Indirect Inference: A Local Projection Approach

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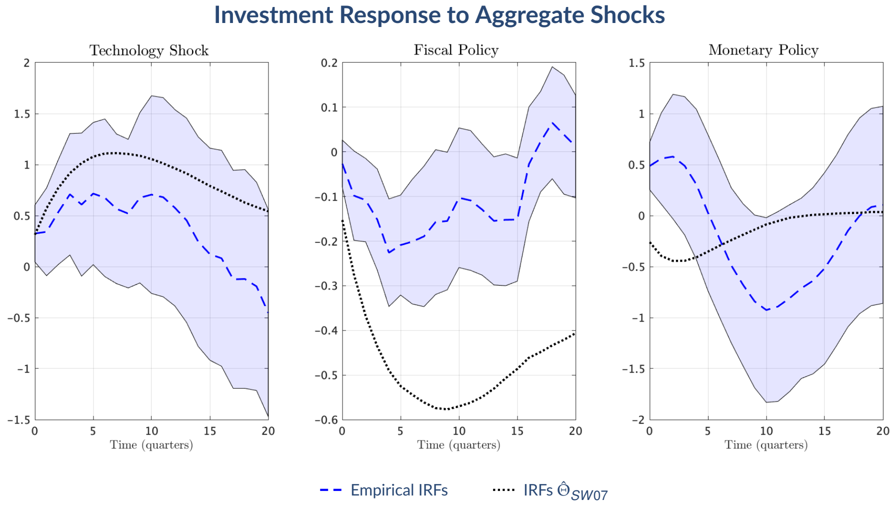
Motivation

- Can the Smets & Wouters (2007) model, a leading representation of the economy as a whole, capture the responses of key macro variables to aggregate shocks?
  - **The model IRFs do no match those from the data very well**
  - Take estimated local projections (LP) responses from other studies, e.g. Ramey (2016), Tenreyro & Thwaites (2016)

- There are at least three possible explanations:
  1. **Parameterization**
  2. **Model structure**
  3. **Empirical IRFs are wrong**

- This paper focuses on estimation and introduces a *new methodology* that consists in using **local projection coefficients** as moments in an **indirect inference** exercise.

Empirical IRFs vs. Smets & Wouters (2007)



What We Do

- To validate our methodology, we carry out a **Monte Carlo analysis** that compares our LP approach to indirect inference to that in Smith (1993) which uses VAR coefficients.
  * VARs and LPs estimate the same IRFs in population, but not in finite samples!
  * We analyze the small sample properties of our indirect inference estimator
  * Data generating process (DGP) $\rightarrow$ Smets & Wouters (2007) model
- Using our methodology, we **re-estimate the Smets & Wouters (2007) model** by targeting *empirically estimated* impulse responses to technology, fiscal or monetary shocks.
  * We compare our parameters to those reported in their paper
  * And evaluate if under our estimates we are able to get closer to the responses of key macro aggregates to aggregate shocks

What We Find

- Monte-Carlo results show that our LP approach to indirect inference produces **consistent** and **computationally efficient** estimates, but there are some trade-offs:
    - *RMSE*: some parameters are better identified through the VAR approach
    - *J-statistic*: maximum values are significantly lower, but average is larger than for the VARs
    - *Distance to structural IRFs*: at the LP estimates we are strikingly close! Not true for VARs

  $\Longrightarrow$ **The *LP approach* picks up much better the relevant parameters for the IRFs**

- The re-estimation of the Smets & Wouters (2007) model reveals that
    - The **small differences** we obtain in **parameter estimates**, although sometimes useful, are **not enough** to explain the disagreements between empirical and theoretical IRFs
    - Need a better model to study the dynamic responses to fiscal and monetary innovations

A Monte-Carlo Study

The Design

The DGP & the hyper-parameters

- The **log-linearized version of the Smets and Wouters (2007)** model is used to generate **100 repeated samples** of macroeconomic aggregates.
- The model is simulated each time at the estimated values from their paper using a sample of **300 observations**.
- We concentrate in **8 structural parameters** of the model:
  - σ_c : intertemporal elasticity of substitution
  - h : habit parameter
  - σ_l : elasticity of labor supply
  - φ : investment adjustment cost parameter
  - ξ_w, ξ_p : Calvo adjustment probabilities
  - ι_w, ι_p : Degree of indexation to past inflation
- **Simulated series are 10 times larger** than the sample size during the optimization.
- The importance of the coefficients used to summarize the data is weighted by the **inverse of its variance covariance matrix**.

