### Local Projections vs. VARs for Structural Parameter Estimation

#### Juan Castellanos

European University Institute

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- Starting with Jordà (2005), **local projections** (LP) have become a common tool to understanding the dynamic effects of economic shocks
  - \* An alternative to vector autorregresions (VARs) when estimating impulse responses
- Other studies analyze the performance of these two models when estimating IRFs
  - \* VARs and LPs estimate the same impulse responses in population (Plagborg-Møller and Wolf, 2020)
  - \* However, there is a bias-variance trade off in finite samples (Li et al., 2021)
- Our focus is instead on the structural parameters of any DSGE model
  - \* Follow Smith (1993) in estimating structural parameters through an indirect inference exercise in which the auxiliary model is a macro-econometric model
- How should we **choose between VARs and LPs** when estimating via **indirect inference** the structural parameters of our DSGE model?

### What I Do: Monte-Carlo Analysis



- Data Generating Process  $\rightarrow$  Smets & Wouters (2007) DSGE model (different sample lenghts)
- Estimation technique  $\rightarrow$  minimum distance
  - \* Indirect Inference: estimating IRFs on model simulated data
  - \* IRF matching: analytical IRFs from ABCD representation
- Moment generating / binding functions  $\, \rightarrow \, {\sf IRF}$  estimators
  - \* Vector Autoregression (VARs)
  - \* Local Projections (LP)
  - \* Bias correction and various lag length also considered
- Shocks & Identification
  - \* Three shocks: technology, fiscal and monetary
  - \* Three identifications: recursive, observed and noisily measured shocks

### What I Find



- Observed shock case
  - \* IRF matching  $\rightarrow$  LP responses (low bias)
  - \* Indirect Inference ightarrow robust to misspecification ightarrow SVAR responses (low variance)
  - \* However, the lag length p used in LP and VAR estimators matters a lot
    - If *p* is small, then use LP for IRF matching, while use VAR for Ind. Inf.
    - As *p* gets large, bias shrinks for VAR but at the cost of higher variance (Olea et al., 2024)
    - Hence, when *p* is large, the LP and SVAR have similar performance
  - \* Ind. Inf. is robust to **small sample** bias in estimated responses, *IRF matching* benefits from bias correction
- Recursive identification & shock proxies
  - \* When recursive assumptions are incorrect, IRF matching struggles but Ind. Inf. is robust to it
  - \* Both estimation techniques and econometric estimators suffer from miss-measured shocks
  - \* Unit normalization seems to be a good fix to deal with it



## DATA GENERATING PROCESS



- The discussion about which binding function to use, VAR or LP, is best made in the context of a specific model, but **which model to use?**
- Many applications that estimate their economies by matching impulse responses concern linearized models, e.g. Rotemberg and Woodford (1998), Christiano et al. (2005), laccoviello (2005), etc.
  - \* Indirect inference was initially proposed as a method to estimate non-linear models
  - \* Nonetheless we still need to understand how to choose the binding function in this simpler set up
- The responses to monetary, fiscal and technology shocks are the most widely studied in empirical applications (Ramey, 2016). Hence, we want a model that is able to speak about the responses to these aggregate shocks
- Given the relevance in the academic literature and in policy circles, the **Smets and Wouters** (2007) model seems a sensible choice



- Representative household with habit formation and preference for leisure

$$c_{t} = c_{1}c_{t-1} + (1 - c_{1})\mathbb{E}_{t}[c_{t+1}] + c_{2}(l_{t} - \mathbb{E}_{t}[l_{t+1}]) - c_{3}\left(r_{t} - \mathbb{E}_{t}[\pi_{t+1}] - \varepsilon_{t}^{b}\right)$$

- Households **invest in capital** given the **capital adjustment cost** they face

$$i_t = i_1 i_{t-1} + (1 - i_1) \mathbb{E}[i_{t+1}] + i_2 q_t + \varepsilon_t^i$$

where

$$\begin{aligned} q_t &= q_1 \mathbb{E}[q_{t+1}] + (1 - q_1) \mathbb{E}r_{t+1}^k - \left(r_t - \mathbb{E}_t[\pi_{t+1}] - \varepsilon_t^b\right) &: \text{ value of capital} \\ k_t &= k_1 k_{t-1} + (1 - k_1)i_t + k_2 \varepsilon_t^i &: \text{ installed capital LoM} \end{aligned}$$

