

Indirect Inference: A Local Projection Approach

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- Starting with Jordà (2005), **local projections** (LP) have become a common tool to understanding the dynamic effects of economic shocks
 - * An alternative to vector autorregresions (VARs)
- Other studies analyze the performance of these two models when estimating IRFs
 - * VARs and LPs estimate the same impulse responses in population (Plagborg-Møller and Wolf, 2020)
 - * However, there is a bias-variance trade off in finite samples (Li et al., 2021)
- Our focus is instead on the **structural parameters** of any DSGE model
 - * Follow Smith (1993) in estimating structural parameters through an indirect inference exercise in which the auxiliary model is a macro-econometrics model
- How should we **choose between VARs and LPs** when estimating, via **indirect inference**, the structural parameters of our DSGE model?

- Monte Carlo analysis

- * Data generating process (DGP) → Smets & Wouters (2007) model
- * Analyze the small sample properties of our LP indirect inference estimator
- * Compare our LP approach to indirect inference to that in Smith (1993) which uses VARs

- Re-estimate the Smets & Wouters (2007) model

- * Target *empirically estimated* impulse responses to technology, fiscal or monetary shocks
- * Compare our parameters to those reported in their paper
- * Evaluate if at our estimates we are able to get closer to the responses of key macro aggregates to aggregate shocks

- Monte-Carlo results show that our LP approach to indirect inference produces **consistent** and **computationally efficient** estimates, but there are some trade-offs:
 - * RMSE: some parameters are better identified through the VAR approach
 - * J-statistic: maximum values are significantly lower, but average is larger than for the VARs
 - * Distance to structural IRFs: at the LP estimates we are strikingly close! Not true for VARs

⇒ **The LP approach picks up much better the relevant parameters for the IRFs**
- The re-estimation of the Smets & Wouters (2007) model reveals that
 - * The model at **Smets and Wouters (2007)** mean parameters **do not match** the recent **LP evidence**
 - * The **small differences** we obtain in **parameter estimates** are **not enough** to explain the disagreements between empirical and theoretical IRFs
 - * Need a better model to study the dynamic responses to fiscal and monetary innovations

A MONTE-CARLO STUDY

THE DESIGN

- The **log-linearized version of the Smets and Wouters (2007)** model is used to generate **100 repeated samples** of macroeconomic aggregates.
- The model is simulated each time at the estimated values from their paper using a sample of **300 observations**.
- We concentrate in **8 structural parameters** of the model:
 - * σ_c : intertemporal elasticity of substitution
 - * h : habit parameter
 - * σ_l : elasticity of labor supply
 - * φ : investment adjustment cost parameter
 - * $\tilde{\zeta}_w, \tilde{\zeta}_p$: Calvo adjustment probabilities
 - * ι_w, ι_p : Degree of indexation to past inflation
- **Simulated series are 10 times larger** than the sample size during the optimization.
- The importance of the coefficients used to summarize the data is weighted by the **inverse of its variance covariance matrix**.

THE MOMENT GENERATING FUNCTIONS

- We focus on the **estimated impulse responses** of four variables:

- * y_t : output

- * i_t : investment

- * c_t : consumption

- * hw_t : hours worked

to one (or a selection) of three following **shocks**:

- * ε_t^a : total factor productivity (TFP) shock

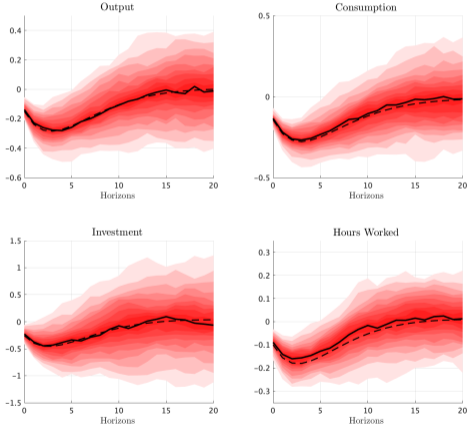
- * ε_t^g : fiscal policy (FP) shock

- * ε_t^m : monetary policy (MP) shock

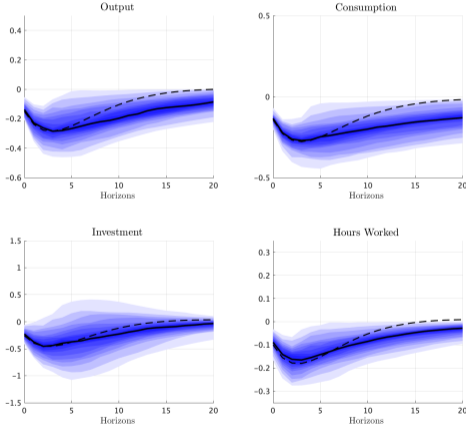
- The IRFs are estimated using the traditional **VAR + Cholesky decomposition** (SVAR - IRFs) or the more recent **Local Projections** (LP - IRFs) approach.
- In either case, the econometrician still needs to decide on at least two more things:
 - * The impulse response horizon, H . We set $H = 20$.
 - * The number of lags, p . We set $p = 4$.

