Durables and Portfolio Choice: Response to Aggregate Shocks*

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Abstract

This paper addresses the relevance of non-linearities at the individual level, in the form of non-convex adjustment costs, borrowing constraints and a non-linear productivity process, for aggregate outcomes. We build a life-cycle incomplete markets model with durables in general equilibrium to address this issue. At the moment we look at a stationary equilibrium, but we will introduce aggregate uncertainty in the future. This will allow us to shed some light on the effects of aggregate variations, including changes in household's income distributions, the magnitude and nature of borrowing constraints and monetary innovations, on household consumption and durable expenditure.

Keywords: *consumption, durable goods, borrowing constraints, adjustment costs, lifecycle, non-linearities, aggregate shocks.*

JEL classification: E10, E21, E52

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1. Introduction

This paper studies the effects of aggregate shocks in a life-cycle economy with durable goods. To do so it is important to highlight the dual role of durables. On the one hand, they can be used as collateral for borrowing; while on the other hand, they can also be used to smooth consumption since we will assume that markets are incomplete. Within this set-up, the paper contributes along two dimensions.

First, the paper studies the household choice over durables in a setting with costs of adjustment. This feature of the model brings it considerably closer to micro evidence on infrequent adjustment, compared to a model without these costs. Moreover, this additional feature allows us to study the household's portfolio composition where the durable good acts as an illiquid asset. The interesting question is how the consequent cross sectional distribution of durable holdings and income influences aggregate outcomes.

Thus, second, the paper also provides guidance on the household consumption and durable expenditure response to aggregate variations, including changes in the distribution of household income, the magnitude and nature of borrowing constraints and monetary innovations. While there is now a sizable literature on these topics, almost all of it ignores a principal dimension of household expenditures, the purchase and sale of durables. This is both an empirically prominent aspect of aggregate variations, along with firm investment, and also a key part of the monetary transmission channel. The paper addresses these issues through both a comparison of steady states for different income distributions and in a dynamic stochastic setting.[1]

There is existing literature, discussed below, that studies these pieces of the model in isolation. The contribution of the paper is in putting them together.

Section 2 lays out the basic framework. The focus there is on an individual choice problem with non-convex adjustment costs and borrowing constraints. The interaction of these features provides rich lifecycle patters along with state contingent policy functions that operate on both the extensive and intensive margins.

Section 3 describes our empirical approach and presents our findings. For this section, there are no aggregate shocks, but we still solve for a stationary equilibrium. Nevertheless,

[1] This second exercise may be developed in a subsequent paper, focusing the analysis here on the effects of variations in borrowing constraints and income distribution.

the focus is on the individual policy functions, lifecycle paths as well as other features of the steady state equilibrium such as the interest rate that clears the capital market.

Section 4 describes aggregate effects. The initial focus is on the effects of changes in the distribution of income shocks, say through changes in income uncertainty, on durable and non-durable spending. Here is where the costly adjustment, along with the borrowing constraints, plays a major role. This is supplemented with an analysis of borrowing constraints and, in particular, their interaction with variations in income distribution. The analysis of monetary policy innovations appears here.

1.1. Literature Review

The starting point for understanding our contribution would be the dynamic general equilibrium model with durables in Fernández-Villaverde and Krueger (2011). That paper studies the stationary equilibrium of an heterogeneous agent life-cycle economy with different borrowing constraints. It goes without saying that the presence of such constraint highlights the role of durables as collateral, a key feature in our modeling framework. However, Fernández-Villaverde and Krueger's model is frictionless in the sense that they do not allow for non-convex costs of adjustment associated to the purchase of these goods. This is an important feature in our model because it generates large and infrequent adjustments in the durable stock in a (S,s) rule fashion as in Grossman and Laroque (1990).

A more recent branch of this literature has shown that the presence of this type of costs are not only consistent with micro-economic evidence, but also have important aggregate implications. For example, Berger and Vavra (2015) show that these type of micro founded durable frictions lead to much lower responsiveness of durable expenditures during recessions. Similarly, McKay and Wieland (2019) develop a model of lumpy durable demand that predicts that the intertemporal shifting of durable demand after a monetary stimulus has a negative impact on the real natural interest rate, $r^{\star}$, in the following periods. In Berger and Vavra's and McKay and Wieland's world, the household's durable adjustment decision is a crucial mechanism, which in turn depends on the joint distribution of income and wealth. In this regards, a distinctive feature of our model is that we allow for such distribution to also depend on age since we are interested on how income, wealth, durable's adjustment and borrowing constraints interact along the lifecycle.

Our exercise of studying changes in the distribution of income is motivated by a number of recent papers that both document these changes and their effects on consumption/saving decisions. Here it is important to move away from the standard Gaussian model to allow forms of non-normality as in De Nardi et al. (2020). In fact, their richer labor productivity process has important aggregate implications in our general equilibrium framework with durable goods. As De Nardi et al. document using a partial equilibrium lifecycle model in the tradition of Bewley (1986), the richer process implies a higher degree of self-insurance of persistent shocks. In our framework, such behavior is going to translate into lower equilibrium interest rates though an increase in capital supply for precautionary reasons[2]. Moreover, changes in distributions interact with the non-convex adjustment cost as in the work of Bloom (2009) and Bayer et al. (2019); and, at least for borrowing constraints that are incentive based, increasing the likelihood of low income realizations implies tighter borrowing restrictions with consequent implications for non-durable consumption and durable expenditures.

In terms of monetary policy, we follow Ampudia et al. (2020) and introduce monetary effects on income and interest rates into our model. We study the individual household and aggregate response to monetary policy innovations. In contrast to Ampudia et al., the focus is on durable spending. Therefore, by studying monetary policy in our model economy, we contribute to the extensive literature that has focused on the transmission of monetary interventions to the real economy through durable demand. See for example Sterk and Tenreyro (2018), Cloyne et al. (2020), or McKay and Wieland (2020), among others.