- Aggregate production uses installed capital  $(k_t^s = k_{t-1} + z_t)$  and labor services

$$y_t = \phi_p \left( \alpha k_t^s + (1 - \alpha) I_t + \varepsilon_t^a \right)$$



- **Price stickiness** as in Calvo (1983) and **partial indexation** to lagged inflation gives rise to New-Keynesian Phillips curve

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 \mathbb{E}[\pi_{t+1}] - \pi_3 \mu_t^p + \varepsilon_t^p$$

- Nominal wage stickiness and partial indexation of wages to inflation

$$w_{t} = w_{1}w_{t-1} + (1 - w_{1})\mathbb{E}[w_{t+1} + \pi_{t+1}] - w_{2}\pi_{t} + w_{3}\pi_{t-1} - w_{4}\mu_{t}^{w} + \varepsilon_{t}^{w}$$

- Government spending is exogenous and correlated with technology

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a$$

- The central bank sets the short-term interest rate according to the monetary policy rule

$$r_{t} = \rho_{r}r_{t-1} + (1 - \rho_{r})\left(r_{\pi}\pi_{t} + r_{y}\left(y_{t} - y_{t}^{p}\right)\right) + r_{\Delta y}\left[\left(y_{t} - y_{t}^{p}\right) - \left(y_{t-1} - y_{t-1}^{p}\right)\right] + \epsilon_{t}^{r}$$



## **ESTIMATION STRATEGY**

### Indirect Inference / Impulse Response Matching

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- When estimating a subset of the structural parameters  $\Theta$  of any DSGE model by matching impulse responses, there are two approaches:
  - \* Target empirical responses but match model impulse responses

$$J^{irf} = \min_{\Theta} \left(\beta - \mathsf{IRF}(\Theta)\right)' W \left(\beta - \mathsf{IRF}(\Theta)\right)$$
(1)

- It doesn't require a simulated dataset, only structural IRFs
- \* Target and match empirical responses

$$J^{smm} = \min_{\Theta} \left(\beta - \beta(\Theta)\right)' W \left(\beta - \beta(\Theta)\right)$$

- It uses the same econometric approach in the real and simulated data

- How does the choice of the econometric model affects parameter estimates?
  - \* J<sup>irf</sup> speaks about potential misspecification of the model economy
  - \* J<sup>smm</sup> relates to misspecification of both the model and the binding function

(2)



## MONTE-CARLO EXPERIMENTS

### The DGP & the hyper-parameters



- The log-linearized version of the Smets and Wouters (2007) model is used to generate S repeated samples of macroeconomic aggregates
- The model is simulated each time at the estimated values from their paper using a sample of **T** observations
  - \* T = 300 used as baseline
  - \* T = 100 to address the issue of small sample bias of LPs (Herbst & Johannsen, 2023)
- We concentrate in 8 structural parameters of the model:
  - \*  $\sigma_c$ : intertemporal elasticity of substitution
  - \* *h* : habit parameter
  - \*  $\sigma_l$ : elasticity of labor supply

- \*  $\varphi$  : investment adjustment cost parameter
- \*  $\xi_w, \xi_p$ : Calvo adjustment probabilities
- \*  $\iota_W$ ,  $\iota_p$ : Degree of indexation to past inflation
- Simulated series are 10 times larger than the sample size during the optimization stage
- The importance of the coefficients used to summarize the data is weighted by a squared matrix W
  - \* Identity matrix: *I*<sub>m</sub>