Monetary Policy: estimated IRFs ($S = 100, T = 300$)

LP - Impulse Responses



SVAR - Impulse Responses

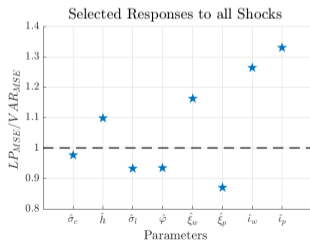
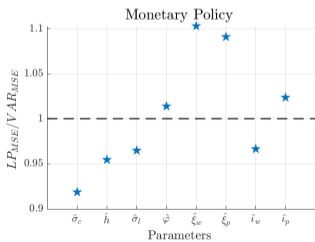
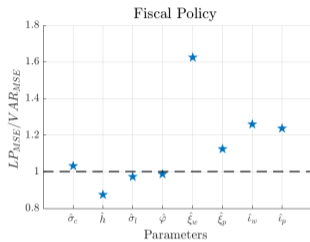
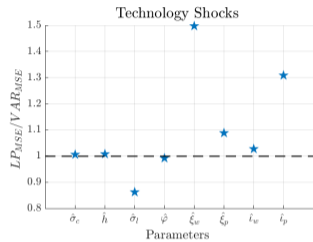


▷ Technology Shock ▷ Fiscal Policy

— Monte Carlo Median IRFs - - Structural IRFs

RESULTS

Relative performance: Local Projections vs SVAR



- The value of the loss function at the estimated parameters $\hat{\Theta}$, which is given by:

$$J(\hat{\Theta}) = \left(\mu^S(x_t; \hat{\Theta}) - \mu(x_t) \right)' W \left(\mu^S(x_t; \hat{\Theta}) - \mu(x_t) \right) \quad (1)$$

is a good measure to assess the overall performance of the estimation.

	Local Projections			Vector Autoregression		
	Avg. $J(\hat{\Theta})$	Max. $J(\hat{\Theta})$	Elapsed Time	Avg. $J(\hat{\Theta})$	Max. $J(\hat{\Theta})$	Elapsed Time
<i>Technology Shock</i>	87.23	117.00	28.62	82.80	247.94	67.98
<i>Fiscal Policy</i>	87.72	129.96	24.68	86.28	251.90	52.49
<i>Monetary Policy</i>	88.58	121.48	23.38	82.77	221.87	53.04
<i>Selected Responses</i>	86.56	128.65	20.03	82.63	240.07	72.42

- Neither of the two previous measures, RMSE and J-statistic, can inform us about how close we are from the structural impulse responses.
- Thus, we also look at the weighted distance between theoretical IRFs coming from the model at the estimated parameter values $\hat{\Theta}$ and at the true values Θ^* .

	Local Projections		Vector Autoregression	
	Avg. J^*	Max. J^*	Avg. J^*	Max. J^*
<i>Technology Shock</i>	2.57	9.43	34.67	228.41
<i>Fiscal Policy</i>	3.05	13.88	58.12	692.14
<i>Monetary Policy</i>	2.71	16.89	178.17	853.72
<i>Selected Responses</i>	8.37	44.69	230.46	1130.58

⇒ **The LP-IRF approach to indirect inference does a significantly better job at picking those parameters that are relevant for the shape of the structural impulse response function**

MEASUREMENT ERROR

- Assume that the **econometrician does not observe the true innovation** of the shock. She observes $\varepsilon_t^{m,obs} \neq \varepsilon_t^m$. We consider two possible specifications for the observed shock:

* **Uncorrelated** with any other shocks $\rightarrow \varepsilon_t^{m,obs} = \varepsilon_t^m + \sigma_v v_t$ where $v_t \sim \mathcal{N}(0, 1)$

	Avg. J	Max. J	Avg. J*	Max. J*
<i>Monetary Policy</i>	88.58	121.48	2.71	16.89
<i>Measurement Error ($\sigma_v = 0.25$)</i>	87.23	118.58	3.61	9.07
<i>Measurement Error ($\sigma_v = 0.5$)</i>	80.56	113.54	4.98	29.46

- Illusion of a better fit

* **Correlated** with the technology shock $\rightarrow \varepsilon_t^{m,obs} = \varepsilon_t^m + \rho_{a,m} \varepsilon_t^a$ where $\rho_{a,m} \in [0, 1]$

	Avg. J	Max. J	Avg. J*	Max. J*
<i>Technology & Monetary Policy</i>	91.15	124.33	5.01	89.67
<i>Measurement Error ($\rho_{a,m} = 0.25$)</i>	90.62	124.81	4.74	90.44
<i>Measurement Error ($\rho_{a,m} = 0.5$)</i>	90.74	127.86	4.25	21.00

- Technology shocks are better at identifying parameters

RE-ESTIMATING THE MODEL

- We target **empirically estimated IRFs** that have been identified using **local projections**.
- Technology shocks \implies Francis, Owyang, Roush, and DiCecio (2014)
 - * We use Ramey's (2016) estimates of the responses of *real GDP, consumption, non-residential investment* and *hours* to an unanticipated TFP shock.
- Fiscal policy shocks \implies Blanchard and Perotti (2002)
 - * We also use Ramey's (2016) estimates of the responses of *GDP, non-durables + services consumption* and *non-residential investment* to a government spending shock.
- Monetary policy shocks \implies Romer and Romer (2005)
 - * We use the estimates reported in Tenreyro and Thwaites (2016). They report the responses of *GDP, non-durable and services consumption* and *fixed business investment*.