The Moment Generating Functions

The auxiliary econometric model

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- We focus on the ***estimated* impulse responses** of four variables:
  * y_t : output
  * c_t : consumption
  * i_t : investment
  * hw_t : hours worked

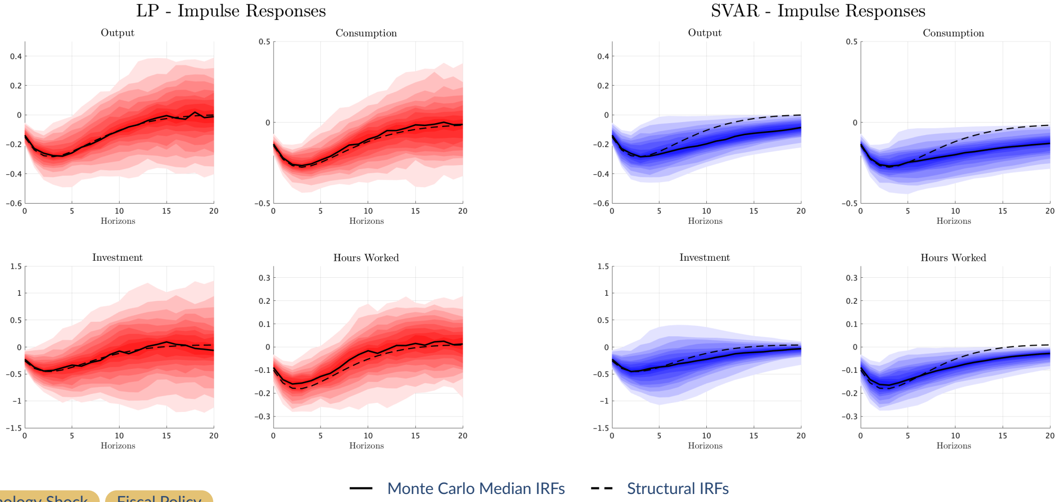
  to one (or a selection) of three following **shocks**:
  * ε_t^a : total factor productivity (TFP) shock
  * ε_t^g : fiscal policy (FP) shock
  * ε_t^m : monetary policy (MP) shock

- The IRFs are estimated using the traditional ***VAR + Cholesky decomposition*** (SVAR - IRFs) or the more recent ***Local Projections*** (LP - IRFs) approach.

- In either case, the econometrician still needs to decide on at least two more things:
  * The impulse response horizon, H. We set $H = 20$.
  * The number of lags, p. We set $p = 4$.

LP - IRFs SVAR - IRFs

Monetary Policy: estimated IRFs ($S = 100$, $T = 300$)



Technology Shock | Fiscal Policy

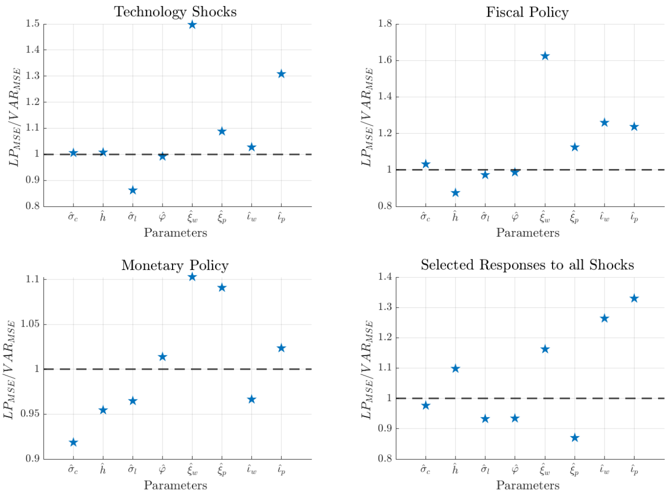
Results

The Local Projection approach

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
	Technology shock, ε_t^a							
Mean	1.23	0.82	2.81	5.91	0.57	0.62	0.48	0.15
Bias	-0.03	0.02	0.29	-0.40	-0.13	-0.04	-0.10	-0.09
Std dev.	0.26	0.10	0.61	1.74	0.15	0.07	0.17	0.05
RMSE	0.26	0.10	0.67	1.78	0.20	0.08	0.19	0.10
	Fiscal Policy, ε_t^g							
Mean	1.40	0.80	2.60	5.90	0.54	0.46	0.52	0.17
Bias	0.14	0.00	0.08	-0.41	-0.16	-0.20	-0.06	-0.07
Std dev.	0.23	0.09	0.70	1.85	0.11	0.14	0.17	0.05
RMSE	0.27	0.09	0.70	1.89	0.19	0.25	0.18	0.09
	Monetary Policy, ε_t^m							
Mean	1.38	0.79	2.36	5.52	0.62	0.53	0.47	0.16
Bias	0.12	-0.01	-0.16	-0.79	-0.08	-0.13	-0.11	-0.08
Std dev.	0.26	0.06	0.77	1.60	0.14	0.14	0.17	0.05
RMSE	0.28	0.06	0.79	1.78	0.15	0.19	0.20	0.09
	Selected Responses to All Shocks							
Mean	1.29	0.81	2.56	5.75	0.56	0.59	0.47	0.15
Bias	0.03	0.01	0.04	-0.56	-0.14	-0.07	-0.11	-0.09
Std dev.	0.25	0.09	0.74	1.40	0.14	0.10	0.17	0.05
RMSE	0.25	0.10	0.74	1.50	0.19	0.12	0.20	0.10

Definitions VAR Results Identification

Relative performance: Local Projections vs SVAR



Across Different Shocks

Overall performance: Local Projections vs SVAR

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- The value of the loss function at the estimated parameters $\hat{\Theta}$, which is given by:

$$J(\hat{\Theta}) = \left(\mu^S(x_t;\hat{\Theta}) - \mu(x_t)\right)' W \left(\mu^S(x_t;\hat{\Theta}) - \mu(x_t)\right) \quad (1)$$

is a good measure to assess the overall performance of the estimation.