\* Diagonal matrix with 1/h elements:  $I_d$ 

\* Inverse of the VCM of the moments:  $\Omega^{-1}$ 

### **Targeted Responses**



- We focus on the *estimated* impulse responses of four variables: *output*, *consumption*, *investment* and *hours* worked to one of three main aggregate shocks: monetary policy, fiscal policy and technology
- Shocks are treated by the econometrician as
  - \* observed, i.e.  $\tilde{x}_t = \eta_t^i$
  - \* inferred via recursive ordering
  - \* observed with error, i.e.  $\tilde{x}_t = \eta_t^i + \sigma_v v_t$
- The IRFs are estimated using a VAR or a Local Projections.
  - \* If the sample size is small (T = 100), we also consider the bias-corrected LP (Herbst & Johannsen, 2023) or the procedure by Killian (1998) for the SVAR
- In either case, the econometrician still needs to decide on at least two more things:
  - \* The impulse response horizon, *H*. We set H = 20.
  - \* The number of lags, p. We experiment with various p's, i.e.  $p \in \{2, 4, 8, 12\}$ .



### **PERFORMANCE METRICS**



- Overall performance

$$J^{*} = \left(\mathsf{IRF}(\Theta^{*}) - \mathsf{IRF}(\hat{\Theta})\right)' \left(\mathsf{IRF}(\Theta^{*}) - \mathsf{IRF}(\hat{\Theta})\right)$$
(3)

$$J^{smm} = \left(\beta(\Theta^*) - \beta(\hat{\Theta})\right)' \left(\beta(\Theta^*) - \beta(\hat{\Theta})\right)$$
(4)

$$J^{irf} = \left(\beta(\Theta^*) - \mathsf{IRF}(\hat{\Theta})\right)' \left(\beta(\Theta^*) - \mathsf{IRF}(\hat{\Theta})\right)$$
(5)

- Parameter-by-parameter performance

$$\mathcal{L}_{\omega}(\hat{\Theta}_{i},\Theta_{i}^{*}) = \omega \times \underbrace{\left(\mathbb{E}\left[\hat{\Theta}_{i}\right] - \Theta_{i}^{*}\right)^{2}}_{\text{bias}} + (1-\omega) \times \underbrace{\operatorname{Var}(\hat{\Theta}_{i})}_{\text{variance}}$$
(6)

- Model fit
  - \* Similar to (3), compute the unweighted distance between the structural IRFs but to other non-targeted shocks in the economy
  - \* For example, if targeting monetary policy shocks, look at fiscal and technology



# MONTE-CARLO RESULTS (OBSERVED SHOCK)

### Targeted Impulse Responses (S=100, T=300, p=4)











Consumption



0.8

0.6

0.4

-0.2

-0.4

-0.6

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20

20



SVAR



- Simplifying assumptions for comparison:
  - $^*$  Target IRFs estimated with a LP or SVAR model and  $\mathcal{T}=$  300 observations
  - \* Weight all responses equally during the estimation stage, i.e. W = I
- Overall performance measures are averaged across estimations using different lag lengths (p ∈ {2, 4, 8, 12}) and shocks (TFP, fiscal, monetary)

|                  |                  | IRF matching |          |                               |  | Indirect Inference |       |           |             |  |
|------------------|------------------|--------------|----------|-------------------------------|--|--------------------|-------|-----------|-------------|--|
|                  | J <sub>irf</sub> | <b>J</b> *   | Time     | J <sup>*</sup> <sub>unt</sub> |  | J <sub>smm</sub>   | $J^*$ | Time      | $J_{unt}^*$ |  |
| Local Projection | 35.10            | 0.27         | 3.49 min | 18.70                         |  | 32.54              | 0.39  | 42.88 min | 17.91       |  |
| Structural VAR   | 35.23            | 0.41         | 3.93 min | 17.93                         |  | 33.87              | 0.33  | 14.47 min | 18.39       |  |

#### Parameter by parameter performance





 $\boldsymbol{Z} = \left( \mathcal{L}_{\omega}(\hat{\boldsymbol{\Theta}}_{i}^{LP}, \boldsymbol{\Theta}_{i}^{*}) - \mathcal{L}_{\omega}(\hat{\boldsymbol{\Theta}}_{i}^{SVAR}, \boldsymbol{\Theta}_{i}^{*}) \right) / \boldsymbol{\Theta}_{i}^{*}$ 