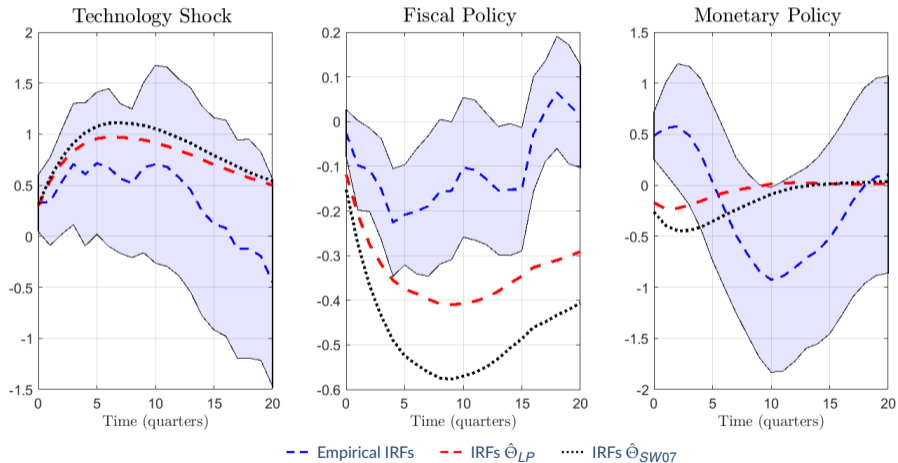
ALL SHOCKS

THE RESPONSE OF INVESTMENT

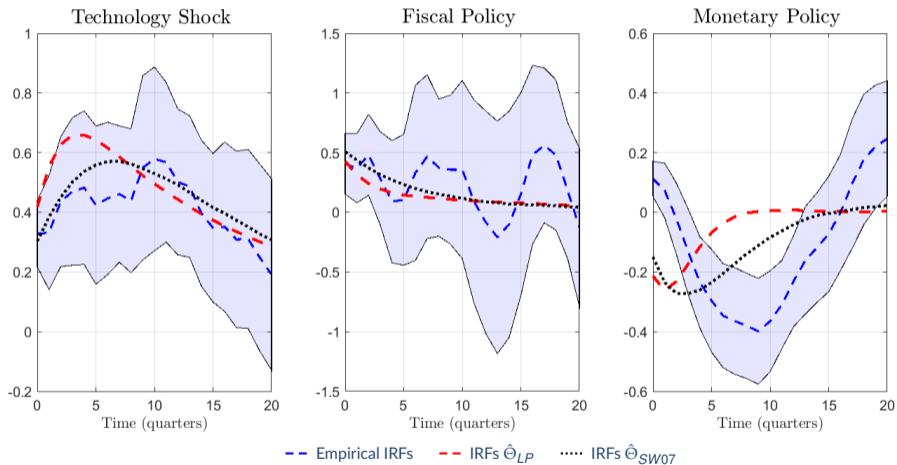
- We target the response of **only investment** to **all three shocks**: i) technology, ii) fiscal policy and iii) monetary policy shocks.
- We use the a diagonal weighting matrix whose entries are the inverse of the IRFs' standard deviation.
- Confidence intervals are obtained through bootstrapping.

	$\hat{\sigma}_c$	\hat{h}	$\hat{\sigma}_l$	$\hat{\phi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	\hat{l}_w	\hat{l}_p
S&W 2007	1.26	0.80	2.52	6.31	0.70	0.66	0.58	0.24
Median	1.00	0.62	3.15	5.89	0.42	0.40	0.43	0.14
10th <i>pctl.</i>	0.76	0.48	1.51	3.79	0.42	0.40	0.35	0.14
90th <i>pctl.</i>	1.57	0.90	3.15	7.89	0.86	0.82	0.72	0.30

Targeted Empirical vs. Estimated Investment IRFs



Untargeted Empirical vs. Estimated Output IRFs



▷ Untargeted Consumption Response

▷ Untargeted Hours Response

CONCLUSION

- The Monte Carlo results show that LP approach produces **consistent** and **computationally efficient** estimates
- It **outperforms VARs as auxiliary models** in an indirect inference exercise
 - * Despite the mixed evidence regarding RMSE and J-statistic . . .
 - * The structural IRFs at the estimated parameters implied by the LP are closer to the truth.
 - * Implication: LP approach picks better those parameters that are most relevant for the IRFs
- Overall, the LP approach to indirect inference is a valid alternative to estimation of any DSGE model

- Smets and Wouters (2007) does a good job in matching the responses to **technology shocks**, either at their parameters and ours.
- For **fiscal policy shocks**, our parameters reduce the crowding out effect on investment, bringing it closer to the data. However, the consumption response has a different sign.
 - * Identification in the data? Recursive vs. Narrative
 - * Model missing elements: heterogenous households and distortionary taxation
- For **monetary policy shocks**, it captures the effects of contractions during expansions but not during recessions.
 - * This result is independent of the parametrization considered
 - * Need a model that generates state-dependent responses to monetary policy
 - * LP approach to indirect inference will be very useful to estimate such model

APPENDIX

MOMENT GENERATING FUNCTIONS

- Some notation:

- * Let $\tilde{y}_t \in \{y_t, c_t, i_t, hw_t\}$ denote one of response variables of interest.
- * Let $\tilde{x}_t \in \{\varepsilon_t^a, \varepsilon_t^g, \varepsilon_t^m\}$ denote the innovation of one of the three aggregate shocks.
- * Define the vector of controls $w_t = \{\tilde{x}_t, \tilde{y}_t\}$.