	Local Projections			Vector Autoregression		
	Avg. $J(\hat{\Theta})$	Max. $J(\hat{\Theta})$	Elapsed Time	Avg. $J(\hat{\Theta})$	Max. $J(\hat{\Theta})$	Elapsed Time
Technology Shock	87.23	117.00	28.62	82.80	247.94	67.98
Fiscal Policy	87.72	129.96	24.68	86.28	251.90	52.49
Monetary Policy	88.58	121.48	23.38	82.77	221.87	53.04
Selected Responses	86.56	128.65	20.03	82.63	240.07	72.42

An alternative measure of overall performance

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- Neither of the two previous measures, RMSE and J-statistic, can inform us about how close we are from the structural impulse responses.
- Thus, we also look at the weighted distance between theoretical IRFs coming from the model at the estimated parameter values $\hat{\Theta}$ and at the true values Θ^*.

	Local Projections		Vector Autoregression	
	Avg. J^*	Max. J^*	Avg. J^*	Max. J^*
Technology Shock	2.57	9.43	34.67	228.41
Fiscal Policy	3.05	13.88	58.12	692.14
Monetary Policy	2.71	16.89	178.17	853.72
Selected Responses	8.37	44.69	230.46	1130.58

$\Longrightarrow$ **The LP-IRF approach to indirect inference does a significantly better job at picking those parameters that are relevant for the shape of the structural impulse response function**

Re-Estimating the Model

Targeting the estimated LP responses

- We target **empirically estimated IRFs** that have been identified using **local projections**.
- *Technology shocks* $\Longrightarrow$ Francis, Owyang, Roush, and DiCecio (2014)
  - $*$ We use Ramey's (2016) estimates of the responses of *real GDP*, *consumption*, *non-residential investment* and *hours* to an unanticipated TFP shock.
- *Fiscal policy shocks* $\Longrightarrow$ Blanchard and Perotti (2002)
  - $*$ We also use Ramey's (2016) estimates of the responses of *GDP*, *non-durables + services consumption* and *non-residential investment* to a government spending shock.
- *Monetary policy shocks* $\Longrightarrow$ Romer and Romer (2005)
  - $*$ We use the estimates reported in Tenreyro and Thwaites (2016). They report the responses of *GDP*, *non-durable and services consumption* and *fixed business investment*.

Ramey (2016) Tenreyro and Thwaites (2016)

All shocks

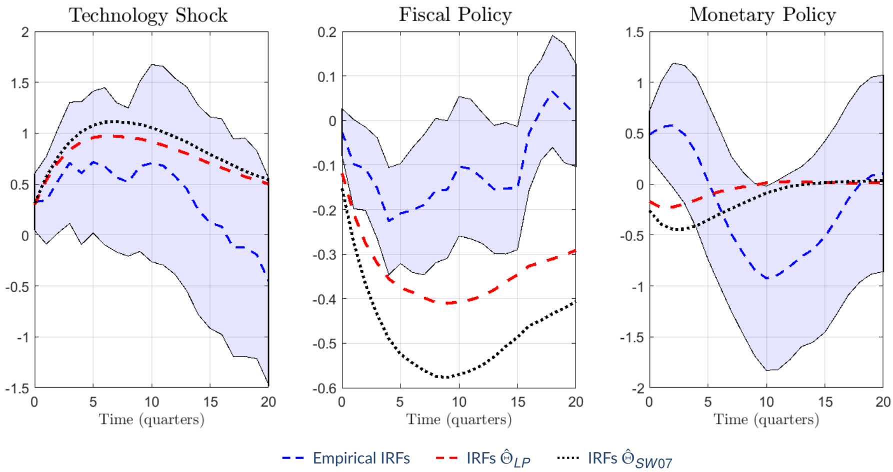
The Response of Investment

Estimation Strategy & Results

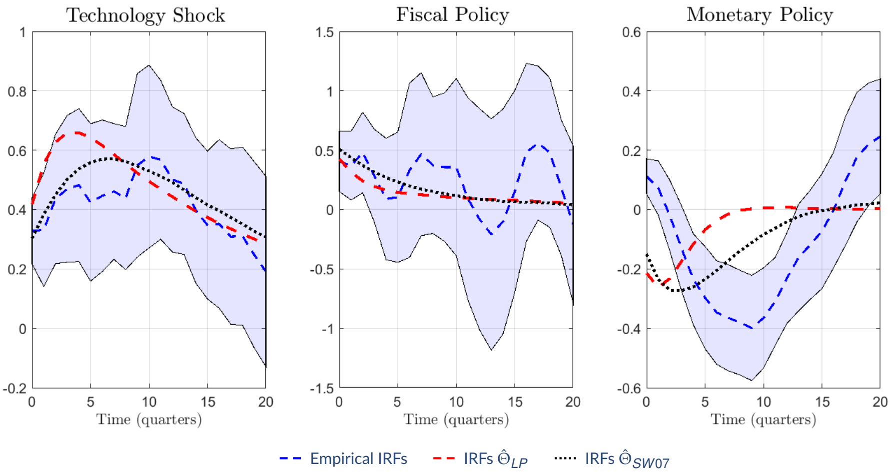
- We target the response of **only investment** to **all three shocks**: i) technology, ii) fiscal policy and iii) monetary policy shocks.
- We use the a diagonal weighting matrix whose entries are the inverse of the IRFs' standard deviation.
- Confidence intervals are obtained through bootstrapping.

