this signal, link their prices to past inflation. In other words, there is **partial indexation**.
- **Sticky Wages**:
  - Households act as a price-setters in the labor market due to their **differentiated labor types**.
  - Wages are set in a **similar fashion to prices**. There are a random wage-change signals and partial indexation.
- **Monetary Policy**:
  - The Central Bank sets short term interest rates according to a **Taylor-type rule**, in which the interest rate depends on *last period's rate*, *inflation* and the level and the *growth rate* of the *output gap*.

Model's Equations

Following the footsteps of Smith (1993, J Appl. Econom.)

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- We solve and simulate the log-linearized version of the Smets and Wouters (2007) model to obtain repeated samples of macroeconomic aggregates $\{x_t^s\}_{s=1}^S$ that then we use to estimate a *subset of the parameters* of the model, Θ.

- We focus on the following **8 parameters**:

  * σ_c : intertemporal elasticity of substitution
  * h : habit parameter
  * σ_l : elasticity of labor supply
  * φ : investment adjustment cost parameter
  * ξ_w, ξ_p : Calvo adjustment probabilities
  * ι_w, ι_p : Degree of indexation to past inflation

- The remaining parameters are set at the estimated values from Smets and Wouters (2007).

- Hyper-parameter choices:

  * Observed sample size, $T = 300$
  * Number of "observed" time series, S = 100
  * Simulated sample size, $T^S = 3,000 \implies \tau = T^S/T = 10$

The Moment Generating Functions

The Auxiliary Econometric Model

- We focus on the ***estimated* impulse responses** of four variables:
  - y_t : output
  - c_t : consumption
  - i_t : investment
  - hw_t : hours worked

  to one (or a selection) of three following **shocks**:
  - ε_t^a : total factor productivity (TFP) shock
  - ε_t^g : fiscal policy (FP) shock
  - ε_t^m : monetary policy (MP) shock
- The IRFs are estimated using the traditional ***VAR + Cholesky decomposition*** (SVAR - IRFs) or the more recent Jordà (2005) approach which relies on ***Local Projections*** (LP - IRFs).
- In either case, the econometrician still needs to decide on at least two more things:
  - The impulse response horizon, H. We set $H = 20$.
  - The number of lags, p. We set $p = 4$.
- As a result of our choices, the economic model is ***overidentified*** $m = 21 \times 4 > 8 = k$. Therefore, we need a weighting matrix. We use the inverse of the variance-covariance matrix of the moments $W = \Sigma^{-1}$

Local Projections (LP - IRFs)

- Some notation:
  - Let $\tilde{y}_t \in \{y_t, c_t, i_t, hw_t\}$ denote one of response variables of interest.
  - Let $\tilde{x}_t \in \{\varepsilon_t^a, \varepsilon_t^g, \varepsilon_t^m\}$ denote the innovation of one of the three aggregate shocks.
  - Define the vector of controls $w_t = \{\tilde{x}_t, \tilde{y}_t\}$.
- Then, consider for each horizon $h = 0, 1, 2, \ldots, H$ the *linear projections*:

$$\tilde{y}_{t+h} = \mu_h + \beta_h \tilde{x}_t + \sum_{\ell=1}^{p} \delta'_{h,\ell} w_{t-\ell} + \xi_{h,t} \tag{1}$$

where $\xi_{h,t}$ is the projection residual and $\mu_h, \beta_h, \{\delta'_{h,\ell}\}_{\ell=1}^{p}$ are the projection coefficients.

- **Definition**. *The LP - IRFs of $\tilde{y}_t$ with respect to $\tilde{x}_t$ is given by $\{\beta_h\}_{h\geq 0}$ in the equation above.*

Structural Vector Autoregression (SVAR - IRFs)

- Consider the multivariate linear *VAR(p) projection*:

$$w_t = c + \sum_{\ell=1}^{p} A_\ell w_{t-\ell} + u_t \tag{2}$$

  where u_t is the projection residual and c, $\{A_\ell\}_{\ell=1}^{p}$ are the projection coefficients.

- Let $\Sigma_u \equiv \mathbb{E}\left[u_t u_t'\right]$ and define the *Cholesky decomposition* $\Sigma_u = BB'$ where B is lower triangular with positive diagonal entries.

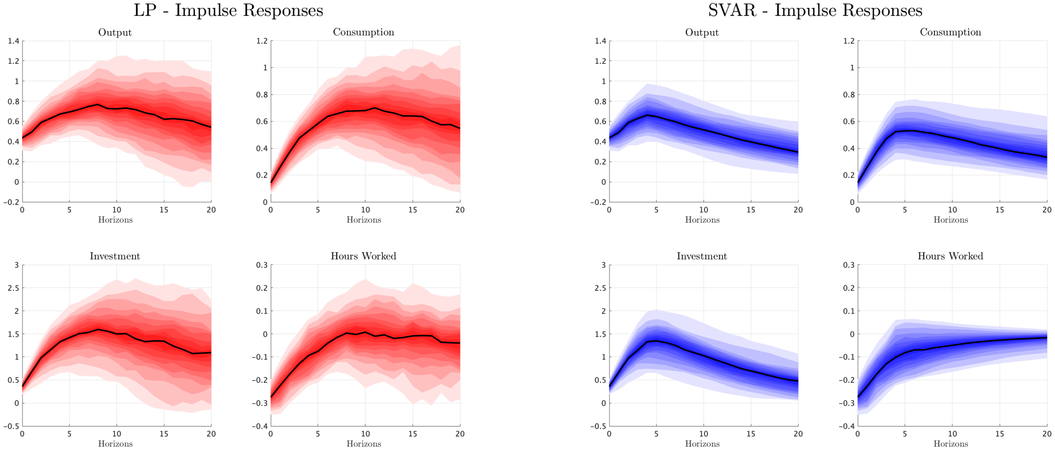
- Consider the corresponding recursive *SVAR representation*:

$$A(L) w_t = c + B\eta \tag{3}$$

  where $A(L) = I - \sum_{\ell=1}^{p} A_\ell L^\ell$ and $\eta = B^{-1} u_t$. Define the lag polynomial $\sum_{\ell=0}^{p} C_\ell L^\ell = C(L) = A(L)^{-1}$.