- Then, consider for each horizon $h = 0, 1, 2, \dots, H$ the *linear projections*:

$$\tilde{y}_{t+h} = \mu_h + \beta_h \tilde{x}_t + \sum_{\ell=1}^p \delta'_{h,\ell} w_{t-\ell} + \zeta_{h,t} \quad (2)$$

where $\zeta_{h,t}$ is the projection residual and $\mu_h, \beta_h, \{\delta'_{h,\ell}\}_{\ell=1}^p$ are the projection coefficients.

- **Definition.** The LP - IRFs of \tilde{y}_t with respect to \tilde{x}_t is given by $\{\beta_h\}_{h \geq 0}$ in the equation above.

- Consider the multivariate linear VAR(p) projection:

$$w_t = c + \sum_{\ell=1}^p A_{\ell} w_{t-\ell} + u_t \quad (3)$$

where u_t is the projection residual and $c, \{A_{\ell}\}_{\ell=1}^p$ are the projection coefficients.

- Let $\Sigma_u \equiv \mathbb{E}[u_t u_t']$ and define the Cholesky decomposition $\Sigma_u = BB'$ where B is lower triangular with positive diagonal entries.
- Consider the corresponding recursive SVAR representation:

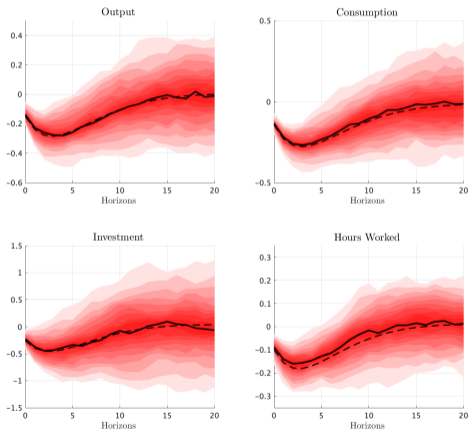
$$A(L)w_t = c + B\eta \quad (4)$$

where $A(L) = I - \sum_{\ell=1}^p A_{\ell} L^{\ell}$ and $\eta = B^{-1} u_t$. Define the lag polynomial $\sum_{\ell=0}^p C_{\ell} L^{\ell} = C(L) = A(L)^{-1}$.

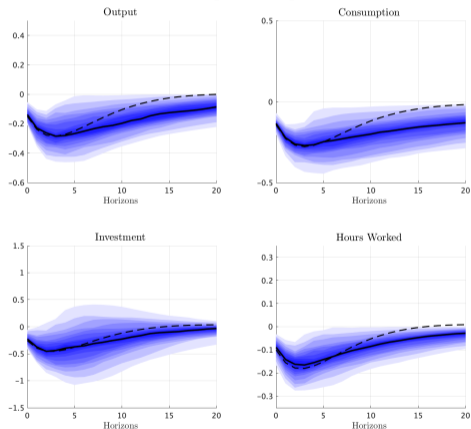
- **Definition.** The SVAR - IRFs of \tilde{y}_t with respect to \tilde{x}_t is given by $\{\theta_h\}_{h \geq 0}$ with $\theta_h \equiv C_{2, \bullet, h} B_{\bullet, 1}$ where $\{C_{\ell}\}$ and B are defined above.

Technology Shock: Estimated IRFs ($S = 100, T = 300$)

LP - Impulse Responses



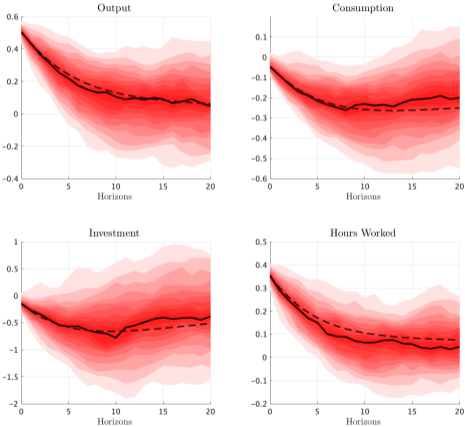
SVAR - Impulse Responses



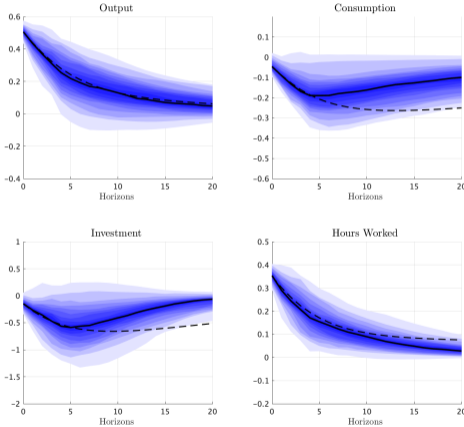
— Monte Carlo Median IRFs - - Model/True IRFs