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
S&W 2007	1.26	0.80	2.52	6.31	0.70	0.66	0.58	0.24
Median	1.00	0.62	3.15	5.89	0.42	0.40	0.43	0.14
10th pctl.	0.76	0.48	1.51	3.79	0.42	0.40	0.35	0.14
90th pctl.	1.57	0.90	3.15	7.89	0.86	0.82	0.72	0.30

Targeted Empirical vs. Estimated Investment IRFs



Untargeted Empirical vs. Estimated Output IRFs



Untargeted Consumption Response | Untargeted Hours Response

Conclusion

Main Takeaways: Monte Carlo

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- The Monte Carlo results show that LP approach produces **consistent** and **computationally efficient** estimates

- It **outperforms VARs as auxiliary models** in an indirect inference exercise
  * Despite the mixed evidence regarding RMSE and J-statistic . . .
  * The structural IRFs at the estimated parameters implied by the LP are closer to the truth.
  * *Implication*: LP approach picks better those parameters that are most relevant for the IRFs

- Overall, the LP approach to indirect inference is a valid alternative to estimation of any DSGE model

Main Takeaways: Re-Estimation

- Smets and Wouters (2007) does a good job in matching the responses to **technology shocks**, either at their parameters and ours.
- For **fiscal policy shocks**, our parameters reduce the crowding out effect on investment, bringing it closer to the data. However, the consumption response has a different sign.
  - Identification in the data? Recursive vs. Narrative
  - Model missing elements: heterogenous households and distortionary taxation
- For **monetary policy shocks**, it captures the effects of contractions during expansions but not during recesions.
  - This result is independent of the parametrization considered
  - Need a model that generates state-dependent responses to monetary policy
  - LP approach to indirect inference will be very useful to estimate such model

Thank You!

Questions/comments more than welcomed. Drop a line at **juan.castellanos@eui.eu**

Local Projections (LP - IRFs)

- Some notation:
  - Let $\tilde{y}_t \in \{y_t, c_t, i_t, hw_t\}$ denote one of response variables of interest.
  - Let $\tilde{x}_t \in \{\varepsilon_t^a, \varepsilon_t^g, \varepsilon_t^m\}$ denote the innovation of one of the three aggregate shocks.
  - Define the vector of controls $w_t = \{\tilde{x}_t, \tilde{y}_t\}$.
- Then, consider for each horizon $h = 0, 1, 2, \dots, H$ the *linear projections*:

$$\tilde{y}_{t+h} = \mu_h + \beta_h \tilde{x}_t + \sum_{\ell=1}^{p} \delta'_{h,\ell} w_{t-\ell} + \xi_{h,t} \tag{2}$$

  where $\xi_{h,t}$ is the projection residual and $\mu_h, \beta_h, \{\delta'_{h,\ell}\}_{\ell=1}^{p}$ are the projection coefficients.
- **Definition**. *The LP - IRFs of $\tilde{y}_t$ with respect to $\tilde{x}_t$ is given by $\{\beta_h\}_{h\geq 0}$ in the equation above.*

Structural Vector Autoregression (SVAR - IRFs)

- Consider the multivariate linear *VAR(p) projection*:

$$w_t = c + \sum_{\ell=1}^{p} A_\ell w_{t-\ell} + u_t \tag{3}$$

  where u_t is the projection residual and c, $\{A_\ell\}_{\ell=1}^{p}$ are the projection coefficients.

- Let $\Sigma_u \equiv \mathbb{E}\left[u_t u_t'\right]$ and define the *Cholesky decomposition* $\Sigma_u = BB'$ where B is lower triangular with positive diagonal entries.

- Consider the corresponding recursive *SVAR representation*:

$$A(L) w_t = c + B\eta \tag{4}$$

  where $A(L) = I - \sum_{\ell=1}^{p} A_\ell L^\ell$ and $\eta = B^{-1} u_t$. Define the lag polynomial $\sum_{\ell=0}^{p} C_\ell L^\ell = C(L) = A(L)^{-1}$.

- **Definition**. *The SVAR - IRFs of $\tilde{y}_t$ with respect to $\tilde{x}_t$ is given by $\{\theta_h\}_{h\geq 0}$ with $\theta_h \equiv C_{2,\bullet,h} B_{\bullet,1}$ where $\{C_\ell\}$ and B are defined above.*