There is also an important computational dimension of this paper due to the complexity of the household optimization problem, and the characterization of an equilibrium. For the individual optimization, we built upon Druedahl (2020), who provides a useful guide on how to solve non-convex consumption savings models. For the equilibrium with aggregate uncertainty, we will experiment with different algorithms to check the robustness to aggregate non-linearities. In this respect, we separate ourselves from the bast majority of the discrete choice literature and abstract from smoothing shocks since they may mask the transmission of individual non-linearities to the aggregate, a key dimension that we would like to explore.

[2] For a targeted equilibrium real interest rate, based for example on empirical capital returns and inflation, this will translate into a lower discount factor.

2. The Model

In this section we present a dynamic general equilibrium model of life-cycle consumption in which households maximize expected discounted utility of a consumption aggregate. They are subject to idiosyncratic labor productivity shocks and can borrow subject to a borrowing constraint. In particular, we extend the traditional life-cycle, incomplete markets model of Huggett (1996) by separating between non-durable and durable consumption goods as well as introducing proportional adjustment costs associated with the purchase of durable goods (Berger and Vavra, 2015, Harmenberg and Öberg, 2021). Moreover, we allow for different specifications of the borrowing constraint to highlight the role of durables as a collateral in which we follow the approach in Luengo-Prado (2006) and Fernández-Villaverde and Krueger (2011). Finally, we also explore the role played by two different labor productivity processes: the canonical linear model with persistent and transitory shocks as in Storesletten et al. (2004), and a much richer process that allows for age-dependence of moments, non-normality, and non-linearity in previous earnings and age as in De Nardi et al. (2020)[3].

2.1. Demographics

There is a continuum of individuals of measure 1 at each point in time in our economy. Each individual lives at most J periods. In each period $j \leq J$ they face a conditional probability of surviving and living in period $j+1$ of $s_j \in (0,1)$ with $s_0 = 1$ and $s_J = 0$. Each year a positive measure $\lambda_1 = (1 + \sum_{j=1}^{J-1} \Pi_{i=1}^{j} s_j + g_n)$ is born, thus population grows at a rate g_n. People start life as workers and work until retirement age $J^{ret} \leq J$. After that there is a mandatory transition to retirement, which acts as an absorbing state. In other words, we do not allow retired workers to return to the labor market[4].

The fraction of people in the economy is defined recursively as:

$$\tilde{\lambda}_{j+1} = \frac{s_j \tilde{\lambda}_j}{1 + g_n} \tag{1}$$

which we then normalize such that it sums up to 1. We denote this normalized measure by $\lambda_j = \tilde{\lambda}_j / \sum_{j=1}^{J} \tilde{\lambda}_j$ for each $j \in \{0, 1 \ldots, J\}$.

[3] The discretized version of these two processes is taken from De Nardi et al. (2020).

[4] This is indeed a more realistic modelling framework but it is not relevant for our analysis.

2.2. Technology

There is single good in this economy, which is produced using a constant return to scale technology that converts the aggregate capital stock K_t and aggregate labor L_t into output Y_t. That is, $Y_t = F(K_t, L_t)$. We also assume that F is strictly increasing in both inputs, is strictly concave, has decreasing marginal products and is homogeneous of degree one. It is well-known that with constant returns to scale the number of firms is indeterminate, thus, without loss of generality, we can assume that there is a single representative firm in this economy.

The final good produced by this firm can either be consumed or invested into physical capital or consumer durables. Each period the physical capital stock depreciates at a rate δ, while the aggregate stock of consumer durables, K_t^d, depreciates at a rate δ^d. Then, the aggregate resource constraint in this economy is given by:

$$C_t + I_t + I_t^d = F(K_t, L_t) \tag{2}$$

where C_t denotes aggregate consumption expenditures, $I_t = K_{t+1} - (1-\delta)K_t$ denotes investment into physical capital and $I_t^d = K_{t+1}^d - (1-\delta^d)K_t^d$ corresponds to investment into durable goods.

2.3. Preferences and Endowments

Households have time-separable preferences with discount factor β, and their utility flow comes from consumption of both non-durable goods, c_t, and durable goods d_t. In other words, we introduce the service flows from consumer durables into the utility function U. In particular, the utility flow from the two types of consumption is given by:

$$U\left(g\left(c,d\right)\right) = \frac{\left(c^{\alpha}\left(d+\underline{d}\right)^{1-\alpha}\right)^{1-\rho}}{1-\rho} \tag{3}$$

where $g : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a Cobb-Douglas aggregator function and $U : \mathbb{R} \rightarrow \mathbb{R}$ is strictly increasing and concave[5]. The parameter α is the share of expenditures devoted to non-durable

[5] Notice that $g(c,d)$ could have taken a more general form, e.g. a CES aggregator. However, we have decided to study the case in which the *intratemporal* substitution is such that agents hold constant expenditure shares in the two types of consumption.

consumption, while the parameter ρ captures the *inter-temporal* elasticity of substitution. The remaining parameter in the utility function, $\underline{d}$, is a durable consumption floor that guarantees that U is still bounded even if d is equal to zero.

Overall, households value their discounted expected lifetime utility according to:

$$\mathbb{E}_0 \left\{ \sum_{j=1}^{J} \beta^{j-1} U(g(c_j, d_j)) \right\} \tag{4}$$

where expectations are taken with respect to the survival probabilities and the stochastic labor productivity. Moreover, the instantaneous utility at death is normalized to zero since we do not model bequests motives.