Fiscal Policy: Estimated IRFs ($S = 100, T = 300$)

LP - Impulse Responses



SVAR - Impulse Responses



— Monte Carlo Median IRFs - - Model/True IRFs

MONTE CARLO RESULTS

- Following Smith (1993), we compute for each of the estimated parameters $\hat{\theta}_i \in \hat{\Theta}$, the following *statistics*:

$$\text{Bias}_i \equiv \mathbb{E} [\hat{\theta}_i] - \theta_i$$

$$\text{Std dev}_i \equiv \sqrt{\text{Var}(\hat{\theta}_i)}$$

$$\text{RMSE}_i \equiv \sqrt{\text{Bias}_i^2 + \text{Var}(\hat{\theta}_i)}$$

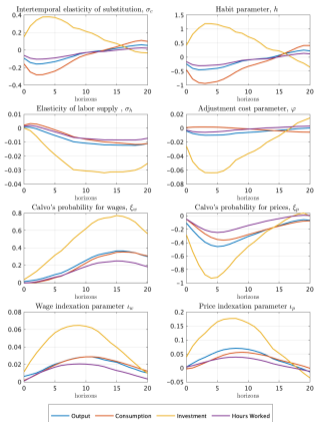
- Expectations are taken over the S Monte Carlo draws.
- We mainly focus on the RMSE as it summarizes the information of both bias and variance.

	$\hat{\sigma}_c$	\hat{h}	$\hat{\sigma}_l$	$\hat{\phi}$	$\hat{\zeta}_w$	$\hat{\zeta}_p$	\hat{i}_w	\hat{i}_p
Technology shock, ε_t^a								
<i>Mean</i>	1.23	0.82	2.81	5.91	0.57	0.62	0.48	0.15
<i>Bias</i>	-0.03	0.02	0.29	-0.40	-0.13	-0.04	-0.10	-0.09
<i>Std dev.</i>	0.26	0.10	0.61	1.74	0.15	0.07	0.17	0.05
<i>RMSE</i>	0.26	0.10	0.67	1.78	0.20	0.08	0.19	0.10
Fiscal Policy, ε_t^g								
<i>Mean</i>	1.40	0.80	2.60	5.90	0.54	0.46	0.52	0.17
<i>Bias</i>	0.14	0.00	0.08	-0.41	-0.16	-0.20	-0.06	-0.07
<i>Std dev.</i>	0.23	0.09	0.70	1.85	0.11	0.14	0.17	0.05
<i>RMSE</i>	0.27	0.09	0.70	1.89	0.19	0.25	0.18	0.09
Monetary Policy, ε_t^m								
<i>Mean</i>	1.38	0.79	2.36	5.52	0.62	0.53	0.47	0.16
<i>Bias</i>	0.12	-0.01	-0.16	-0.79	-0.08	-0.13	-0.11	-0.08
<i>Std dev.</i>	0.26	0.06	0.77	1.60	0.14	0.14	0.17	0.05
<i>RMSE</i>	0.28	0.06	0.79	1.78	0.15	0.19	0.20	0.09
Selected Responses to All Shocks								
<i>Mean</i>	1.29	0.81	2.56	5.75	0.56	0.59	0.47	0.15
<i>Bias</i>	0.03	0.01	0.04	-0.56	-0.14	-0.07	-0.11	-0.09
<i>Std dev.</i>	0.25	0.09	0.74	1.40	0.14	0.10	0.17	0.05
<i>RMSE</i>	0.25	0.10	0.74	1.50	0.19	0.12	0.20	0.10

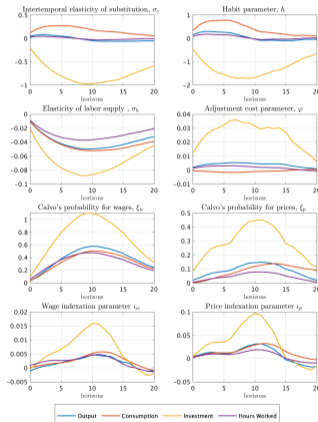
	$\hat{\sigma}_c$	\hat{h}	$\hat{\sigma}_l$	$\hat{\phi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	\hat{i}_w	\hat{i}_p
Technology shock, ε_t^a								
<i>Mean</i>	1.28	0.82	2.36	6.81	0.64	0.62	0.43	0.14
<i>Bias</i>	0.02	0.02	-0.16	0.50	-0.06	-0.04	-0.15	-0.10
<i>Std dev.</i>	0.20	0.08	0.77	1.46	0.10	0.07	0.15	0.05
<i>RMSE</i>	0.20	0.08	0.78	1.54	0.12	0.08	0.21	0.11
Fiscal Policy, ε_t^g								
<i>Mean</i>	1.32	0.82	2.65	5.44	0.68	0.47	0.52	0.18
<i>Bias</i>	0.06	0.02	0.13	-0.87	-0.02	-0.19	-0.06	-0.06
<i>Std dev.</i>	0.25	0.11	0.68	1.82	0.07	0.16	0.16	0.05
<i>RMSE</i>	0.25	0.11	0.70	2.02	0.08	0.25	0.17	0.07
Monetary Policy, ε_t^m								
<i>Mean</i>	1.32	0.79	2.39	5.54	0.66	0.48	0.50	0.17
<i>Bias</i>	0.06	-0.01	-0.13	-0.77	-0.04	-0.18	-0.08	-0.07
<i>Std dev.</i>	0.27	0.06	0.80	1.63	0.09	0.14	0.17	0.05
<i>RMSE</i>	0.27	0.06	0.81	1.80	0.10	0.23	0.18	0.08
Selected Responses to All Shocks								
<i>Mean</i>	1.21	0.85	2.57	6.45	0.58	0.58	0.50	0.15
<i>Bias</i>	-0.05	0.05	0.05	0.14	-0.12	-0.08	-0.08	-0.09
<i>Std dev.</i>	0.24	0.08	0.75	1.39	0.12	0.10	0.16	0.05
<i>RMSE</i>	0.24	0.09	0.75	1.40	0.17	0.13	0.18	0.10