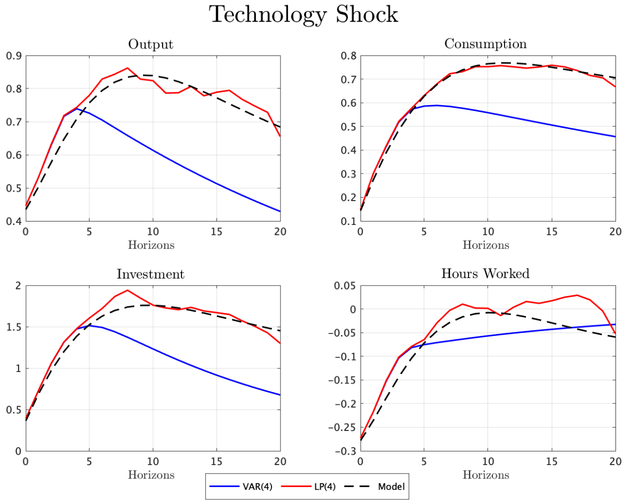
- **Definition**. *The SVAR - IRFs of $\tilde{y}_t$ with respect to $\tilde{x}_t$ is given by $\{\theta_h\}_{h\geq 0}$ with $\theta_h \equiv C_{2,\bullet,h} B_{\bullet,1}$ where $\{C_\ell\}$ and B are defined above.*

Technology Shock: Estimated IRFs for $S = 100$, $T = 300$

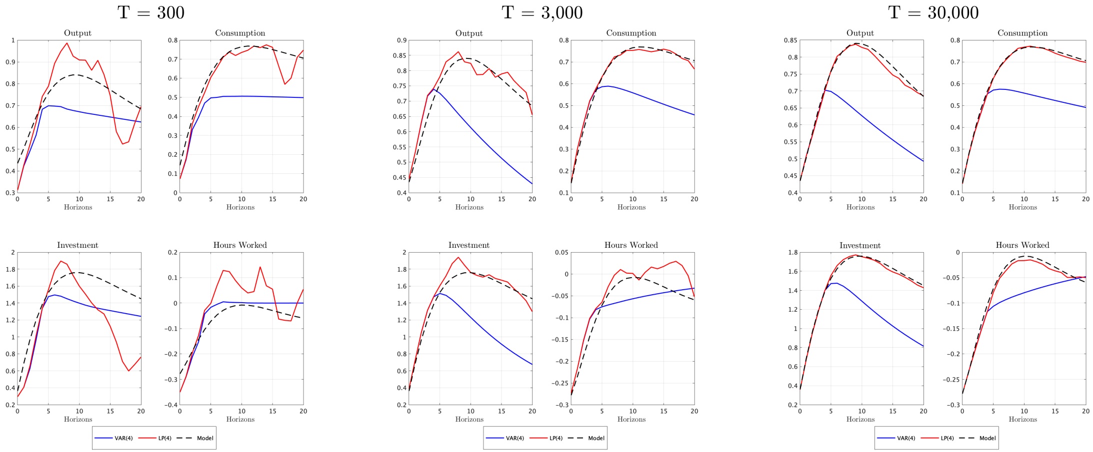


Fiscal Policy Monetary Policy

Technology Shock: Estimated IRFs for $T^S = 3,000$



Technology Shock: The Role of the Sample Size



Fiscal Policy Monetary Policy

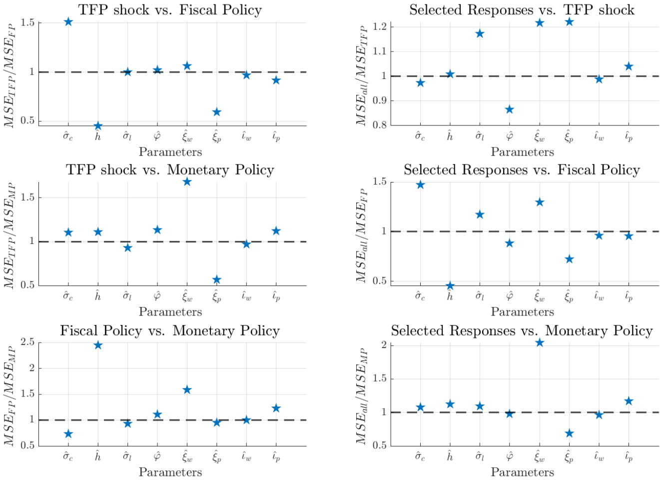
Results

The Local Projection Approach

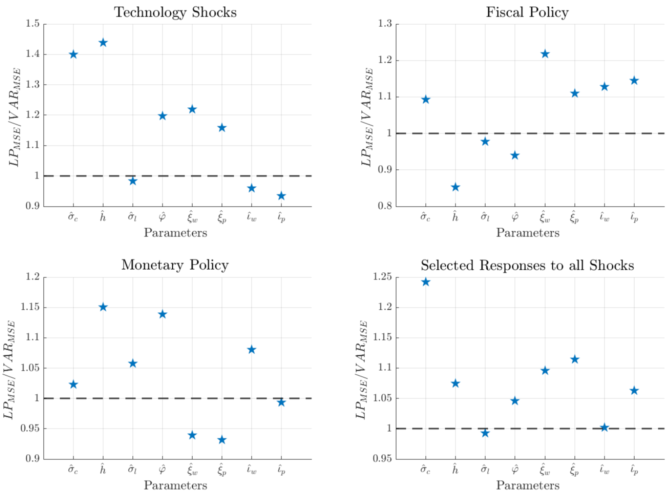
Table 2: SMM estimates using LP - IRFs

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
	Technology shock, ε_t^a							
Mean	1.35	0.79	2.66	5.73	0.67	0.59	0.48	0.16
Bias	0.08	-0.02	0.14	-0.58	-0.10	0.06	-0.05	-0.02
Std dev.	0.28	0.06	0.70	1.79	0.17	0.08	0.16	0.05
RMSE	0.29	0.06	0.71	1.89	0.19	0.10	0.17	0.06
	Fiscal Policy, ε_t^g							
Mean	1.40	0.80	2.74	6.41	0.63	0.43	0.50	0.15
Bias	0.13	-0.01	0.22	0.10	-0.14	-0.10	-0.03	-0.03
Std dev.	0.14	0.13	0.68	1.84	0.12	0.12	0.17	0.05
RMSE	0.19	0.13	0.71	1.84	0.18	0.16	0.18	0.06
	Monetary Policy, ε_t^m							
Mean	1.44	0.79	2.57	5.68	0.70	0.43	0.48	0.18
Bias	0.17	-0.02	0.05	-0.63	-0.06	-0.10	-0.06	-0.00
Std dev.	0.20	0.05	0.77	1.54	0.10	0.14	0.17	0.05
RMSE	0.26	0.05	0.77	1.66	0.12	0.17	0.18	0.05
	Selected Responses to All Shocks							
Mean	1.36	0.80	2.11	5.45	0.60	0.51	0.49	0.15
Bias	0.10	-0.01	-0.41	-0.86	-0.17	-0.02	-0.04	-0.02
Std dev.	0.26	0.06	0.73	1.38	0.17	0.12	0.16	0.05
RMSE	0.28	0.06	0.84	1.63	0.24	0.12	0.17	0.06

Relative Performance Across Different Shocks



Relative Performance: Local Projections vs SVAR



Overall Performance: Local Projections vs SVAR

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- The value of the loss function at the estimated parameters $\hat{\Theta}$, which is given by:

$$J(\hat{\Theta}) = \left(\mu^S(x_t;\hat{\Theta}) - \mu(x_t)\right)' W \left(\mu^S(x_t;\hat{\Theta}) - \mu(x_t)\right) \quad (4)$$

  is a good measure to assess the overall performance of the estimation.
  - ∗ Note that we fixed the maximum number of iterations in the optimization algorithm to 5000 and the tolerance for changes in Θ across consecutive iterations ('TolX') to $1e^{-4}$.