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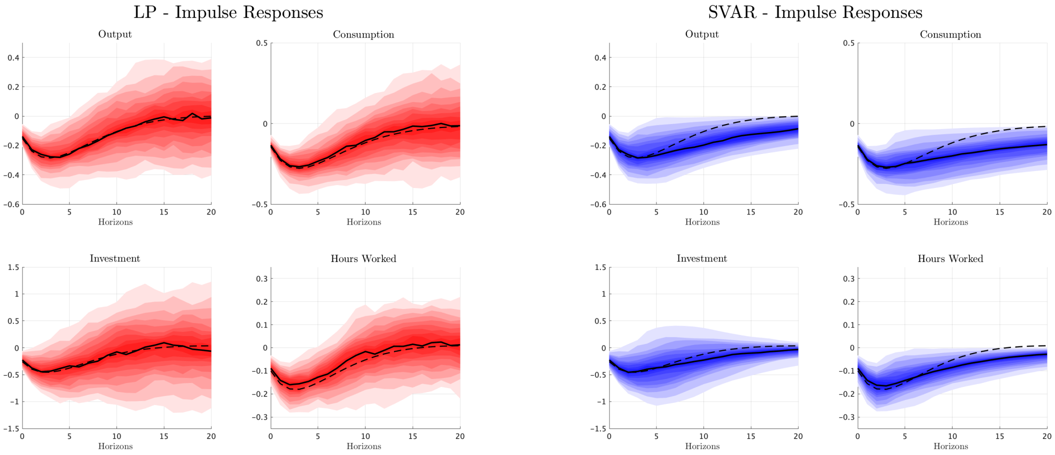
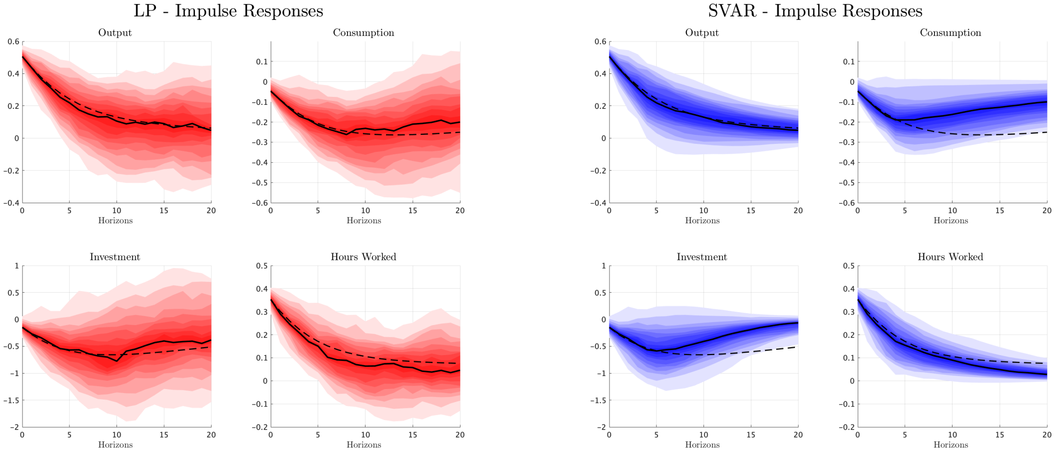
Technology Shock: Estimated IRFs ($S = 100$, $T = 300$)



— Monte Carlo Median IRFs - - Model/True IRFs

Fiscal Policy: Estimated IRFs ($S = 100$, $T = 300$)



— Monte Carlo Median IRFs - - Model/True IRFs

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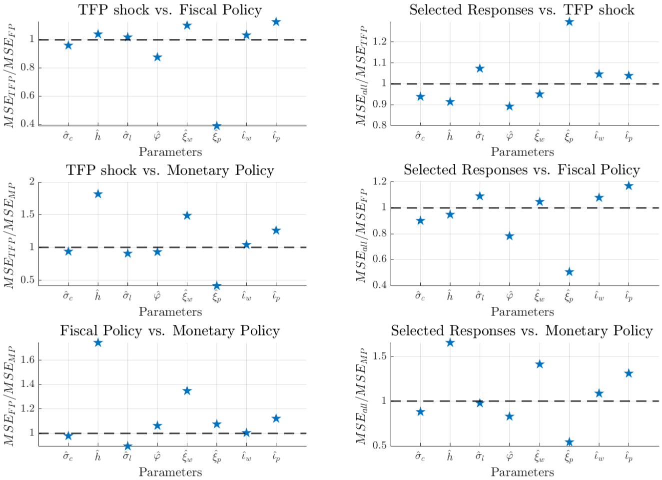
Relative Performance Across Different Shocks



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Monte Carlo results: what do we report?

- Following Smith (1993), we compute for each of the estimated parameters $\hat{\theta}_i \in \hat{\Theta}$, the following *statistics*:

$$\text{Bias}_i \equiv \mathbb{E}\left[\hat{\theta}_i\right] - \theta_i$$

$$\text{Std dev}_i \equiv \sqrt{\text{Var}(\hat{\theta}_i)}$$

$$RMSE_i \equiv \sqrt{\text{Bias}_i^2 + \text{Var}(\hat{\theta}_i)}$$

- Expectations are taken over the S Monte Carlo draws.
- We mainly focus on the RMSE as it summarizes the information of both bias and variance.