Moments' Sensitivity: Local Projection IRFs

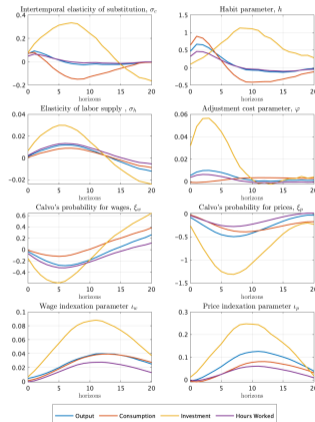
Technology Shock



Fiscal Policy

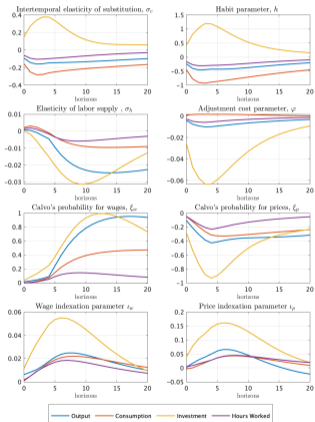


Monetary Policy

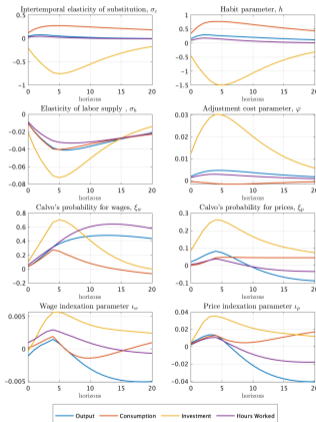


Moments' Sensitivity: Vector Autoregression IRFs

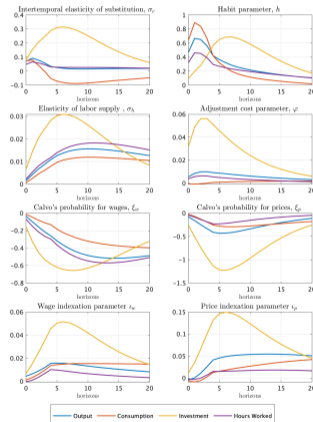
Technology Shock

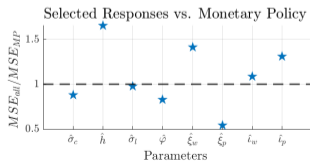
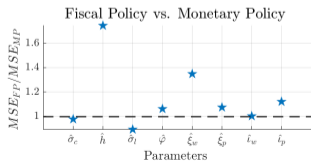
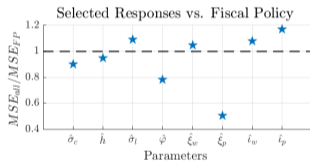
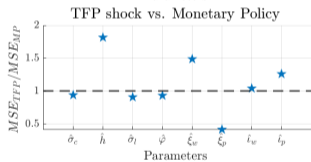
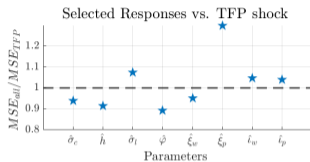
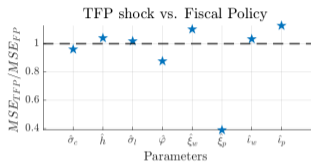


Fiscal Policy



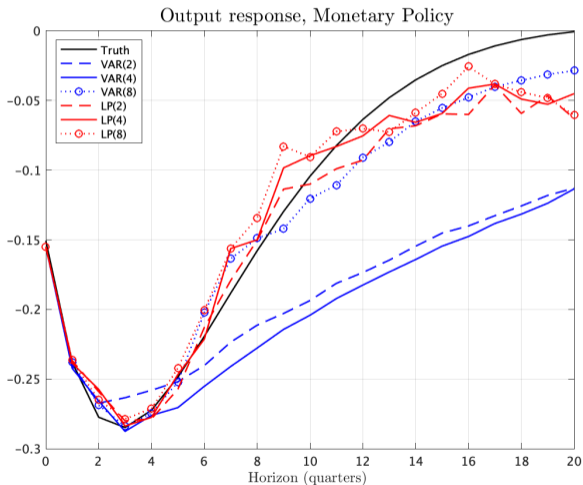
Monetary Policy





LAG LENGTH

- The choice of p does not matter for the LP if the identified shock is the true one.
- However, it does for the VAR! For short lag lengths, VAR misspecification may be an issue.
- Thus, one needs to be careful when choosing p if estimating the response with SVAR. Not the case for LP.
- What does it imply for the structural parameters?