- We report below its average and its maximum value across all the S Monte Carlo draws.

Table 2: Overall performance ($S = 100$, $p = 4$, $T = 300$)

	Local Projections			Vector Autoregression		
	Avg. $J(\hat{\Theta})$	Max. $J(\hat{\Theta})$	Elapsed Time	Avg. $J(\hat{\Theta})$	Max. $J(\hat{\Theta})$	Elapsed Time
Technology Shock	86.93	117.61	17.38	83.71	245.40	69.90
Fiscal Policy	88.02	127.94	21.63	83.62	250.59	68.03
Monetary Policy	89.68	123.26	20.31	82.69	210.79	68.42
Selected Responses	86.28	123.03	20.11	81.97	237.03	62.49

Estimation: Empirical Applications

Applying the Local Projection Approach with Real Data

- We are interested in empirical estimates of the **dynamic response** of certain macroeconomic aggregates, e.g. output, consumption, investment and hours worked, **to empirically identified shocks**, e.g. technology, fiscal policy and monetary policy shocks, within Jordà's (2005) **local projection** framework.
- *Technology shocks*:
  - We use Ramey's (2016) estimates of the responses of *real GDP*, *consumption*, *non-residential investment* and *hours* to an unanticipated TFP shock, measured as in Francis, Owyang, Roush, and DiCecio (2014) (FORD).
- *Fiscal policy shocks*:
  - We also use Ramey's (2016) estimates of the responses of *GDP*, *non-durables + services consumption* and *non-residential investment* to a government spending shock identified using Blanchard and Perotti's (2002) method.
- *Monetary policy shocks*:
  - (TBC) : We will use the estimates reported in Tenreyro and Thwaites (2016).

Technology Shocks

Estimation Strategy

- **Fitted Parameters.** We try to identify the same eight parameters from the Monte Carlo study plus the two parameters governing the TFP shock. That makes a total of 10 *structural parameters*, $\Theta_{tfp} = \{\sigma_c, h, \sigma_l, \varphi, \xi_p, \xi_w, \iota_p, \iota_w, \rho_a, \sigma_a\}$.

- **Targeted Data Moments.** We use the response of real GDP, consumption, non-residential investment and hours to an unanticipated TFP shock as estimated in Ramey (2016). In particular, she specifies the following local projection for each of the dependent variables:

$$z_{t+h} = \alpha_h + \theta_h \cdot \text{ shock }_t + \varphi_h(L) y_{t-1} + \text{ quadratic trend } + \varepsilon_{t+h} \tag{5}$$

  This gives us $4 \times 21 = 84$ *moments*.

- **Auxiliary Econometric Model.** To obtain the model counterparts of our targeted moments, we simulate the model $T^s = 300,000$ times and use the simulated paths for output, consumption, investment and hours worked as well as the TFP shock to estimate equation (1), in which we set $\tilde{y}_t \in \{y_t, c_t, i_t, hw_t\}$, $\tilde{x}_t = \varepsilon_t^a$ and $p = 2$. The coefficients $\{\beta_h\}_{h=0}^{20}$ are then our simulated moments.

- Note that eq. (1) is slightly different from eq. (5). However, we think this is not problematic because the TFP series from the model is not subject to potential measurement error (which may lead endogeneity issues), so no additional controls are really needed.

Estimation Results

- **Bootstrapping.** We repeat the estimation described above 100 times to get a *distribution* over the structural parameter estimates.

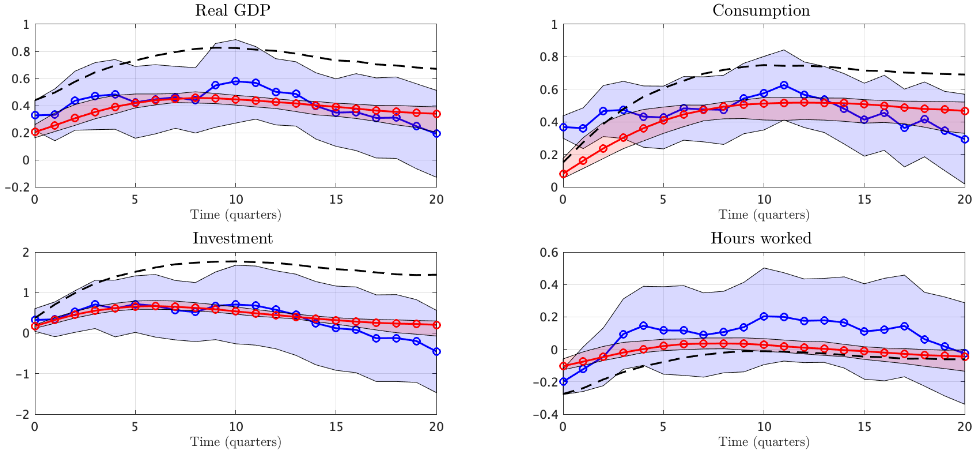
Table 3: Technology Shocks – SMM using the LP approach

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$	ρ_a	σ_a
Median	1.57	0.80	3.15	3.79	0.87	0.32	0.66	0.21	1.00	0.20
5th pctl.	0.77	0.57	1.54	3.79	0.82	0.32	0.56	0.12	1.00	0.17
95th pctl.	1.57	0.85	3.15	4.56	0.95	0.32	0.66	0.21	1.00	0.24

  - Note that the variation comes from the different draws of the shocks that we use to simulate the Smets and Wouters model.
  - Issue: we hit the upper bound for some parameters. When increasing the bounds, Blanchard-Kahn conditions are not satisfied. Need to find what is the problematic parameter combination.

- **Overall performance.** The mean J-statistic is 1.934, while its maximum value is 2.6325 across the 100 draws.