Individuals are endowed with an initial financial position $a_0 \geq 0$ and initial stock of durable goods $n_0 \geq 0$. We will assume that these initial conditions are drawn from a log-normal distribution with mean μ_{a_0} and variance $\sigma^2_{a_0}$ for the financial asset, and mean μ_{n_0} and variance $\sigma^2_{n_0}$ for the durable's stock.

In addition to their initial asset position, an individual in this economy is endowed with one indivisible unit of labor that supplies inelastically to the labor market. Individuals differ, however, in their labor productivity due to their age and idiosyncratic uncertainty. In particular, the stochastic labor productivity of an individual of age j is given by $\xi_j \eta_j$, where ξ_j corresponds with the transitory component of labor productivity and η_j with the persistent one. As in De Nardi et al. (2020), the former is characterized by an age-dependent state space $\xi_j \in \{\xi_j^1, \ldots, \xi_j^M\}$ and a vector of weights $p^\xi \in \{p_1^\xi, \ldots, p_M^\xi\}$; while the latter follows a finite-state (age dependent) Markov chain with state space $\eta_j = \{\eta_j^1, \ldots, \eta_j^N\}$ and transition probabilities given by the matrix Π^j of size $N \times N$. In other words, we assume that all individuals of a given age and independent of other characteristics face the same transition probabilities. Thus, the fraction of people transitioning from η to η' at any given age j is given by $\Pi^j(\eta' \mid \eta)$. As a result, using the law of large numbers and the model's demographic structure we ensure that the aggregate labor demand is fixed in this economy.

2.4. The Household Problem

In any given period, an agent i of age j chooses non-durable consumption $c_{i,j}$ and durable consumption $d_{i,j}$ to maximize their expected discounted life-time utility subject to their budget constraint and a borrowing constraint. That is, each household solves:

$$
\begin{aligned}
&\max_{c_{i,j}, d_{i,j} \geq 0} \mathbb{E}_0 \left\{ \sum_{j=1}^{J} \beta^{j-1} U(g(c_{i,j}, d_{i,j})) \right\} \qquad \text{s.t.} \\
&c_{i,j} + d_{i,j} + a_{i,j+1} = \mathcal{I}_{\{j \leq J^{ret}\}} \omega \xi_{i,j} \eta_{i,j} + \mathcal{I}_{\{j > J^{ret}\}} p + R a_{i,j} + (1-\delta^d) d_{i,j-1} - A(d_{i,j}, d_{i,j-1}) \\
&a_{i,j+1} \geq -(1-\theta) \left[(1-\delta^d) d_{i,j} - A(0, d_{i,j}) \right]
\end{aligned} \tag{5}
$$

where $\mathcal{I}$ is an indicator function that establishes if an agent is in his working age or retired, p is a pension entitlement that will depend on previous labor income and ω and R are the aggregate wage and interest rate, respectively. The parameter $\theta \in [0, 1]$ captures the down payment requirement associated to durable purchases and the function $A(d_{i,j}, d_{i,j-1})$ is the fixed adjustment cost that households face when adjusting their durable stock. We assume that A takes the form:

$$
A(d, d_-) = \begin{cases} 0 & \text{if } d = (1-\delta^d) d_- \\ \tau (1-\delta^d) d_- & \text{if } d \neq (1-\delta^d) d_- \end{cases} \tag{6}
$$

Therefore, households face two market frictions that create non-linearities at the individual level. First, adjusting the durable stock is associated with a proportional adjustment cost controlled by τ. And second, households cannot borrow more than a fraction $(1-\theta)$ of the pledgeable part of their next-period stock of durables. In short, they face a credit constraint.

2.4.1. Recursive Formulation

Given this structure, the sequential problem can be recast into recursive form in the following way. Let a denote current financial assets, n denote the current stock of durable goods, η represent the persistent component of labor productivity, p be the pension entitlement for retired agents and j denote age. These variables compose the state space of either a working or retired agent.

In terms of choices, a household, independently of being a worker or a retiree, decides on non-durable consumption: c; whether to adjust or not their durable stock, and in the former case, by how much: d. In particular, these optimal decision rules result from solving the dynamic programming problems described below.

(i) A *working agent*'s discrete choice is given by:

$$V^w(a,n,\eta,j) = \max\left\{V^w_{NA}(a,n,\eta,j), V^w_A(a,n,\eta,j)\right\} \tag{7}$$

where the $V^w_{NA}(\cdot)$ denotes the value of a worker that decides not to adjust its stock of durables, while $V^w_A(\cdot)$ corresponds to the value of a worker that adjusts it. These two value functions are the result of solving the following two sub-problems. On the one hand, the value of keeping the stock of durables is given by:

$$\begin{aligned} V^w_{NA}(a,n,\eta,j) &= \max_c u(c,n) + s_j\beta\mathbb{E}\left[V^w(a',n',\eta',j+1)\right] \\ \text{s.t.} \quad & a' = Ra + \omega\xi_j\eta_j - c \\ & n' = (1-\delta^d)n \\ & a' \geq -(1-\theta)(1-\tau)n' \end{aligned} \tag{8}$$

where a worker of age j only decides how much to spend on the non-durable good given its portfolio/asset position and labor productivity. On the other hand, the value of adjusting results from the workers's decision of how much to spend in both non-durable and durable consumption as shown in the problem below:

$$\begin{aligned} V^w_A(a,n,\eta,j) &= \max_{c,d} u(c,d) + s_j\beta\mathbb{E}\left[V^w(a',n',\eta',j+1)\right] \\ \text{s.t.} \quad & a' = Ra + \omega\xi_j\eta_j + (1-\tau)n - c - d \\ & n' = (1-\delta^d)d \\ & a' \geq -(1-\theta)(1-\tau)n' \end{aligned} \tag{9}$$