	Local Projections		Vector Autoregression	
	Avg. J^*	Max. J^*	Avg. J^*	Max. J^*
	p=2			
Technology Shock	2.60	8.69	284021.78	2587907.99
Fiscal Policy	3.14	13.07	86762.92	1003678.62
Monetary Policy	2.77	13.44	-1090391.15	361979.19
Selected Responses	9.10	47.94	2617507.84	16254584.36
	p=4			
Technology Shock	2.57	9.43	34.67	228.41
Fiscal Policy	3.05	13.88	58.12	692.14
Monetary Policy	2.71	16.89	178.17	853.72
Selected Responses	8.37	44.69	230.46	1130.58
	p=8			
Technology Shock	2.47	13.33	2.87	19.93
Fiscal Policy	2.93	15.79	3.21	20.77
Monetary Policy	2.71	15.82	5.26	20.57
Selected Responses	9.66	50.95	11.84	79.72
	p=12			
Technology Shock	2.33	8.67	2.43	14.75
Fiscal Policy	2.74	16.47	2.43	14.68
Monetary Policy	2.63	15.34	3.20	23.26
Selected Responses	8.58	52.95	10.79	53.69

- As we increase the lag length, the differences between the two approaches become smaller and smaller.
- The J^* is very similar for the LPs regardless of the chosen lag length.
- For the SVAR approach, J^* sharply decreases and gets closer to the LP counterpart as p increases.

RAMEY (2016)

TECHNOLOGY & FISCAL POLICY SHOCKS

- **Structural Parameters.** Same eight from the Monte Carlo study, $\Theta = \{\sigma_c, h, \sigma_l, \varphi, \zeta_p, \zeta_w, l_p, l_w\}$.

- **Local Projection Regression:**

$$z_{t+h} = \alpha_h + \theta_h \cdot \text{shock}_t + \varphi_h(L)y_{t-1} + \text{quadratic trend} + \varepsilon_{t+h} \quad (5)$$

- * Technology shocks: medium run restrictions as in FORD (2014)

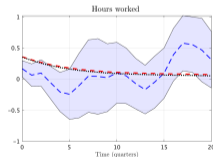
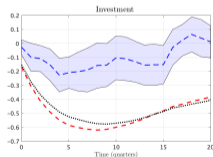
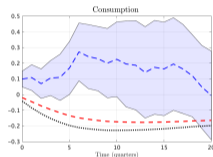
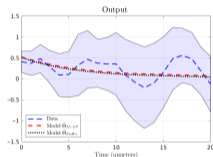
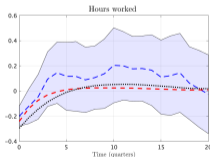
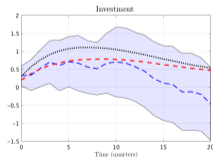
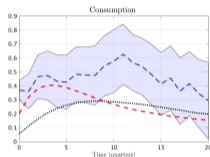
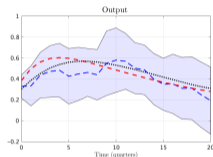
- * Fiscal policy shocks: government spending pre-determined as in Blanchard and Perotti (2006)

- **Targeted Coefficients:** only those identifying the responses, i.e. θ_h
- **Auxiliary Econometric Model.** Local Projection regression of the dependent variable on the shock and p lags each of the shock and the dependent variable
 - * No need for extra controls because shock comes directly from the model. No measurement error.
- **Weighting Matrix.** We use a diagonal matrix whose entries coincide with the inverse of the IRFs' standard deviation

	$\hat{\sigma}_c$	\hat{h}	$\hat{\sigma}_l$	$\hat{\phi}$	$\hat{\xi}_w$	$\hat{\xi}_p$	\hat{l}_w	\hat{l}_p
S&W 07	1.26	0.80	2.52	6.31	0.70	0.66	0.58	0.24
Technology Shocks								
<i>Median</i>	0.85	0.69	3.28	8.20	0.44	0.59	0.47	0.14
<i>10th pctl.</i>	0.76	0.48	1.51	3.79	0.42	0.40	0.35	0.14
<i>90th pctl.</i>	1.36	0.89	3.28	8.20	0.84	0.86	0.75	0.31
Fiscal Policy								
<i>Median</i>	1.23	0.85	1.51	5.14	0.52	0.59	0.41	0.15
<i>10th pctl.</i>	0.90	0.55	1.51	3.79	0.42	0.40	0.35	0.14
<i>90th pctl.</i>	1.57	0.97	2.18	7.89	0.85	0.82	0.72	0.30