Model Validation: Empirical vs. Model Estimated IRFs



Only y_t, c_t, i_t Only c_t, i_t, hw_t

Fiscal Policy

Estimation Strategy

- **Fitted Parameters**. We try to identify the same eight parameters from the Monte Carlo study plus the two parameters controlling the government spending shock. That makes a total of 10 *structural parameters*, $\Theta_{fp} = \{\sigma_c, h, \sigma_l, \varphi, \xi_p, \xi_w, \iota_p, \iota_w, \rho_g, \sigma_g\}$.

- **Targeted Data Moments**. Ramey (2016) uses the same local projection approach as in the TFP shock case, with obviously different dependent variables and controls.
  - We match the response of GDP, non-durables + services consumption, non-residential investment and hours worked to an unanticipated government spending shock.
  - The total number of *moments* is $4 \times 21 = 84$.

- **Auxiliary Econometric Model**. We follow the same approach to estimate equation (1), in which we still set $\tilde{y}_t \in \{y_t, c_t, i_t, hw_t\}$ and $p = 2$, but $\tilde{x}_t = \varepsilon_t^g$. Hence, the vector of coefficients $\{\beta_h\}_{h=0}^{20}$ coming from that regression are our simulated moments.

Estimation Results

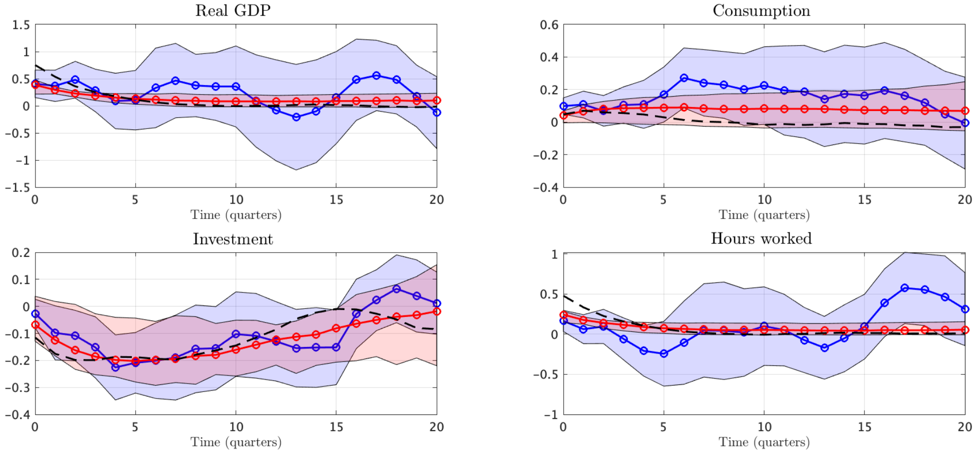
- **Bootstrapping.** As for the technology shock we repeat the estimation described above 100 times to get a *distribution* over the structural parameter estimates.

Table 4: Fiscal Policy – SMM using the LP approach

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$	ρ_g	σ_g
Median	1.57	0.90	1.51	7.89	0.93	0.66	0.53	0.20	0.75	0.33
5th pctl.	1.01	0.48	1.51	3.79	0.46	0.32	0.32	0.10	0.52	0.20
95th pctl.	1.57	1.00	1.53	7.89	0.95	0.66	0.66	0.21	1.00	0.39

- **Overall performance.** The mean value of the *J-statistic* is 3.036 while its maximum value is 4.397. Therefore, there is still some room for improvement.

Model Validation: Empirical vs. Estimated IRFs



Only y_t, c_t, i_t Only c_t, i_t, hw_t

Monetary Policy (TBC)

Concluding Remarks

What We Find

- The **local projection approach** to indirect inference can be a **valid alternative** for estimation of the economic parameters of any economic model.

- However, the traditional **VAR approach** to indirect inference is **superior**, at least for the particular application in this paper, i.e. the dgp being the Smets and Wouters (2007) model with sample size $T = 300$.
  * Misspecification is not an issue for indirect inference.
  * SVAR consistently get it wrong, while LP is more sensible to different draws. In other words, averaging out is more important for LPs that for VARs.

- When we test our local projection approach against real data, it does a good job and the model is still able to pick up the response of selected macro aggregates to technology and fiscal shocks.
  * Some of our parameters estimates hit the bounds. Thus, the reported confidence intervals are underestimated. Still need to do some more work here.
  * Key estimated parameters differ from those obtained by Smets and Wouters (2007).

Next steps

1. **Monte Carlo**
   - ∗ Indirect inference vs. IRFs matching
   - ∗ What are the implications of using different sample sizes $\{T, T^S\}$ and lag length p?
   - ∗ Non-linear data generating process, e.g. DSGE model with a ZLB constraint.

2. **Empirical Application**
   - ∗ Real world application using monetary policy shocks
   - ∗ How to combine the information of the three shocks? Which responses to match?

Appendix

The Log-Linearized Equilibrium Conditions I

- The aggregate resource constraint:

$$\hat{y}_t = c_y \hat{c}_t + i_y \hat{i}_t + z_y \hat{z}_t + \varepsilon_t^g$$

- The consumption Euler equation:

$$\hat{c}_t = \frac{h/\gamma}{1+h/\gamma}\hat{c}_{t-1} + \frac{1}{1+h/\gamma}\mathbb{E}_t\hat{c}_{t+1} + \frac{wl_c(\sigma_c-1)}{\sigma_c(1+h/\gamma)}\left(\hat{l}_t - \mathbb{E}_t\hat{l}_{t+1}\right) - \frac{1-h/\gamma}{(1+h/\gamma)\sigma_c}\left(\hat{r}_t - \mathbb{E}_t\hat{\pi}_{t+1}\right) - \frac{1-h/\gamma}{(1+h/\gamma)\sigma_c}\varepsilon_t^b$$

- The investment Euler equation:

$$\hat{i}_t = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\hat{i}_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}\mathbb{E}_t\hat{i}_{t+1} + \frac{1}{\varphi\gamma^2\left(1+\beta\gamma^{(1-\sigma_c)}\right)}\hat{q}_t + \varepsilon_t^i$$

- The arbitrage equation for the value of capital:

$$\hat{q}_t = \beta(1-\delta)\gamma^{-\sigma_c}\mathbb{E}_t\hat{q}_{t+1} - \hat{r}_t + \mathbb{E}_t\hat{\pi}_{t+1} + \left(1-\beta(1-\delta)\gamma^{-\sigma_c}\right)\mathbb{E}_t\hat{r}_{t+1}^k - \varepsilon_t^b$$