(ii) A *retired agent*'s discrete choice is given by:

$$V^r(a,n,p,j) = \max\left\{V^r_{NA}(a,n,p,j), V^r_A(a,n,p,j)\right\} \tag{10}$$

where $V^r_{NA}(\cdot)$ and $V^r_A(\cdot)$ denote the value of non-adjusting or adjusting the stock of durables for a retired agent, respectively. In particular, the value of non-adjusting for a retiree is given by:

$$
\begin{aligned}
V^{r}_{NA}\left(a,n,p,j\right) &= \max_{c} u\left(c,n\right) + s_j \beta V^{r}\left(a',n',p,j+1\right) \\
\text{s.t.} \quad & a' = Ra + p - c \\
& n' = (1-\delta^{d})n \\
& a' \geq 0
\end{aligned} \tag{11}
$$

while the value of adjusting is:

$$
\begin{aligned}
V^{r}_{A}\left(a,n,p,j\right) &= \max_{c,d} u\left(c,n\right) + s_j \beta V^{r}\left(a',n',p,j+1\right) \\
\text{s.t.} \quad & a' = Ra + p + (1-\tau)n - c - d \\
& n' = (1-\delta^{d})n \\
& a' \geq 0
\end{aligned} \tag{12}
$$

A few things are worth discussing regarding the household problem. First, note from the second constraint in problems (8) and (11) that tomorrow's stock of durables, if the agent does not adjust, corresponds to the non-depreciated part of the current stock. That is, he/she does not sale and then buys a durable good of the same size, but rather keeps it. That's indeed more realistic than other specifications in which that was the case. Second, from the budget constraint in problems (9) and (12) one can observe that adjusting the stock of durables requires selling your current stock, and such action also implies losing a fix fraction of the value of your current stock $(1-\tau)n$. These costs correspond for example to broker's fees, titling costs, etc. As mentioned above, this will generate some non-linearities at the individual level which will complicate enormously the solution of the problem. On these grounds, we follow the approach proposed by Druedahl (2020), which has also been used in Harmenberg and Öberg (2021)[6]. Third, note that p has replaced η in the retiree's state space since he/she does no longer work and his/her pension depends on his/her pre-retirement labor productivity at age J^{ret} (De Nardi et al., 2020)[7]. As it will become clear in Section 3.1, the pre-retirement level of both, transitory and permanent, components of labor productivity, which summarize the entire working history of an agent, will be used to compute the pension entitlement.

[6] For more details on how to solve this model see Appendix A.

[7] See Section 3.1 for a complete description of the retirement scheme.

2.5. Equilibrium

We study an equilibrium in which prices, wages and interest rates are constant across time. In particular, we normalize the price of the final good to 1 and let $r = R - 1$ and ω denote the interest rate and wage rate for efficiency units of labor.

Two remarks should be made before stating the equilibrium definition. First, the relative prices of the durable and non-durable "goods" is unity because we assumed that there is single good in this economy. And second, households take these prices as given when solving their individual optimization problems, i.e. they are price-takers.

DEFINITION. *A stationary equilibrium is a pair of value functions* $\{V^\star_{NA}, V^\star_A\}$*; a set of policy functions for the household* $\{c^\star_{NA}, c^\star_A, d^\star_A\}$*; labor and capital demand for the representative firm* $\{K, L\}$*; and prices* $\{\omega, r\}$ *such that:*

(i) *Given* $\{\omega, r\}$, $\{V^\star_{NA}, V^\star_A\}$ *solve the household problem and* $\{c^\star_{NA}, c^\star_A, d^\star_A\}$ *are the associated policy functions.*

(ii) *Input prices satisfy:*

$$r = F_K(K, L) - \delta$$
$$\omega = F_L(K, L)$$

(iii) *Markets clear:*

$$\sum_{j=1}^{J} \sum_{i=1}^{N} \lambda_j a^\star_{ij} = K \qquad \text{(Capital Market)}$$
$$\sum_{j=1}^{J^{ret}} \sum_{i=1}^{N} \lambda_j \xi_{ij} \eta_{ij} = L \qquad \textit{(Labor Market)}$$
$$\sum_{j=1}^{J} \sum_{i=1}^{N} \lambda_j c^\star_{i,j} + \delta \sum_{j=1}^{J} \sum_{i=1}^{N} \lambda_j a^\star_{i,j} + \delta^d \sum_{j=1}^{J} \sum_{i=1}^{N} \lambda_j n^\star_{i,j} = F(K, L) \qquad \textit{(Goods Market)}$$

where $c^\star$ *is determined in such way that it is equal to* $c^\star_{NA}$ *if* $V^\star_{NA} \geq V^\star_A$ *and* $c^\star_A$ *otherwise. This same reasoning also applies for* $a^\star$ *and* $n^\star$*.*

It is also important to discuss at least three modeling decisions, mostly motivated to reduce the computational burden, which are evident from the equilibrium definition. First, there is no government despite we distinguish between working and retirement age. That is the pension entitlement is exogenous to the model. However, we still compute it based on previous working histories, which are themselves summarized by the persistent and transitory components of labor productivity at the terminal working age: $\eta_{J^{ret}}$ and $\xi_{J^{ret}}$. As discussed in Section 3.1, these correspond with after tax labor earnings; thus, in order to compute the pension entitlement we need to recover the pre-tax earnings. Here we follow the approach in Kaplan and Violante (2010). Second, there is a positive flow of accidental bequests in each period associated with the mortality risk and the absence of annuity markets. We do not redistribute them back to the households in the form of transfers and thus they get lost. And third, we do not solve explicitly for the distribution of individuals over assets and productivities but rather simulate the lifecycle of N households and use this panel to aggregate over while taking into account the age distribution of the economy.[8]

3. Quantitative Analysis

This section discusses how the model is taken to the data and presents results based on the stationary equilibrium defined above. In Section 4 we will extend this framework to allow for aggregate uncertainty.