(a) Technology Shocks

(b) Fiscal Policy



-- Empirical IRFs -- IRFs $\hat{\Theta}_{LP}$ IRFs $\hat{\Theta}_{SW07}$

TENREYRO & THWAITES (2016)

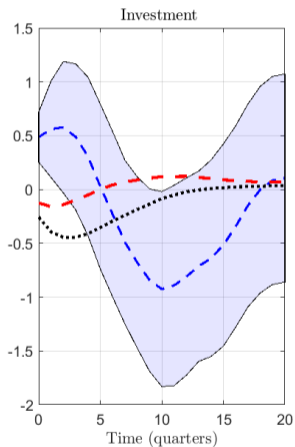
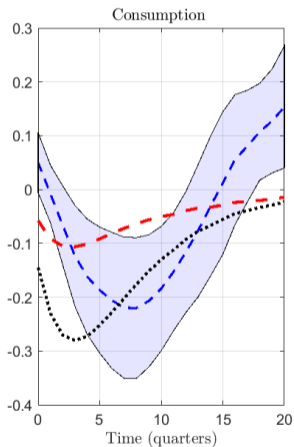
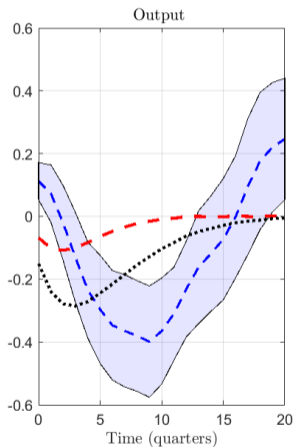
MONETARY POLICY SHOCKS

- The **responses** of output, consumption and investment to **monetary policy** are **state-dependent**.
- The Smets and Wouters model does not contain the relevant non-linearities to capture these effects, so why should we care?
- Because it is still constructive to match the non-linear responses since **results may inform us about which parts of the model are more reactive to this state-dependance**, i.e. those in which the parameters differ the most across the two scenarios.
- Tenreyro and Thwaites (2016) empirical specification:

$$y_{t+h} = \tau_t + F(z_t) \left(\alpha_h^b + \beta_h^b \varepsilon_t + \gamma^{\mathbf{b}'} \mathbf{x}_t \right) + (1 - F(z_t)) \left(\alpha_h^r + \beta_h^r \varepsilon_t + \gamma^{\mathbf{r}'} \mathbf{x}_t \right) + u_t \quad (6)$$

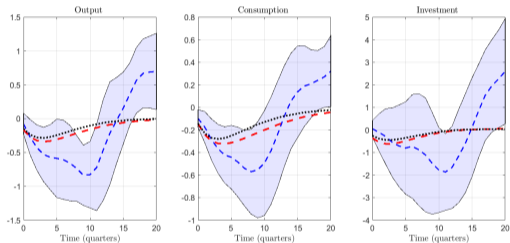
- We target β_h^b and β_h^r in two independent estimation exercises using a linear LP model on model simulated data.

	$\hat{\sigma}_c$	\hat{h}	$\hat{\sigma}_l$	$\hat{\varphi}$	$\hat{\zeta}_w$	$\hat{\zeta}_p$	\hat{i}_w	\hat{i}_p
S&W 2007	1.26	0.80	2.52	6.31	0.70	0.66	0.58	0.24
Linear Model								
<i>Median</i>	1.26	0.91	3.15	7.89	0.46	0.32	0.32	0.10
<i>10th pctl.</i>	0.85	0.80	1.51	5.94	0.46	0.32	0.32	0.10
<i>90th pctl.</i>	1.57	0.98	3.15	7.89	0.76	0.66	0.66	0.21
Non-Linear Model: Expansion								
<i>Median</i>	1.57	0.76	1.51	4.06	0.72	0.66	0.32	0.10
<i>10th pctl.</i>	0.76	0.64	1.51	3.79	0.46	0.32	0.32	0.10
<i>90th pctl.</i>	1.57	0.94	3.15	7.89	0.90	0.66	0.66	0.21
Non-Linear Model: Recession								
<i>Median</i>	1.57	0.91	3.15	7.89	0.46	0.32	0.66	0.21
<i>10th pctl.</i>	0.90	0.79	1.51	4.83	0.46	0.32	0.32	0.10
<i>90th pctl.</i>	1.57	0.98	3.15	7.89	0.77	0.56	0.66	0.21

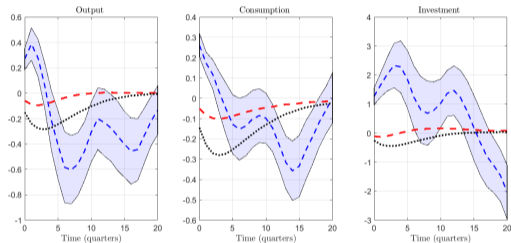


--- Empirical IRFs --- IRFs $\hat{\Theta}_{LP}$ IRFs $\hat{\Theta}_{SW07}$

(a) Non-Linear: Expansion



(b) Non-Linear: Recession



-- Empirical IRFs -- IRFs $\hat{\Theta}_{LP}$ IRFs $\hat{\Theta}_{SW07}$