- The aggregate production function:

$$\hat{y}_t = \Phi\left(\alpha\hat{k}_t^s + (1-\alpha)\hat{l}_t + \varepsilon_t^a\right)$$

The Log-Linearized Equilibrium Conditions II

- Capital services:

$$\hat{k}_t^s = \hat{k}_{t-1} + \hat{z}_t$$

- Capital utilization:

$$\hat{z}_t = \frac{1-\psi}{\psi} \hat{r}_t^k$$

- The accumulation of installed capital:

$$\hat{k}_t = \frac{(1-\delta)}{\gamma} \hat{k}_{t-1} + (1-(1-\delta)/\gamma)\hat{i}_t + (1-(1-\delta)/\gamma)\varphi\gamma^2 \left(1+\beta\gamma^{(1-\sigma_c)}\right) \varepsilon_t^i$$

- Cost minimization by firms implies that the price mark up:

$$\hat{\mu}_t^p = \alpha \left(\hat{k}_t^s - \hat{l}_t\right) - \hat{w}_t + \varepsilon_t^a$$

- New Keynesian Phillips curve:

$$\hat{\pi}_t = \frac{\beta\gamma^{(1-\sigma_c)}}{1+\iota_p\beta\gamma^{(1-\sigma_c)}} \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\iota_p}{1+\beta\gamma^{(1-\sigma_c)}} \hat{\pi}_{t-1} - \frac{\left(1-\beta\gamma^{(1-\sigma_c)}\xi_p\right)(1-\xi_p)}{\left(1+\iota_p\beta\gamma^{(1-\sigma_c)}\right)(1+(\Phi-1)\varepsilon_p)\,\xi_p} \hat{\mu}_t^p + \varepsilon_t^p$$

The Log-Linearized Equilibrium Conditions III

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- Cost minimization by firms implies that the rental rate of capital:

$$\hat{r}_t^k = \hat{l}_t + \hat{w}_t - \hat{k}_t^s$$

- In the monopolistically competitive labor market, the wage mark-up

$$\hat{\mu}_t^w = \hat{w}_t - \sigma_l \hat{l}_t - \frac{1}{1-h/\gamma}\left(\hat{c}_t - h/\gamma \hat{c}_{t-1}\right)$$

- Wage adjustment:

$$\hat{w}_t = \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}\left(\mathbb{E}_t \hat{w}_{t+1} + \mathbb{E}_t \hat{\pi}_{t+1}\right) + \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\left(\hat{w}_{t-1} - \iota_w \hat{\pi}_{t-1}\right)$$
$$- \frac{1+\beta\gamma^{(1-\sigma_c)}\iota_w}{1+\beta\gamma^{(1-\sigma_c)}}\hat{\pi}_t - \frac{\left(1-\beta\gamma^{(1-\sigma_c)}\xi_w\right)\left(1-\xi_w\right)}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)\left(1+\left(\lambda_w - 1\right)\epsilon_w\right)\xi_w}\hat{\mu}_t^w + \varepsilon_t^u$$

- Empirical monetary policy reaction function:

$$\hat{r}_t = \rho \hat{r}_{t-1} + (1-\rho)\left(r_\pi \hat{\pi}_t + r_y\left(\hat{y}_t - \hat{y}_t^*\right)\right) + r_{\Delta y}\left(\left(\hat{y}_t - \hat{y}_t^*\right) - \left(\hat{y}_{t-1} - \hat{y}_{t-1}^*\right)\right) + \varepsilon_t^r.$$

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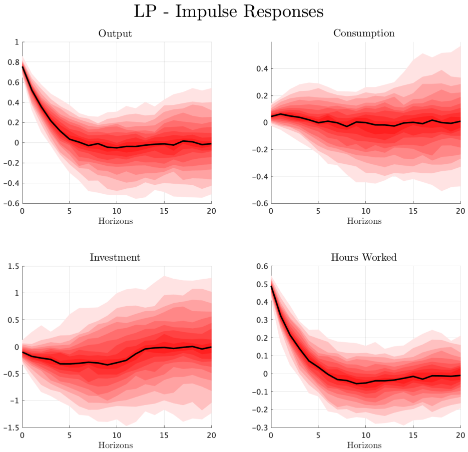
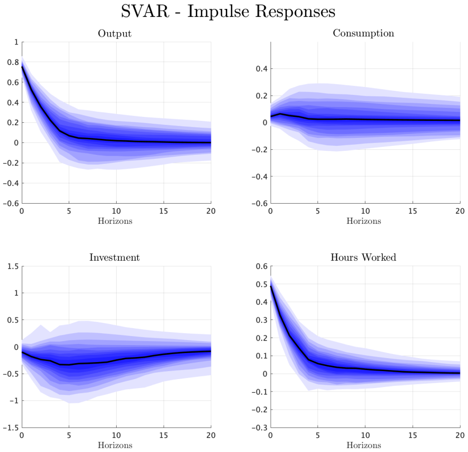
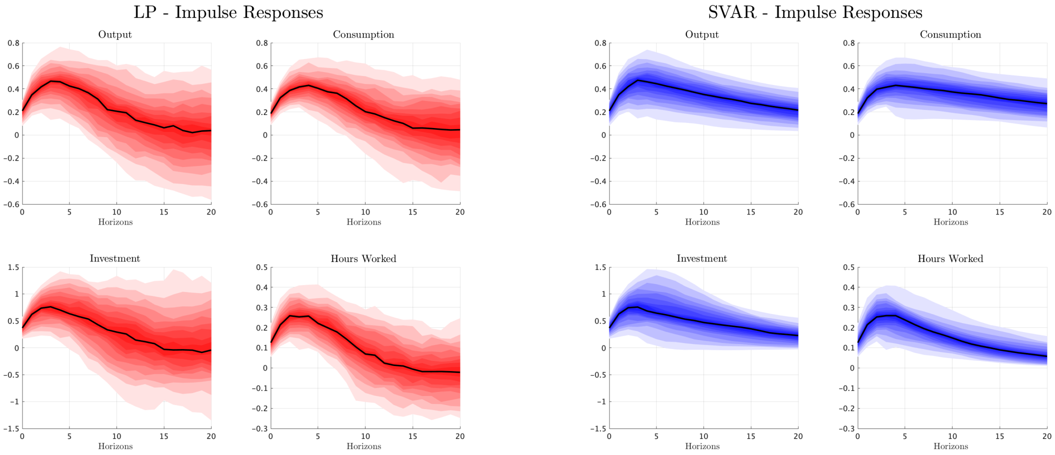
Fiscal Policy: Estimated IRFs for $S = 100$, $T = 300$