3.1. Calibration

The model period is one year. Agents enter the economy at age 25, they retire with certainty at age 60 and live until age 85. That is model age 0 corresponds to 25 years old and model's retirement and terminal ages are $J^{ret} = 36$ and $J = 60$, respectively. The population growth rate is set to 1.2% per year, and the survival probabilities are taken from Bell et al. (1992).

We use a Cobb-Douglas production function $F(K_t, L_t) = AK_t^{\alpha^f} L_t^{1-\alpha^f}$ for the technology that produces the final good. We normalize $A = 1$ and set $\alpha^f = 0.3$, which are the standard choices in the literature to capture the long-run labor share of national income for the US economy of approximately $1 - \alpha^f = 0.7$. We set the depreciation rate of physical capital to 0.1125 (Fernández-Villaverde and Krueger, 2011).

[8] We make sure that N is sufficiently large.

In regards to household's parameters, we set the coefficient of risk aversion to 2.0, a standard value in the literature. The discount factor is set to 0.965 as in Druedahl (2020) but we should calibrate it to match the wealth to income ratio. There is not a consensus on what the value for the *intratemporal* elasticity of substitution between non-durable and durable consumption should be, but most of the literature assumes a constant expenditure share. We set the non-durable share to 0.9 as in Druedahl (2020). For the durable good's depreciation rate we also follow Druedahl (2020), but we should probably calibrate it to match the investment share as in Fernández-Villaverde and Krueger (2011). The proportional adjustment cost is set to 10% of the durables' stock (Druedahl, 2020). Finally, the down payment requirement is set free in our computation of the equilibrium because our definition of durable goods includes different commodities: houses, cars, furniture, etc., and, as argued by Luengo-Prado (2006), down-payments will likely be very different for different categories of durable goods.

In the parametrization of the stochastic labor productivity process we follow De Nardi et al. (2020). They use PSID data on after-tax household earnings to estimate two different earning processes: a richer (non-linear and non-normal) earnings process along the lines of Arellano et al. (2017) and a traditional linear earnings process with permanent and transitory components as well as normal innovations as in for example Storesletten et al. (2004). The latter can be described using our notation as follows:

$$\eta_{i,j} = \rho\eta_{i,j-1} + \psi_{i,j}, \tag{13}$$

$$\eta_{i,0} \overset{id}{\sim} \mathcal{N}\left(0, \sigma_{\eta_1}\right), \psi_{i,j} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{\psi}\right),\ \xi_{i,j} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{\xi}\right) \tag{14}$$

while the former can be written by replacing the process for η, equation (13), with:

$$\eta_{i,j} = Q_\eta\left(\nu_{i,j} \mid \eta_{i,j-1}, j\right), \nu_{i,j} \overset{iid}{\sim} U(0,1), j > 1 \tag{15}$$

where $Q_\eta(\nu \mid \cdot)$ denotes a conditional quantile function for the variable η at the ν^{th} conditional quantile of η. In both cases, total labor productivity is the result of the product of the persistent component η and the transitory one ξ.

For the social security benefits, we follow De Nardi et al. (2020) who draw on Kaplan and Violante (2010) themselves. Consequently, we let the pension entitlement to be a function of lifetime earnings, summarized by the persistent and transitory components of labor productivity. In particular, we specify that the benefits are equal to 90 percent of past gross earnings up to a given bend point (0.18 times the cross-sectional average of gross earnings), 32 percent from this first bend point to a second bend point (1.10 times the cross-sectional average of gross earnings) and 15 percent beyond that. Consequently, computing the pension benefits requires a measure of gross earnings. For that purpose, we follow also De Nardi et al. (2020), but with the subtle difference that we use the process for labor productivity instead of pre-tax earnings because ours is a general equilibrium model.

In regards to the simulation of the life-cycle profiles we make the following choices. We generate idiosyncratic labor productivity shocks using the discretized after-tax earnings process in De Nardi et al. (2020). For such purpose, we need to specify an initial distribution for labor productivity at the first age we consider, i.e. age 25 in reality and age 0 in the model. Ideally this should come from an empirical distribution, but for now it is drawn from a log-normal distribution with zero mean and variance equal to 0.2. The initial endowments of physical capital and durables are specified in a similar way, i.e. they are drawn from a log-normal distribution with means equal to 0.1, 0.8 and variance of 0.2 as shown in the table below.

3.2. Model Restrictions

At this stage and for a given vector of parameters, we are able to solve the model presented in Section 2 using computational methods.[9] Unfortunately, as of today, we have only been able to solve the model above for the workers. Thus, results presented below correspond to this simplified version of the model where retirement is not yet included. Moreover, we also impose that $\theta = 1$, i.e. no borrowing.

3.3. Individual choices, Life-Cycle Moments and Simulated Profiles

This section depicts the policy functions of the household choices. There are a couple of key features. First, as always, we are interested in the life-cycle patterns of spending, both on durable and non-durables. Second, given the non-convexity, it is interesting to understand the life-cycle patterns of adjustment but also how the adjustment choice depends on the individual state, principally on wealth and income, whose sum is represented here as cash-on-hand.