# MONTE-CARLO RESULTS (RECURSIVE IDENTIFICATION)



- Shocks are not observable in applied work. Thus, there is a need for identification
- The most commonly used identification method in macroeconomics imposes recursive zero restrictions on contemporaneous coefficients
- As shown by Ramey (2016), there are two widely used alternatives:
  - \* Policy variable does not respond within the period to the other endogenous variable
    - We use this assumption to identify **technology shocks** and **government spending shocks** within the Smets and Wouters model
    - TFP and government spending are the policy variables, ordered first. Output, consumption, investment and hours worked are included in the VAR or as lagged controls in the LP
  - \* Other endogenous variables do not respond to the policy shock within the period
    - We use this assumption to identify monetary policy shocks within the Smets and Wouters model
    - We order the policy rate last in a VAR that also includes output, consumption, investment, hours worked, wages, and inflation. Similarly, these variables are added as contemporaneous controls in the LP



### **TECHNOLOGY SHOCKS**

#### Recursive assumption is correct in Sm & Wo (2007)







### If assumptions are right, identification does not matter **EUI** EUI DIAL EUI



- If recursive assumptions are correct, the identification strategy does not play a role in the estimation
- Main lesson still holds: use LPs for IRF matching exercises and VARs for Ind. Inf.
- Model fit:  $J_{unt}^*$  is large  $\implies$  not great idea to target just TFP shocks in the Smets-Wouters model

|                  |                         | IRF matching |          |             |  | Indirect Inference |       |           |             |  |
|------------------|-------------------------|--------------|----------|-------------|--|--------------------|-------|-----------|-------------|--|
|                  | <b>J</b> <sub>irf</sub> | $J^*$        | Time     | $J_{unt}^*$ |  | J <sub>smm</sub>   | $J^*$ | Time      | $J_{unt}^*$ |  |
| Local Projection | 1.05                    | 0.67         | 2.87 min | 37.30       |  | 0.70               | 0.84  | 42.41 min | 35.92       |  |
| Structural VAR   | 2.53                    | 1.07         | 3.11 min | 35.74       |  | 0.97               | 0.00  | 14.34 MIN | 37.31       |  |

#### It is all about the investment adjustment cost $\hat{\varphi}$







### MONETARY POLICY SHOCKS

### Real variables respond at t = 0 in the Sm & Wo model **EUI** UNVERSITY

20

20







27/38

### **Overall performance**



- Main results still hold when targeting IRFs to monetary policy shock and it is a **better idea** in the Smets-Wouters world: lower  $J_{unt}^*$
- When identification assumption are incorrect, then Ind. Inf. is robust to such misspecification
  - \* Targeting consistently wrong responses helps with parameter identification as long as they have low variance

|                  |                  | IRF n | natching |             | Indirect Inference |       |           |                               |
|------------------|------------------|-------|----------|-------------|--------------------|-------|-----------|-------------------------------|
|                  | J <sub>irf</sub> | $J^*$ | Time     | $J_{unt}^*$ | J <sub>smm</sub>   | $J^*$ | Time      | J <sup>*</sup> <sub>unt</sub> |
|                  |                  |       |          | Observe     | ed Shock           |       |           |                               |
| Local Projection | 50.65            | 0.07  | 3.46 min | 9.36        | 48.46              | 0.31  | 41.39 min | 9.40                          |
| Structural VAR   | 54.07            | 0.11  | 4.38 min | 9.26        | 53.60              | 0.30  | 14.65 min | 9.44                          |
|                  |                  |       |          | Recursiv    | ve Shock           |       |           |                               |
| Local Projection | 48.11            | 0.29  | 3.34 min | 9.60        | 56.91              | 0.18  | 78.57 min | 9.34                          |
| Structural VAR   | 47.09            | 0.34  | 3.78 min | 9.31        | 58.70              | 0.12  | 11.44 min | 9.34                          |