Monte Carlo Results: Vector Autoregression

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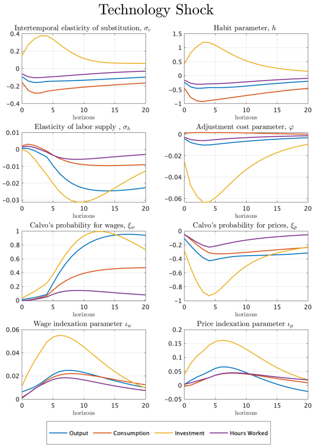
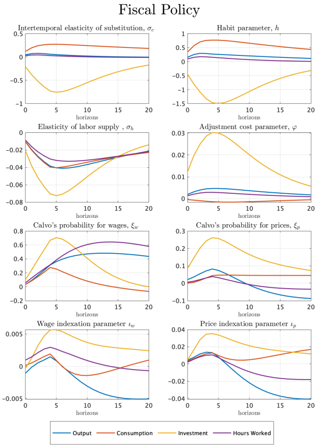
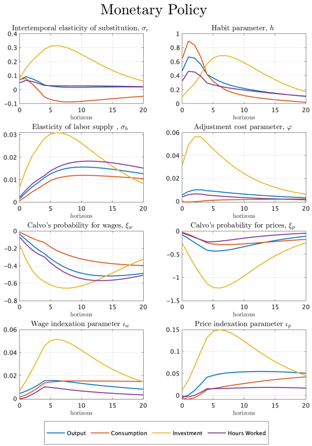
	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
	Technology shock, ε_t^a							
Mean	1.28	0.82	2.36	6.81	0.64	0.62	0.43	0.14
Bias	0.02	0.02	-0.16	0.50	-0.06	-0.04	-0.15	-0.10
Std dev.	0.20	0.08	0.77	1.46	0.10	0.07	0.15	0.05
RMSE	0.20	0.08	0.78	1.54	0.12	0.08	0.21	0.11
	Fiscal Policy, ε_t^g							
Mean	1.32	0.82	2.65	5.44	0.68	0.47	0.52	0.18
Bias	0.06	0.02	0.13	-0.87	-0.02	-0.19	-0.06	-0.06
Std dev.	0.25	0.11	0.68	1.82	0.07	0.16	0.16	0.05
RMSE	0.25	0.11	0.70	2.02	0.08	0.25	0.17	0.07
	Monetary Policy, ε_t^m							
Mean	1.32	0.79	2.39	5.54	0.66	0.48	0.50	0.17
Bias	0.06	-0.01	-0.13	-0.77	-0.04	-0.18	-0.08	-0.07
Std dev.	0.27	0.06	0.80	1.63	0.09	0.14	0.17	0.05
RMSE	0.27	0.06	0.81	1.80	0.10	0.23	0.18	0.08
	Selected Responses to All Shocks							
Mean	1.21	0.85	2.57	6.45	0.58	0.58	0.50	0.15
Bias	-0.05	0.05	0.05	0.14	-0.12	-0.08	-0.08	-0.09
Std dev.	0.24	0.08	0.75	1.39	0.12	0.10	0.16	0.05
RMSE	0.24	0.09	0.75	1.40	0.17	0.13	0.18	0.10

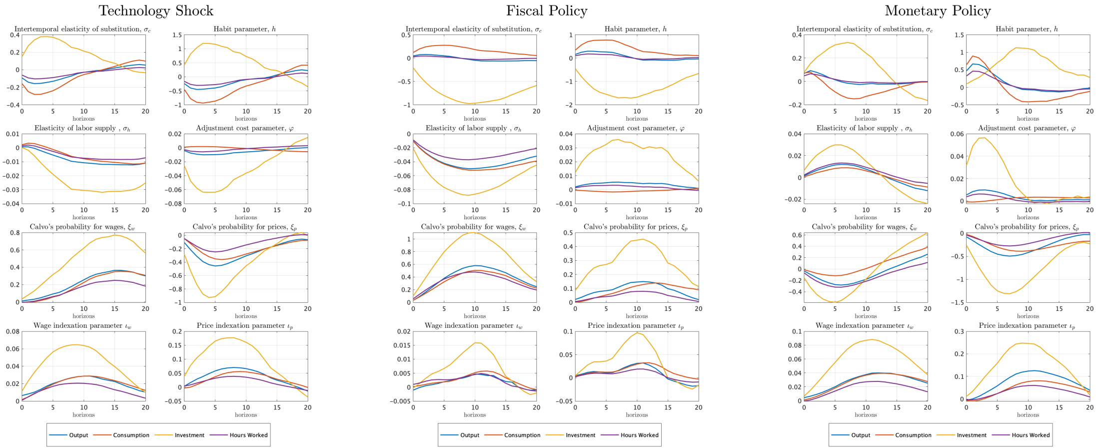
Moments' Sensitivity: Vector Autoregression IRFs







Moments' Sensitivity: Local Projection IRFs



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Ramey (2016)

Technology & Fiscal Policy Shocks

Estimation Strategy

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- **Structural Parameters.** Same eight from the Monte Carlo study, $\Theta = \{\sigma_c, h, \sigma_l, \varphi, \xi_p, \xi_w, \iota_p, \iota_w\}$.
- **Local Projection Regression:**

$$z_{t+h} = \alpha_h + \theta_h \cdot \text{ shock }_t + \varphi_h(L)y_{t-1} + \text{ quadratic trend } + \varepsilon_{t+h} \tag{5}$$