Monetary Policy: Estimated IRFs for $S = 100$, $T = 300$



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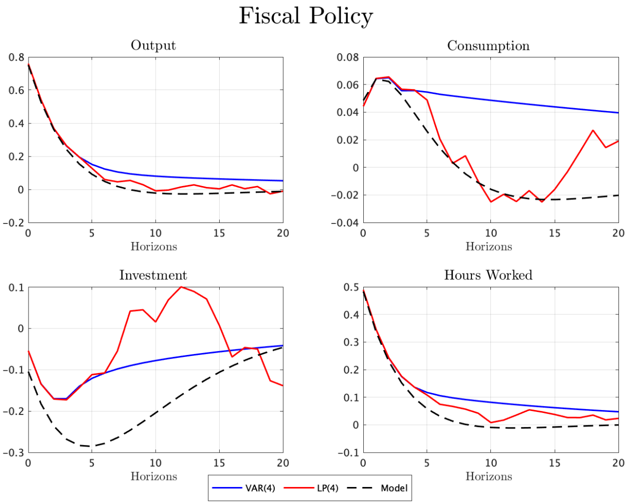
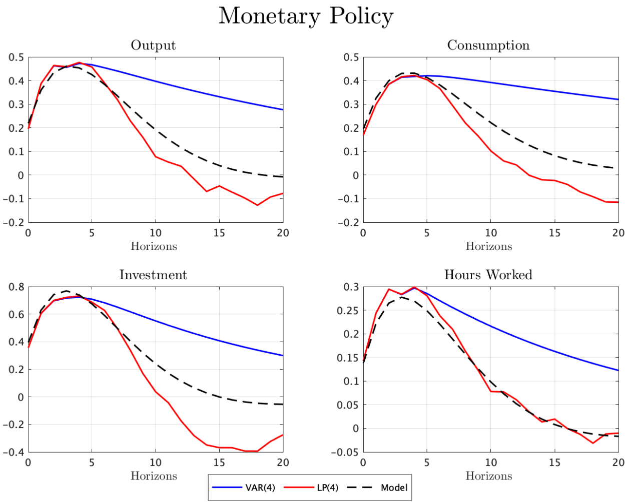
Fiscal Policy: Estimated IRFs for $T^S = 3,000$



Monetary Policy: Estimated IRFs for $T^S = 3,000$



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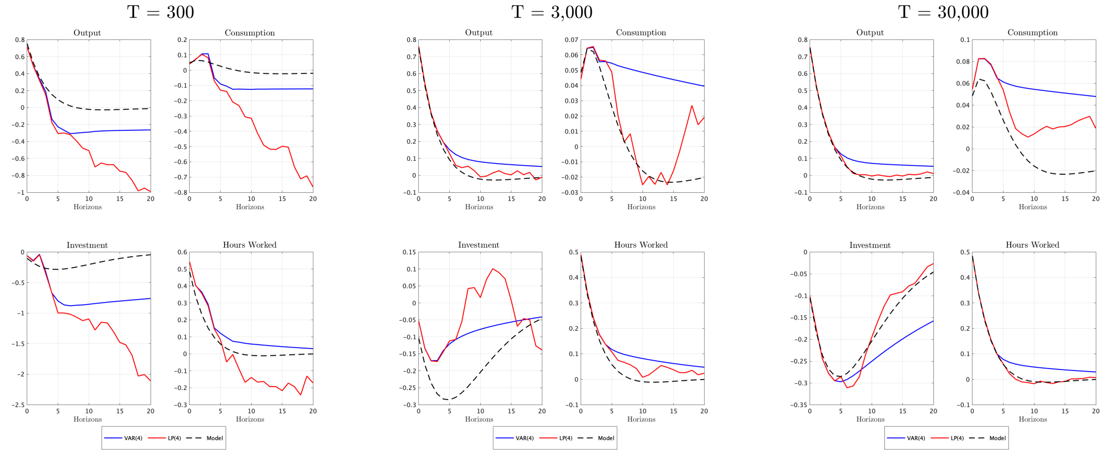
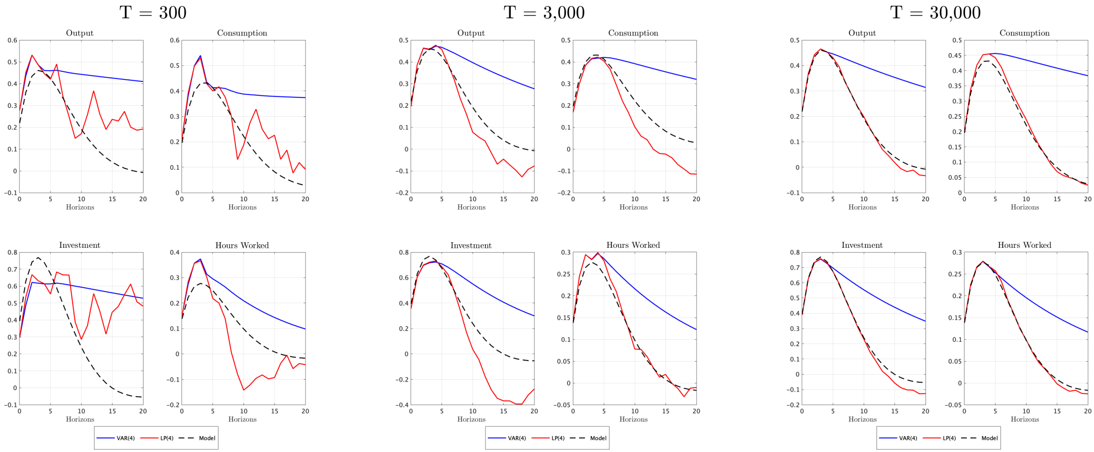
Fiscal Policy: The Role of the Sample Size



Monetary Policy: The Role of the Sample Size



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Monte Carlo Results: What do we report?

- Following Smith (1993), we compute for each of the estimated parameters $\hat{\theta}_h \in \hat{\Theta}$, the following *statistics*:

$$\text{Bias}_h \equiv \mathbb{E}\left[\hat{\theta}_h\right] - \theta_h$$

$$\text{Std dev}_h \equiv \sqrt{\text{Var}(\hat{\theta}_h)}$$

$$MSE_h^{1/2} \equiv \sqrt{\text{Bias}_h^2 + \text{Var}(\hat{\theta}_h)}$$

  where expectations are taken over the S Monte Carlo draws.