#### Parameter by parameter performance







# MONTE-CARLO RESULTS (MEASUREMENT ERROR)



- A popular identification approach consist of constructing a series from historical documents to identify policy shocks, e.g.
  - \* Romer and Romer (2004) monetary shock series based on FOCM meetings
  - \* Ramey (2011) defense news series based on *Business Week* articles
- These series are used in dynamic single equation regressions or embedded in a Choleski decomposition, as we have done for the observed shock scheme
- In practice, there are good reasons to expect that these shocks suffer from **measurement error** or capture only part of the shock. Hence, I distinguish 3 cases:
  - 1. Study classical measurement error case,  $\eta_t^{obs} = \eta_t + \sigma_\nu \nu_t$
  - 2. Proxy is correlated with other shocks, e.g. government spending with technology shock
  - 3. Unit normalization (Stock & Watson, 2018)

### Attenuation bias in IRFs











### Proxy shocks: bad news for the structural estimates



- The estimation outcome is significantly worse for both LPs and VARs as well as for the *IRF matching* and *Ind*. *Inf*. estimators relative to the observed shock case.
- These findings also apply to other sources of variation such as monetary or fiscal policy shocks.
- Does it get worse when the proxy is correlated with other shocks? Does unit normalization of the IRFs help in identifying responses?

|                  |                  | IRF matching |              |             |                      | Indirect Inference   |                                 |             |  |
|------------------|------------------|--------------|--------------|-------------|----------------------|----------------------|---------------------------------|-------------|--|
|                  | J <sub>irf</sub> | <b>J</b> *   | Time         | $J_{unt}^*$ | J <sub>smm</sub>     | $J^*$                | Time                            | $J_{unt}^*$ |  |
|                  |                  |              | Т            | rue technol | ogy shocl            | $\langle (\eta_t^a)$ |                                 |             |  |
| Local Projection | 1.05             | 0.67         | 2.87 min     | 37.30       | 0.70                 | 0.84                 | 42.41 min                       | 35.92       |  |
| Structural VAR   | 2.53             | 1.07         | 3.11 min     | 35.74       | 0.97                 | 0.66                 | 14.34 min                       | 37.31       |  |
|                  |                  |              | Proxied tecl | hnology sh  | ock ( $\eta_t^{a,o}$ | $bs = \eta_t^a$      | $\sigma + \sigma_{\nu} \nu_t$ ) |             |  |
| Local Projection | 1.79             | 1.25         | 3.05 min     | 34.30       | 1.35                 | 1.40                 | 40.23 min                       | 33.31       |  |
| Structural VAR   | 3.41             | 1.70         | 2.80 min     | 33.47       | 1.70                 | 1.18                 | 13.74 min                       | 34.39       |  |

### Govn't spending and its correlation with technology



- The  $J^*$  is again much larger than in the observed shock case or in the proxy measure with classical measurement error, and for both estimation approaches.
  - \* Ind. Inf. is not robust to this type of misspecification, unlike for (misspecified) recursive shocks
- The model fit,  $J_{unt}^*$ , improves in the *IRF matching* because the IRF with the shock (not just the innovation) captures some information about technology shocks.

|                                    |  | IRF matching |                      |              |                  | Indirect Inference |                        |                               |  |
|------------------------------------|--|--------------|----------------------|--------------|------------------|--------------------|------------------------|-------------------------------|--|
|                                    | J <sub>irf</sub>   | <b>J</b> *   | Time                 | $J_{unt}^*$  | J <sub>smm</sub> | <b>J</b> *         | Time                   | J <sup>*</sup> <sub>unt</sub> |  |
|                                    | Government spending innovation $(\eta_t^g)$                        |              |                      |              |                  |                    |                        |                               |  |
| Local Projection<br>Structural VAR | 53.59<br>49.09   | 0.07<br>0.05 | 4.14 min<br>4.32 min | 9.43<br>8.79 | 48.45<br>47.03   | 0.02<br>0.03       | 44.82 min<br>14.42 min | 8.40<br>8.42                  |  |
|                                    | A correlated government spending proxy ( $\varepsilon_t^{g,obs}$ ) |              |                      |              |                  |                    |                        |                               |  |
| Local Projection<br>Structural VAR | 30.82<br>31.45   | 0.34<br>0.34 | 4.09 min<br>4.19 min | 7.80<br>7.78 | 39.05<br>42.42   | 0.35<br>0.40       | 46.13 min<br>14.20 min | 10.15<br>10.53                |  |