  - *Technology shocks*: medium run restrictions as in FORD (2014)
  - *Fiscal policy shocks*: government spending pre-determined as in Blanchard and Perotti (2006)
- **Targeted Coefficients**: only those identifying the responses, i.e. θ_h
- **Auxiliary Econometric Model.** Local Projection regression of the dependent variable on the shock and p lags each of the shock and the dependent variable
  - No need for extra controls because shock comes directly form the model. No measurement error.
- **Weighting Matrix.** We use a diagonal matrix whose entries coincide with the inverse of the IRFs' standard deviation

Estimation Results

	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
S&W 07	1.26	0.80	2.52	6.31	0.70	0.66	0.58	0.24
	Technology Shocks							
Median	0.85	0.69	3.28	8.20	0.44	0.59	0.47	0.14
10th pctl.	0.76	0.48	1.51	3.79	0.42	0.40	0.35	0.14
90th pctl.	1.36	0.89	3.28	8.20	0.84	0.86	0.75	0.31
	Fiscal Policy							
Median	1.23	0.85	1.51	5.14	0.52	0.59	0.41	0.15
10th pctl.	0.90	0.55	1.51	3.79	0.42	0.40	0.35	0.14
90th pctl.	1.57	0.97	2.18	7.89	0.85	0.82	0.72	0.30

Model Validation: Empirical vs. Model Estimated IRFs

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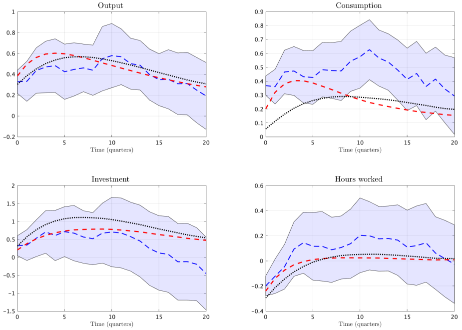
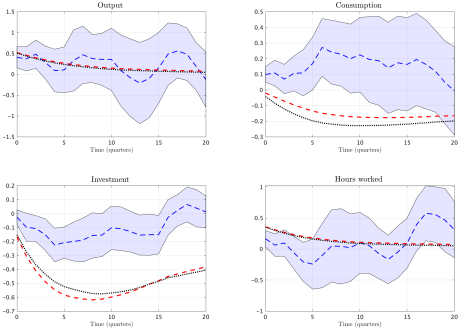
(a) Technology Shocks



(b) Fiscal Policy



Empirical IRFs   IRFs $\hat{\Theta}_{LP}$ IRFs $\hat{\Theta}_{SW07}$

Tenreyro & Thwaites (2016)

Monetary Policy Shocks

Estimation Strategy

- The **responses** of output, consumption and investment **to monetary policy are state-dependent**.
- The Smets and Wouters model does not contain the relevant non-linearities to capture these effects, so why should we care?
- Because it is still constructive to match the non-linear responses since **results may inform us about which parts of the model are more reactive to this state-dependance**, i.e. those in which the parameters differ the most across the two scenarios.
- Tenreyro and Thwaites (2016) empirical specification:

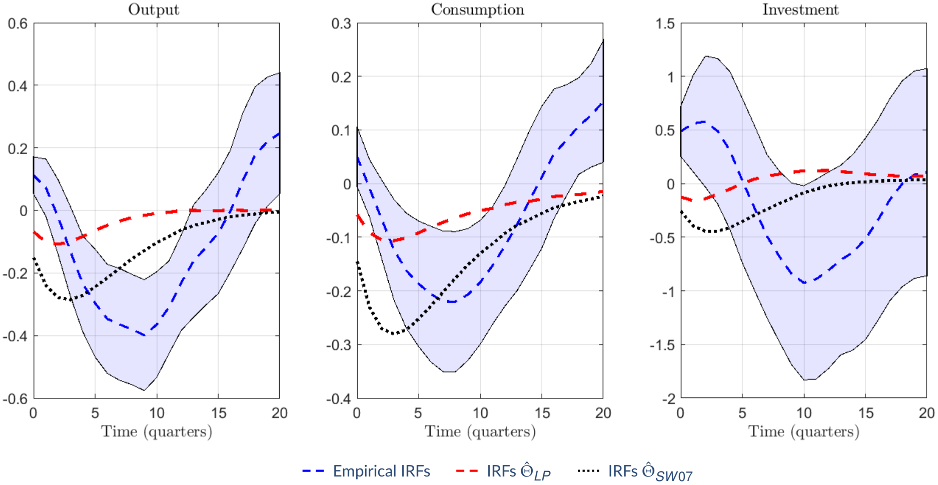
$$y_{t+h} = \tau_t + F(z_t)\left(\alpha_h^b + \beta_h^b \varepsilon_t + \gamma^{\mathbf{b}'} \mathbf{x_t}\right) + (1 - F(z_t))\left(\alpha_h^r + \beta_h^r \varepsilon_t + \gamma^{\mathbf{r}'} \mathbf{x_t}\right) + u_t \qquad (6)$$

- We target β_h^b and β_h^r in two independent estimation exercises using a linear LP model on model simulated data.