- In future work, it may be interesting to explore whether the **bias-variance trade off** documented in Li, Plagborg-Møller and Wolf (2021) for the parameters of our auxiliary models (VAR or LP) is also present in the estimated structural parameters. For example, by means of the following *loss function*:

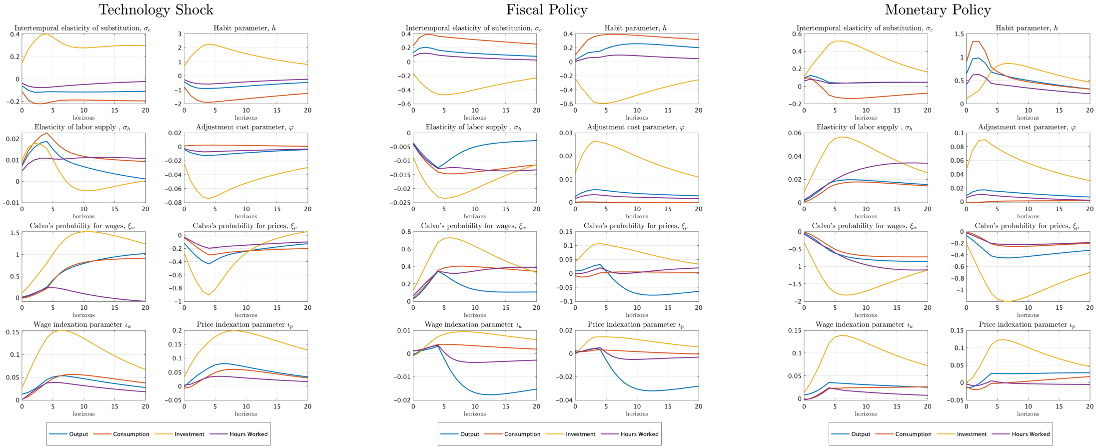
$$\mathcal{L}_\omega\left(\theta_h, \hat{\theta}_h\right) = \omega \times \left(\mathbb{E}\left[\hat{\theta}_h - \theta_h\right]\right)^2 + (1-\omega) \times \text{Var}\left(\hat{\theta}_h\right)$$

Monte Carlo Results: Vector Autoregression

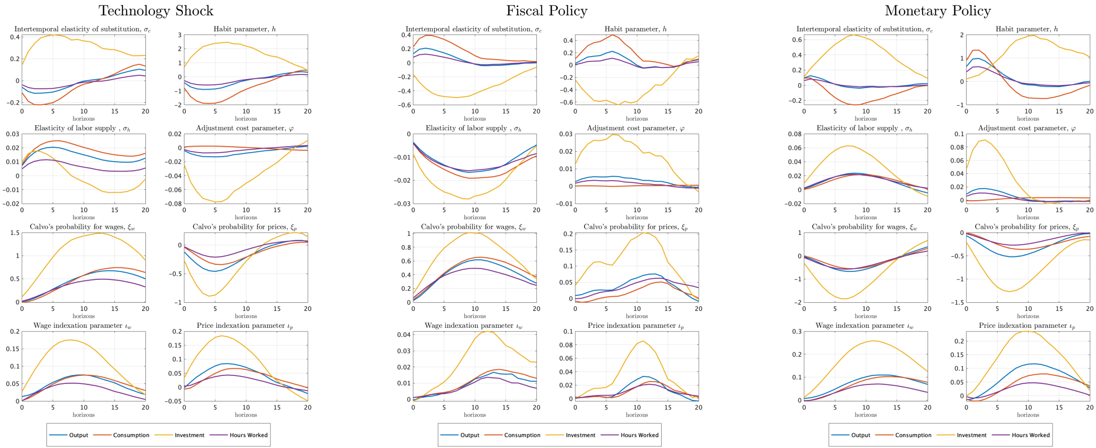
Table 3: SMM estimates using SVAR - IRFs

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
	Technology shock, ε_t^a							
Mean	1.24	0.81	2.59	6.78	0.67	0.52	0.45	0.15
Bias	-0.03	0.01	0.07	0.47	-0.10	-0.01	-0.09	-0.03
Std dev.	0.20	0.04	0.72	1.50	0.12	0.08	0.16	0.05
RMSE	0.21	0.04	0.73	1.57	0.16	0.08	0.18	0.06
	Fiscal Policy, ε_t^g							
Mean	1.35	0.78	2.50	5.90	0.67	0.48	0.56	0.17
Bias	0.08	-0.03	-0.02	-0.41	-0.09	-0.05	0.02	-0.01
Std dev.	0.16	0.15	0.73	1.92	0.12	0.14	0.15	0.05
RMSE	0.18	0.16	0.73	1.96	0.15	0.15	0.16	0.05
	Monetary Policy, ε_t^m							
Mean	1.29	0.80	2.57	5.70	0.69	0.37	0.53	0.17
Bias	0.02	-0.01	0.05	-0.62	-0.08	-0.16	-0.00	-0.00
Std dev.	0.25	0.05	0.72	1.32	0.09	0.08	0.16	0.05
RMSE	0.26	0.05	0.73	1.46	0.12	0.18	0.16	0.05
	Selected Responses to All Shocks							
Mean	1.25	0.81	2.14	5.59	0.60	0.51	0.49	0.16
Bias	-0.02	0.00	-0.38	-0.72	-0.17	-0.02	-0.04	-0.02
Std dev.	0.23	0.06	0.75	1.38	0.14	0.10	0.16	0.05
RMSE	0.23	0.06	0.84	1.56	0.22	0.10	0.17	0.06

Moments' Elasticities: Vector Autoregression IRFs



Moments' Elasticities: Local Projection IRFs



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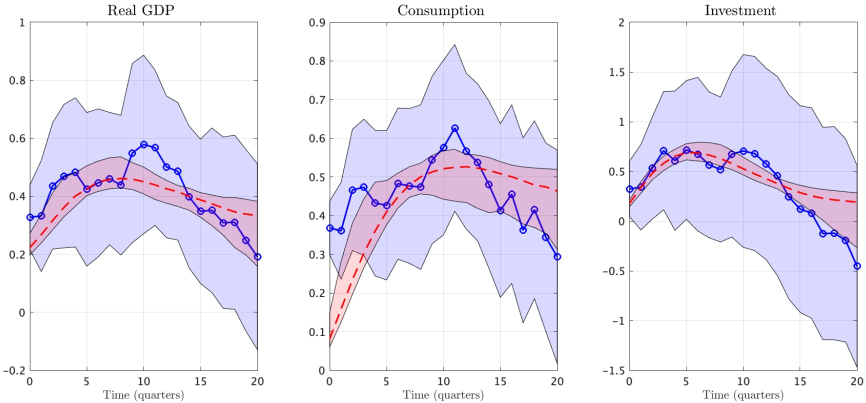
Technology Shock: What responses to match?