### Stock & Watson unit normalization



- Unit normalization corrects the bias in estimated responses through rescaling.
- Great fix for the structural estimation as well, specially for *IRF matching*.

|                  |  | IRF        | matching                 |             |                  | Indirect Inference |               |                               |  |  |
|------------------|--|------------|--------------------------|-------------|------------------|--------------------|---------------|-------------------------------|--|--|
|                  | J <sub>irf</sub>   | <b>J</b> * | Time                     | $J_{unt}^*$ | J <sub>smm</sub> | $J^*$              | Time          | J <sup>*</sup> <sub>unt</sub> |  |  |
|                  |  |            | True                     | Monetary    | policy sho       | ck ( $\eta_t^m$ )  |               |                               |  |  |
| Local Projection | 50.65  | 0.07       | 3.46 min                 | 9.36        | 48.46            | 0.31               | 41.39 min     | 9.40                          |  |  |
| Structural VAR   | 54.07  | 0.11       | 4.38 min                 | 9.26        | 53.60            | 0.30               | 14.65 min     | 9.44                          |  |  |
|                  | Proxied monetary policy shock ( $\eta_t^{a,obs} = \eta_t^a + \sigma_v v_t$ ) |            |                          |             |                  |                    |               |                               |  |  |
| Local Projection | 1.79   | 1.25       | 3.05 min                 | 34.30       | 1.35             | 1.40               | 40.23 min     | 33.31                         |  |  |
| Structural VAR   | 3.41   | 1.70       | 2.80 min                 | 33.47       | 1.70             | 1.18               | 13.74 min     | 34.39                         |  |  |
|                  |  | A 1% in    | crease in r <sub>0</sub> | (Stock ar   | d Watson         | (2018)             | normalization | ı)                            |  |  |
| Local Projection | 50.77  | 0.08       | 3.83 min                 | 19.34       | 49.49            | 0.52               | 49.84 min     | 17.85                         |  |  |
| Structural VAR   | 53.41  | 0.32       | 4.04 min                 | 18.86       | 51.23            | 0.42               | 12.49 min     | 17.93                         |  |  |



## CONCLUSION





- 1. *IRF matching* is more **sensitive to bias** in targeted responses and hence using LP-IRFs is preferable, while *Ind. Inf.* is **robust to misspecification** and hence benefits from the lower variance of VAR-IRFs.
- 2. When the **lag length** *p* **is large**, then IRFs and estimated parameters are **similar** independently of the econometric model. On the other hand, when *p* **is small**, **LP-IRFs** are less biased and hence better for *IRF matching*, while **SVAR-IRFs** have a larger bias but lower variance and hence better for <u>Ind. Inf.</u>
- 3. **Small sample bias** worsens the performance of the estimation specially for *IRF matching* when bias correction partly offsets the problem.
- 4. **Incorrect recursive identifications** are not an issue for parameter estimation when employing *Ind. Inf.*. Not true for *IRF matching*.
- 5. **Measurement error** worsens the structural estimation outcome and unit normalization only ameliorates the problem.



## APPENDIX



- Some notation:
  - \* Let  $\tilde{y}_t \in \{y_t, c_t, i_t, hw_t\}$  denote one of response variables of interest.
  - \* Let  $\tilde{x}_t \in \{\eta_t^a, \eta_t^g, \eta_t^m\}$  denote the innovation of one of the three aggregate shocks.
  - \* Define the vector of contemporaneous  $r_t$  and lagged controls  $w_t = {\tilde{x}_t, \tilde{y}_t}$
- Then, consider for each horizon h = 0, 1, 2, ..., H the linear projections:

$$\tilde{\mathbf{y}}_{t+h} = \mu_h + \beta_h \tilde{\mathbf{x}}_t + \gamma'_h r_t + \sum_{\ell=1}^p \delta'_{h,\ell} \mathbf{w}_{t-\ell} + \xi_{h,t}$$
<sup>(7)</sup>

where  $\xi_{h,t}$  is the projection residual and  $\mu_h$ ,  $\beta_h$ ,  $\gamma_h$ ,  $\{\delta'_{h,\ell}\}_{\ell=1}^p$  are the projection coefficients.