Estimation Results

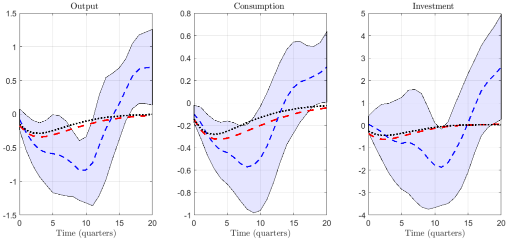
	$\hat{\sigma}_c$	$\hat{h}$	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	$\hat{\iota}_w$	$\hat{\iota}_p$
S&W 2007	1.26	0.80	2.52	6.31	0.70	0.66	0.58	0.24
	Linear Model							
Median	1.26	0.91	3.15	7.89	0.46	0.32	0.32	0.10
10th pctl.	0.85	0.80	1.51	5.94	0.46	0.32	0.32	0.10
90th pctl.	1.57	0.98	3.15	7.89	0.76	0.66	0.66	0.21
	Non-Linear Model: Expansion							
Median	1.57	0.76	1.51	4.06	0.72	0.66	0.32	0.10
10th pctl.	0.76	0.64	1.51	3.79	0.46	0.32	0.32	0.10
90th pctl.	1.57	0.94	3.15	7.89	0.90	0.66	0.66	0.21
	Non-Linear Model: Recession							
Median	1.57	0.91	3.15	7.89	0.46	0.32	0.66	0.21
10th pctl.	0.90	0.79	1.51	4.83	0.46	0.32	0.32	0.10
90th pctl.	1.57	0.98	3.15	7.89	0.77	0.56	0.66	0.21

Linear Model: Empirical vs. Estimated IRFs

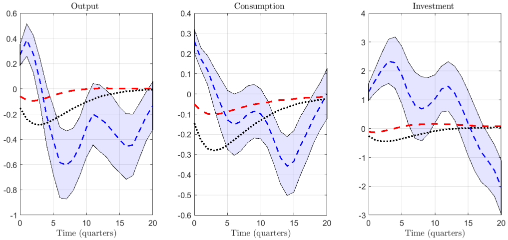


Non-Linear Model: Empirical vs. Estimated IRFs

(a) Non-Linear: Expansion



(b) Non-Linear: Recession



Empirical IRFs   IRFs $\hat{\Theta}_{LP}$ IRFs $\hat{\Theta}_{SW07}$

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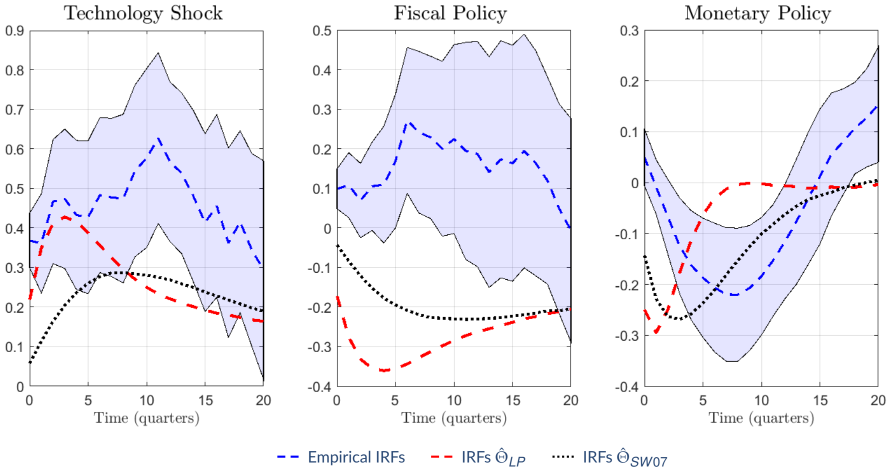
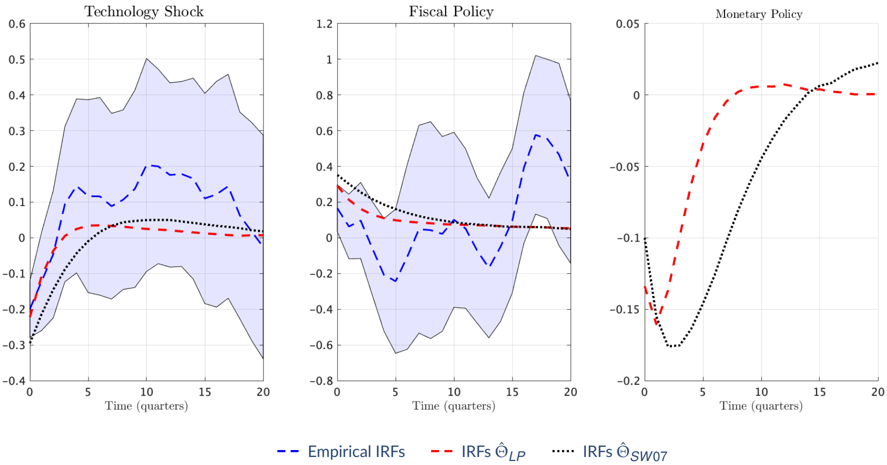
Untargeted Empirical vs. Estimated Consumption IRFs



Untargeted Empirical vs. Estimated Hours Worked IRFs



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