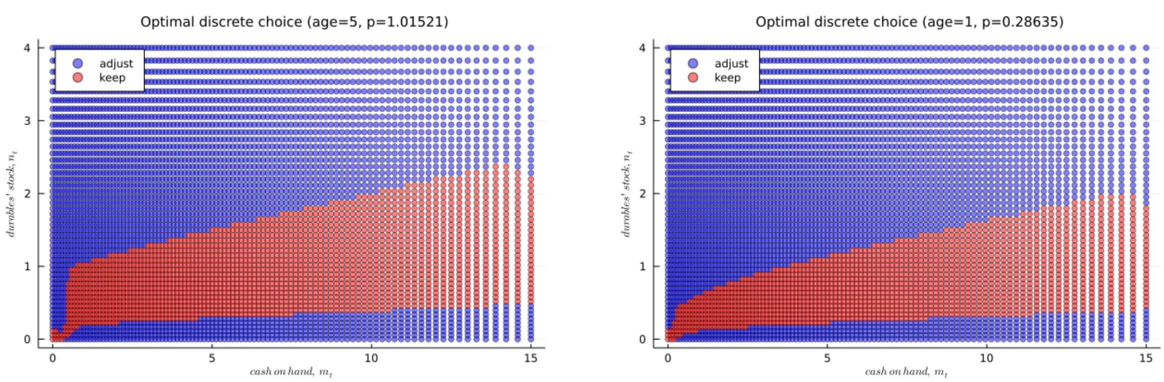
[9] See Appendix A for more details on the numerical approach.

Parameter	Value	Description	Target
Households			
J	61	Model's terminal age	De Nardi et al. (2020)
J^{ret}	36	Model's retirement age	De Nardi et al. (2020)
β	0.965	Discount factor	Druedahl (2020)
ρ	2.00	Relative risk aversion	Standard value
α	0.90	Non-durable consumption share	Druedahl (2020)
τ	0.10	Proportional adjustment cost	Druedahl (2020)
δ^d	0.15	Durable good's depreciation rate	Druedahl (2020)
Firms			
A	1.00	Total factor productivity	Normalized
α^f	0.30	Capital share	Fernández-Villaverde and Krueger (2011)
δ	0.1125	Physical capital's depreciation rate	Fernández-Villaverde and Krueger (2011)
Social Security			
$\{b_1, b_2\}$	$\{0.18, 1.10\}$	Bend points for SS benefits	Kaplan and Violante (2010)
$\{f_1, f_2, f_3\}$	$\{0.90, 0.32, 0.15\}$	Fraction of past earnings for SS benefits	Kaplan and Violante (2010)
Simulation			
N	$10,000$	Number of simulated households	Ad-hoc
μ_{η_0}	0.00	Mean of initial labor productivity	Ad-hoc
$\sigma^2_{\eta_0}$	0.20	Variance of initial labor productivity	Ad-hoc
μ_{a_0}	0.10	Mean of the initial asset position	Druedahl (2020)
$\sigma^2_{a_0}$	0.20	Variance of the initial asset position	Druedahl (2020)
μ_{n_0}	0.80	Mean of the initial durable's stock	Druedahl (2020)
$\sigma^2_{n_0}$	0.20	Variance of the initial durable's stock	Druedahl (2020)

Figure 1 depicts the discrete choice of a household for a given individual state.[10] In particular, the right panel shows the discrete choice for a household given the mid level of permanent labor productivity, $\eta_j = \eta_j^9$; while the left panel plots the same discrete choice but fixing age to the initial age, $j = 1$, and allowing the permanent productivity to vary. To be clear, these decision rules result from the household optimization problem given the equilibrium prices that clear the capital and labor markets.

From these two simple plots, we can already see some of the most important mechanisms in our economy. First, households with very little durable holdings will increase their stock in the following period, and in particular, the maximum level of durables that still induces them to make such adjustment is increasing in the level of cash-on-hand, age and labor productivity. Graphically, this translates into an increase in the (blue) adjusting area below the non-adjusting (red) region as we move along the x-axis and increase age or labor productivity. Second, the inaction (red) region is also increasing in cash-on-hand, age and labor productivity. In words, richer and older agents are more satisfied with their current durable holdings. Third, the durable stock is also used for smoothing consumption, i.e. agents with very little cash-on-hand will sell their durable stock. This can be seen by looking at the blue dots in the left part of both graphs. In short, households employ a three dimensional (S,s) type decision rule due to the presence of adjustment costs.

Figure 1: Household decision rules



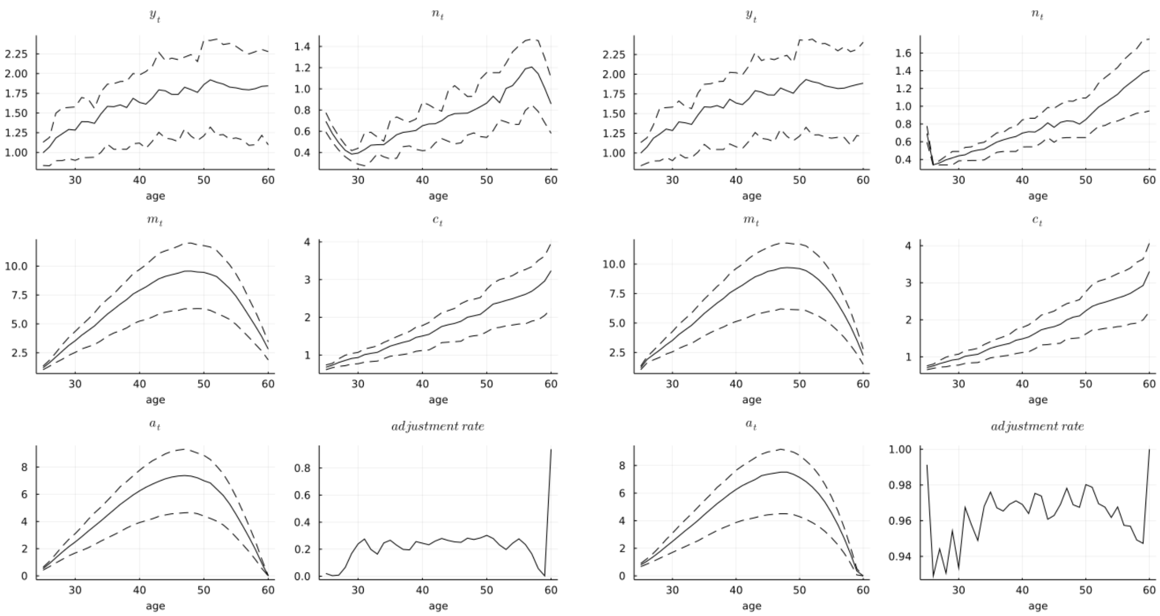
(a) For a given persistent productivity level

(b) For a given age

[10] We have discretized the state space using non-uniform girds for cash-on-hand and the stock of durables of size 100 and 50 points, respectively. The labor productivity process uses the discretized version of De Nardi et al. (2020) with 18 bins for the permanent component and 8 bins for the transitory one.