- **Definition**. The LP - IRFs of  $\tilde{y}_t$  with respect to  $\tilde{x}_t$  is given by  $\{\beta_h\}_{h>0}$  in the equation above.



- Consider the multivariate linear VAR(p) projection:

$$w_{t} = c + \sum_{\ell=1}^{p} A_{\ell} w_{t-\ell} + u_{t}$$
(8)

where  $u_t$  is the projection residual and c,  $\{A_\ell\}_{\ell=1}^p$  are the projection coefficients.

- Let  $\Sigma_u \equiv \mathbb{E} [u_t u'_t]$  and define the Cholesky decomposition  $\Sigma_u = BB'$  where B is lower triangular with positive diagonal entries.
- Consider the corresponding recursive SVAR representation:

$$A(L)w_t = c + B\eta \tag{9}$$

where  $A(L) = I - \sum_{\ell=1}^{p} A_{\ell} L^{\ell}$  and  $\eta = B^{-1} u_{t}$ . Define the lag polynomial  $\sum_{\ell=0}^{p} C_{\ell} L^{\ell} = C(L) = A(L)^{-1}$ .

- **Definition**. The SVAR - IRFs of  $\tilde{y}_t$  with respect to  $\tilde{x}_t$  is given by  $\{\theta_h\}_{h\geq 0}$  with  $\theta_h \equiv C_{2,\bullet,h}B_{\bullet,1}$  where  $\{C_\ell\}$  and B are defined above.





## HYPER-PARAMETER CHOICES: LAG LENGTH

### Does the lag length matter for the IRFs?



#### - Point estimates

- \* Local Projection IRFs are independent of the lag length when the shock is observed
- \* SVAR IRFs approximately agree with LP IRFs up to horizon *p*, then extrapolates using the first *p* sample autocovariances

#### - Confidence Intervals

- \* Local Projection IRFs have a much wider bands, specially at long horizons
- \* SVAR IRFs converge towards the sample uncertainty of LPs as p gets large





|                  |                  | IRF matching |          |             |    | Indirect Inference |            |           |             |
|------------------|------------------|--------------|----------|-------------|----|--------------------|------------|-----------|-------------|
|                  | J <sub>irf</sub> | <b>J</b> *   | Time     | $J_{unt}^*$ |    | J <sub>smm</sub>   | <b>J</b> * | Time      | $J_{unt}^*$ |
|                  |                  |              |          |             | p= | 2                  |            |           |             |
| Local Projection | 35.75            | 0.24         | 3.30 min | 18.97       |    | 25.47              | 0.34       | 18.93 min | 18.02       |
| Structural VAR   | 34.61            | 0.61         | 4.32 min | 17.00       |    | 26.25              | 0.16       | 11.88 min | 19.32       |
|                  | p=4              |              |          |             |    |                    |            |           |             |
| Local Projection | 35.68            | 0.25         | 3.40 min | 18.74       |    | 30.26              | 0.37       | 28.99 min | 17.95       |
| Structural VAR   | 36.01            | 0.39         | 3.89 min | 17.75       |    | 31.49              | 0.26       | 15.35 min | 18.26       |
|                  |                  |              |          |             | p= | 8                  |            |           |             |
| Local Projection | 34.69            | 0.28         | 3.83 min | 18.47       |    | 35.91              | 0.44       | 45.06 min | 17.69       |
| Structural VAR   | 34.92            | 0.34         | 3.85 min | 18.36       |    | 37.26              | 0.49       | 13.35 min | 18.01       |
|                  | p=12             |              |          |             |    |                    |            |           |             |
| Local Projection | 34.27            | 0.29         | 3.44 min | 18.63       |    | 38.52              | 0.41       | 78.53 min | 17.98       |
| Structural VAR   | 35.39            | 0.30         | 3.67 min | 18.61       |    | 40.47              | 0.41       | 17.29 min | 17.98       |



# HYPER-PARAMETER CHOICES: SAMPLE SIZE

### Small sample bias & bias correction



- P-M & W (2023) show that *LP*(*p*) exactly agree with true responses and that *SVAR*(*p*) agrees up to lag *p*
- However, sample uncertainty matters!
  - \* In finite samples, e.g. T = 300, both LP and SVAR are biased after horizon p, with SVARs having a more severe bias as long as the response is persistent
  - The sample size typically found in empirical applications is even shorter and around T=100 (H&J, 2023), which makes these biases worse.
- **Bias correction** partially offsets the small sample bias, but two questions arise in our context
  - \* Q1: Does Indirect Inference improves upon IRF matching when this bias is severe?
  - \* Q2: Does targeting bias corrected responses improve the model estimation?