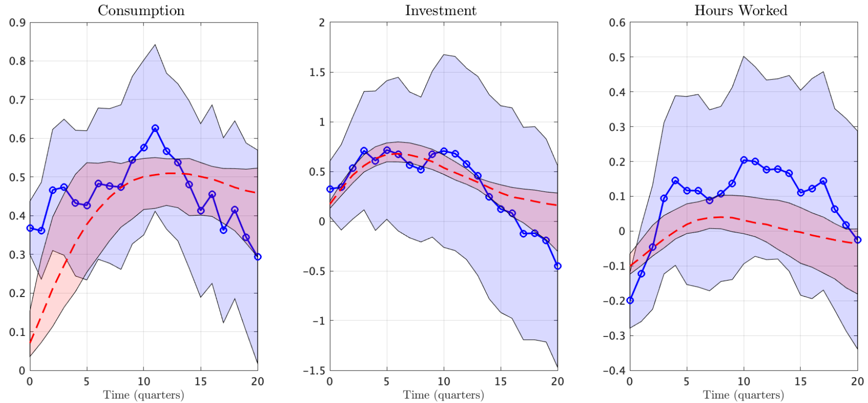
Table B: Technology Shock – SMM using the LP approach										
	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$	ρ_g	σ_g
	Matching the response of y_t, c_t, i_t, hw_t									
Median	1.57	0.80	3.15	3.79	0.87	0.32	0.66	0.21	1.00	0.20
5th pctl.	0.77	0.57	1.54	3.79	0.82	0.32	0.56	0.12	1.00	0.17
95th pctl.	1.57	0.85	3.15	4.56	0.95	0.32	0.66	0.21	1.00	0.24
	Matching the response of y_t, c_t, i_t									
Median	1.64	0.81	1.51	3.79	0.83	0.32	0.69	0.22	1.00	0.24
5th pctl.	0.76	0.72	1.51	3.79	0.77	0.32	0.49	0.11	1.00	0.21
95th pctl.	1.64	0.85	3.28	8.17	0.99	0.35	0.69	0.22	1.00	0.27
	Matching the response of c_t, i_t, hw_t									
Median	1.64	0.82	3.28	3.79	0.89	0.32	0.69	0.22	1.00	0.20
5th pctl.	0.78	0.64	2.56	3.79	0.83	0.32	0.63	0.11	1.00	0.13
95th pctl.	1.64	0.87	3.28	8.20	0.99	0.32	0.69	0.22	1.00	0.23

Technology Shock: Output, Consumption and Investment



- The average and maximum value for the **J-statistic** are 1.433 and 2.102, respectively.

Technology Shock: Consumption, Investment and Hours



- The average and maximum value for the **J-statistic** are 1.591 and 2.327, respectively.

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Fiscal Policy: What responses to match?

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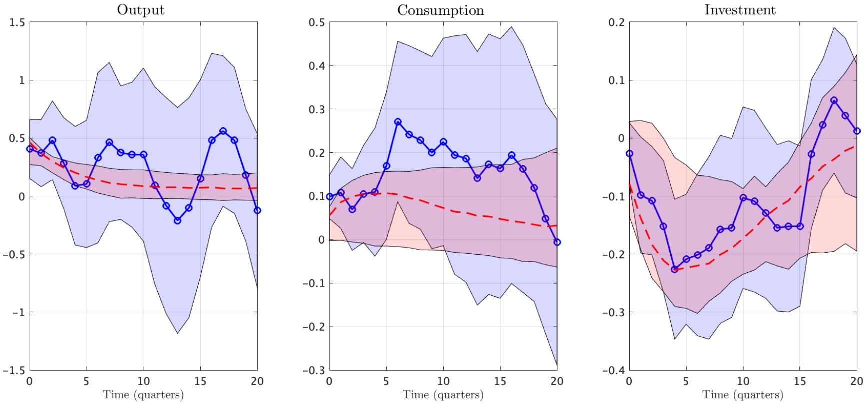
Table C: Fiscal Policy – SMM using the LP approach

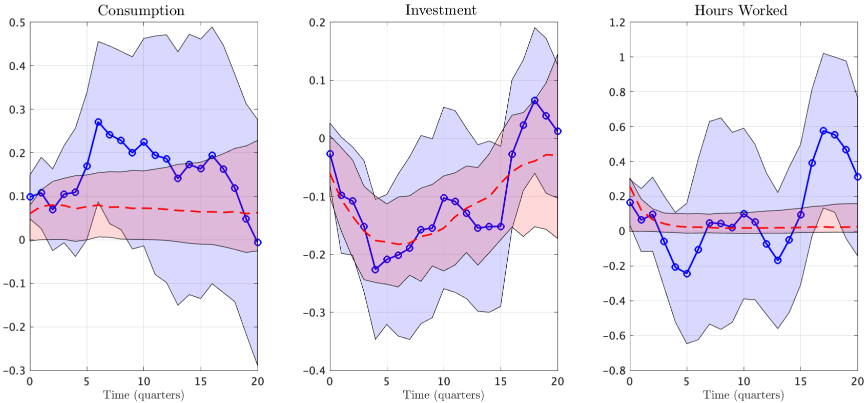
	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$	ρ_g	σ_g
	Matching the response of y_t, c_t, i_t, hw_t									
Median	1.57	0.90	1.51	7.89	0.93	0.66	0.53	0.20	0.75	0.33
5th pctl.	1.01	0.48	1.51	3.79	0.46	0.32	0.32	0.10	0.52	0.20
95th pctl.	1.57	1.00	1.53	7.89	0.95	0.66	0.66	0.21	1.00	0.39
	Matching the response of y_t, c_t, i_t									
Median	1.57	0.86	1.51	7.89	0.93	0.66	0.36	0.19	0.78	0.40
5th pctl.	0.99	0.48	1.51	4.49	0.46	0.32	0.32	0.10	0.60	0.24
95th pctl.	1.57	1.00	1.52	7.89	0.95	0.66	0.66	0.21	1.00	0.44
	Matching the response of c_t, i_t, hw_t									
Median	1.57	0.93	1.51	7.88	0.81	0.55	0.63	0.16	0.49	0.34
5th pctl.	0.88	0.48	1.51	3.79	0.46	0.32	0.32	0.10	0.19	0.00
95th pctl.	1.57	1.00	3.15	7.89	0.95	0.66	0.66	0.21	0.98	0.42

Fiscal Policy: Output, Consumption and Investment



- The average and maximum value for the **J-statistic** are 1.733 and 3.046, respectively.

Fiscal Policy: Consumption, Investment and Hours



- The average and maximum value for the **J-statistic** are 1.594 and 2.205, respectively.

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