Given the optimal policy functions, we simulate the lifecycle of 10,000 households and look at the average lifecycle patterns of labor and financial income, nondurable consumption, the stocks of consumer durables, financial assets as well as the adjustment rate. Figure 2 plots these profiles for both an economy with proportional adjustment costs (left panel) as well as for an economy without these costs (right panel).

Figure 2: Household life-cycle profiles



(a) Proportional Adjustment Costs (b) Zero Adjustment Costs

In both worlds, financial assets exhibit the usual hump shape, however the evolution of the durable's stock differs significantly. Simply, just by looking at the adjustment rate over the lifecycle, we can observe that households make infrequent adjustments in the presence of these costs. Even though these smooth out at the mean, we can still observe that this behavior is still present for the 25^{th} and 75^{th} percentiles of the durable stock, which in turn hints that these costs may generate some non-linear response at the aggregate.

We explore this particular feature, i.e. the transmission of non-linearities at the individual level to the aggregate, a little further by means of a comparison of steady states. In particular, we solve the stationary equilibrium under different regimes for the adjustment costs (proportional vs. zero costs) and the labor productivity process (linear vs. non-linear). Results are summarized in the table below.

	Non-linear Income		Linear Income	
	Zero Costs	*Proportional Costs*	*Zero Costs*	*Proportional Costs*
r	5.24%	5.37%	8.73%	8.80%
ω	0.904	0.902	0.833	0.832
L	1.154	1.154	1.669	1.669
K	4.513	4.492	3.482	3.476
K^d	0.742	0.702	0.578	0.547
$(K+K^d)/Y$	2.308	2.293	2.188	2.175
C/Y	0.728	0.730	0.742	0.744

First, in terms of prices it seems that the role of the adjustment cost is minimal, however, the labor productivity process has a significant impact, mostly through its impact on capital accumulation. And second, the presence of adjustment cost does not seem to affect total investment, but rather its timing.

A word of caution is due, because as mentioned above, these calculations rely on the assumptions of no-borrowing and no retirement period. The inclusion of these two features may very well modify the results significantly. In part, including a retirement phase is motivated by the fact that the equilibrium interest rates are unreasonably too high (above $(1-\beta)/\beta$)). However, the inclusion of borrowing will likely work in the opposite direction, but is undoubtedly a more realistic framework that we wish to explore.

4. Aggregate Implications

Here we describe a couple of exercises related to the understanding the aggregate implications of the model.

4.1. Income Uncertainty and Dispersion

Here we study how the steady state depends on the underlying distribution of household income. In particular, we study both the effects of increased uncertainty on spending as well as the impact of increased dispersion. Here, uncertainty influences belief without actual changes in the distribution of income.

This exercise, related to Berger and Vavra (2015), is enriched by the presence of different forms of durable adjustment costs as well as borrowing constraints. In some cases, the greater dispersion of income can limit borrowing and thus impact the aggregate economy.

4.2. Monetary Policy

The goal here is to assess the effects of both income and return variations induced by monetary policy. The plan is not to build a dynamic general equilibrium model with money, but rather as in Ampudia et al. (2020), to study the effects of monetary innovations through their impact on income and returns at the household level. This analysis will be supplemented by the adjustment frictions at the household level.

5. Conclusions

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A. Solution Method

This part of the appendix presents our computational strategy to solve the model presented in Section 2. We first solve the household problem with no borrowing constraints following closely the approach in Druedahl (2020). Then, we move to a slightly more complicated household problem in which borrowing against your durable stock is allowed. Here we follow the approach in Harmenberg and Öberg (2021) of normalizing the credit constraint in such way that the new normalized measure of assets must be greater or equal than zero. As a result, the same computational tricks can be used to solve the household problem with borrowing constraints. Finally, we integrate the household problem into general equilibrium. For such purpose it is important to remember that the relative price of non-durable and durable consumption is unity, i.e. there is only one good in this economy. Consequently, one can make use of the standard algorithm to find an equilibrium interest rate and an associated wage rate as in Aiyagari (1994) or Huggett (1996).

A.1. Household Problem with No Borrowing

Take the worker's dynamic optimization problem in recursive formulation from Section 2. Reproduced below for readability after imposing no borrowing, i.e. $\theta = 1$.

$$V\left(a,n,\eta,j\right) = \max\left\{V_{NA}\left(a,n,\eta,j\right), V_{A}\left(a,n,\eta,j\right)\right\} \tag{A.1}$$

where

$$\begin{aligned} V_{NA}\left(a,n,\eta,j\right) &= \max_{c} u\left(c,n\right) + s_j\beta\mathbb{E}\left[V\left(a',n',\eta',j+1\right)\right] \\ \text{s.t.} \quad & a' = Ra + \omega\xi_j\eta_j - c \geq 0 \\ & n' = (1-\delta^d)n \end{aligned} \tag{A.2}$$

and

$$\begin{aligned} V_{A}\left(a,n,\eta,j\right) &= \max_{c,d} u\left(c,d\right) + s_j\beta\mathbb{E}\left[V\left(a',n',\eta',j+1\right)\right] \\ \text{s.t.} \quad & a' = Ra + \omega\xi_j\eta_j + (1-\tau)n - c - d \geq 0 \\ & n' = (1-\delta^d)d \end{aligned} \tag{A.3}$$