### IRF matching vs. Indirect Inference in small samples



- Higher sample uncertainty associated with fewer observations (T = 100) leads to a **worse fit** of the model **for both estimation strategies**
- IRF matching suffers more its consequences as **Ind. Inf. is robust to misspecification** of the binding function
- For the same reason, applying bias correction to the targeted IRFs is more useful for IRF matching

|                     | IRF matching<br>$J_{irf}$ $J^*$ Time $J_{unt}^*$ |       |          |             | Indirect Inference |       |           |             |
|---------------------|--|-------|----------|-------------|--------------------|-------|-----------|-------------|
|                     | J <sub>irf</sub>                                 | $J^*$ | Time     | $J_{unt}^*$ | J <sub>smm</sub>   | $J^*$ | Time      | $J_{unt}^*$ |
|                     |  |       |          | T=          | 300                |       |           |             |
| Local Projection    | 35.10  | 0.27  | 3.49 min | 18.70       | 32.54              | 0.39  | 42.88 min | 17.91       |
| Structural VAR      | 35.23  | 0.41  | 3.93 min | 17.93       | 33.87              | 0.33  | 14.47 min | 18.39       |
|                     |  |       |          | T=          | 100                |       |           |             |
| Local Projection    | 29.71  | 0.53  | 3.56 min | 18.13       | 22.00              | 0.46  | 18.46 min | 19.03       |
| Structural VAR      | 31.62  | 0.47  | 3.33 min | 17.98       | 25.16              | 0.36  | 9.78 min  | 19.50       |
| Bias Corrected LP   | 31.55  | 0.32  | 3.26 min | 19.18       | 23.29              | 0.35  | 20.48 min | 19.50       |
| Bias Corrected SVAR | 33.48  | 0.32  | 3.42 min | 18.65       | 26.06              | 0.33  | 11.02 min | 20.11       |





## HYPER-PARAMETER CHOICES: WEIGHTING MATRIX



|                                    |                  | IRF I                    | matching             |                | Indirect Inference |              |                        |                |  |
|------------------------------------|------------------|--------------------------|----------------------|----------------|--------------------|--------------|------------------------|----------------|--|
|                                    | J <sub>irf</sub> | <b>J</b> *               | Time                 | $J_{unt}^*$    | J <sub>smm</sub>   | <b>J</b> *   | Time                   | $J_{unt}^*$    |  |
|                                    |                  | Identity Matrix          |                      |                |                    |              |                        |                |  |
| Local Projection                   | 35.10            | 0.27                     | 3.49 min             | 18.70          | 32.54              | 0.39         | 42.88 min              | 17.91          |  |
| Structural VAR                     | 35.23            | 0.41                     | 3.93 min             | 17.93          | 33.87              | 0.33         | 14.47 min              | 18.39          |  |
|                                    | Diagonal Matrix  |                          |                      |                |                    |              |                        |                |  |
| Local Projection                   | 34.44            | 0.22                     | 3.61 min             | 18.87          | 32.82              | 0.35         | 40.56 min              | 18.22          |  |
| Structural VAR                     | 34.87            | 0.27                     | 3.85 min             | 18.20          | 34.17              | 0.31         | 11.55 min              | 18.62          |  |
|                                    |                  | Optimal Weighting Matrix |                      |                |                    |              |                        |                |  |
| Local Projection<br>Structural VAR | 33.63<br>34.17   | 0.04<br>0.05             | 3.07 min<br>3.20 min | 21.56<br>20.80 | 32.69<br>34.26     | 0.06<br>0.08 | 35.56 min<br>10.69 min | 21.41<br>20.90 |  |