First, following the insights in Judd et al. (2017), compute the continuation value for a given combination of post-decision states: end-of-period assets a', end-of-period durable stock d and the permanent component of labor productivity η. That is, specify the *post-decision value function* using Druedahl (2020)'s terminology as:

$$W(a', d, \eta, j) = s_j \beta \mathbb{E}\left[V\left(m', n', \eta', j+1\right)\right] \tag{A.4}$$

where we have already re-defined the problem in terms of the worker's cash on hand $m' \equiv Ra + \omega \xi' \eta'$.

Then, re-write the non-adjuster problem in the following way:

$$\begin{aligned} V_{NA}\left(m, n, \eta, j\right) = & \max_{c} u\left(c, n\right) + W(a', d, \eta, j) \\ \text{s.t.} \quad & a' = m - c \geq 0 \\ & d = n \end{aligned} \tag{A.5}$$

Second, one can argue that the consumption decision for the adjuster problem is exactly the same as that of the non-adjuster that starts the period with a chosen stock of durables and some level of cash-on-hand. Thus, following the nesting structure in Druedahl (2020), re-write the adjuster problem as:

$$\begin{aligned} V_{A}\left(x, \eta, j\right) = & \max_{d} \; V_{NA}\left(m, d, \eta, j\right) \\ \text{s.t.} \quad & m = x - d \\ & d \in [0, x] \end{aligned} \tag{A.6}$$

where $x \equiv m + (1-\tau)n$ is the post-decision cash-on-hand. As discussed in Druedahl (2020), these two modifications lead to speed improvements because first it reduces the number of interpolations required to solve the non-adjuster problem, and second it reduces the dimensionality of the adjuster problem.

A.2. Normalizing the Borrowing Constraint

Consider the household problem from Section 2, and in particular focus on the collateral constraint, which written in its canonical form is given by:

$$a' + (1-\theta)(1-\tau)n' \geq 0 \tag{A.7}$$

Thus, if we make the following variable substitution $\hat{a}' = a' + (1-\theta)(1-\tau)n'$ as in Harmenberg and Öberg (2021), we can recast the problem in such a way that allow us to solve the model with collateral constraints and still use the same structure as in the no-borrowing case.

First, focus on the non-adjuster problem:

$$\begin{aligned} V_{NA}(m,n,\eta,j) = \max_{c} u(c,n) + W(\hat{a}',d,\eta,j) \\ \text{s.t.} \quad & \hat{a}' = m + (1-\delta-R)(1-\theta)(1-\tau)n - c \\ & d = n \\ & \hat{a}' \geq 0 \end{aligned} \tag{A.8}$$

where on top of the tricks from Section A.1 we have also included the new re-definition of the variable $\hat{a}$ to accommodate the presence of collateral constraints.

For the adjuster problem, we can still use the post-decision cash-on-hand as a state variable to reduce the dimensionality of the state space. However, now it is defined as $\hat{x} = m + (1 + R(1-\theta))(1-\tau)n = x + R(1-\theta)(1-\tau)n$. Therefore, the problem read now as follows:

$$\begin{aligned} V_{A}(\hat{x},\eta,j) = \max_{d} \; V_{NA}(m,d,\eta,j) \\ \text{s.t.} \quad & m = \hat{x} - d \\ & d \in [0,\hat{x}] \end{aligned} \tag{A.9}$$

A.3. General Equilibrium

Finding a stationary equilibrium in this economy amounts to finding an interest rate $r^\star$ that clears the asset market. This is the case because the labor market equilibrium is determined exogenously through the labor endowment process and the remaining market, the goods market, can be ignored by Walras' Law.

Therefore, we implement the following standard algorithm:

(i) Fix an interest rate $r \in (-\delta, \frac{1-\beta}{\beta})$. For a fixed $R = 1 + r$, solve the household problem and obtain the value and policy functions. Note that they will depend upon the interest rate chosen.

(ii) Given the value and the policy functions, simulate the life-cycle for a panel of households. Then, use these simulated paths to aggregate over them using the demographic structure of the model and compute the aggregate/mean level of financial assets: $\mathbb{E}\left[a(r)\right]$.

(iii) Using the firm's F.O.C. and guessed prices, r and ω, compute the aggregate level of capital, $K(r)$[11].

(iv) Compute the excess of demand for capital:

$$d(r) = K(r) - \mathbb{E}\left[a(r)\right]$$

where $d(r)$ is just a number. If this number its close to zero (smaller than some pre-defined tolerance level), then we are done and we have found a quasi-equilibrium. If not we update our guess and start again from (i). Since $\mathbb{E}\left[a(r)\right]$ is increasing in r for our particular application, then $d(r)$ is a strictly increasing function. Then, we will increase our guess for r if $d(r) > 0$ and decrease it otherwise.

As you may have noticed, the theoretical range where $r^\star$ may lie is quite wide. Therefore, in order to avoid some unnecessary computations, we first solve for $K(r)$ and $\mathbb{E}\left[a(r)\right]$ over a coarse grid for r. Then, we plot these two functions and look at where they cross. This give us an "eyeball" equilibrium interest rate which we use to initialize the algorithm above.

[11] Note that one can compute the value of the wage rate, ω, using the given interest rate, aggregate labor demand, L, and some parameters. In particular, $\omega = A(1-\alpha)\left[\frac{A\alpha}{r+\delta}\right]^{\alpha/(1-\alpha)}$