Durables, Portfolio Choice & Aggregate Uncertainty

Progress Summary

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Goals

BORROWING CONSTRAINS $\Longrightarrow$ highlighting the triple role of durable goods

- No borrowing
- Collateral constrain, i.e. down-payment
- Endogenous borrowing constraints

THREE ASSET ECONOMY $\Longrightarrow$ bonds, stocks and durables

- Study the real effects of *Monetary Policy*

AGGREAGATE UNCERTAINTY

- Does individual non-linearities translate to the aggregate?

Status

GENERAL EQUILIBRIUM

- Aiyagari (1994, QJE) $\rightarrow$ SIM + Infinite Horizon ✓
- Hugget (1996, JME) $\rightarrow$ SIM + Lifecycle ✓
- Krueger & Fernandez-Villaverde (2011, Macro Dynamics) $\rightarrow$ Hugget + Durables

PARTIAL EQUILIBRIUM

- Druedahl (2020, CE) $\rightarrow$ Durables + Discrete Choice + Micro Frictions ✓
- Berger & Vavra (2015, Ecta) $\rightarrow$ Druedahl + Down-payment + Aggregate Uncertainty

Literature Review

DURABLE GOODS

Borrowing Constraints

- Luengo-Pardo (2006, JME)
- Krueger & Fernandez-Villaverde (2011, Macro Dynamics)

Monetary Policy

- Sterk & Tenreyro (2018, JME)
- McKay & Wieland (2021, R&R Ecta)
- McKay & Wieland (2021, R&R AER: Insights)

Aggregate Demand

- Berger & Vavra (2015, Ecta)
- Harmenberg & Öberg (2021, JME)

Other Assets

- Paz-Pardo (2020, WP ECB)

Literature Review II

SOLUTION METHODS

For the household problem

- Fella (2014, RED)
- Iskhakov et al. (2017, QE)
- Druedahl & Jørgensen (2017, JEDC)
- Druedahl (2020, CE)

For aggregate uncertainty

- Ampudia et al. (2020, ECB WP) $\rightarrow$ data consistent approach
- Fernandez-Villaverde et al. (2020, WP) $\rightarrow$ approximate PLM w/ a NN

Partial Equilibrium

A. The Model

A Consumption-Savings Model with Durable Goods

States

- m_t : cash-on-hand
- n_t : stock of durable good
- p_t : persistent component of income

Controls

- c_t : non-durable consumption
- d_t : durable consumption

Preferences

$$u(c_t, d_t) = \frac{\left(c_t^{\alpha}\left(d_t + \underline{d}\right)^{1-\alpha}\right)^{1-\rho}}{1-\rho}, \ \alpha \in (0,1), \ \rho > 0$$

Budget Constraint

$$a_t = \begin{cases} m_t - c_t & \text{if } d_t = n_t \\ x_t - c_t & \text{if } d_t \neq n_t \end{cases}$$

$$\text{with} \quad x_t = m_t + (1-\tau)n_t$$

Transition equations

$$m_{t+1} = Ra_t + y_{t+1}$$
$$n_{t+1} = (1-\delta)d_t$$

Stochastic Income Process

$$p_{t+1} = \psi_{t+1} p_t^{\lambda}, \quad \log \psi_{t+1} \sim \mathcal{N}\left(-0.5\sigma_{\psi}^2, \sigma_{\psi}^2\right), \ \lambda \in (0,1]$$

$$y_{t+1} = \xi_{t+1} p_{t+1}, \quad \log \xi_{t+1} \sim \mathcal{N}\left(-0.5\sigma_{\xi}^2, \sigma_{\xi}^2\right)$$

Consumption-Savings Model w/ Durables II

$$v_t(m_t, n_t, p_t) = \max\left\{v_t^{keep}(m_t, n_t, p_t), v_t^{adj}(x_t, p_t)\right\}$$
$$\text{s.t.} \quad x_t = m_t + (1-\tau)n_t$$

where

$$v_t^{keep}(m_t, n_t, p_t) = \max_{c_t} u(c_t, n_t) + \beta \mathbb{E}_t\left[v_{t+1}(m_{t+1}, n_{t+1}, p_{t+1})\right]$$
$$\text{s.t.} \quad a_t = m_t - c_t \geq \bar{b}_t(n_t, p_t)$$
$$m_{t+1} = Ra_t + y_{t+1}$$
$$n_{t+1} = (1-\delta)n_t$$

and

$$v_t^{adj}(x_t, p_t) = \max_{c_t, d_t} u(c_t, d_t) + \beta \mathbb{E}_t\left[v_{t+1}(m_{t+1}, n_{t+1}, p_{t+1})\right]$$
$$\text{s.t.} \quad a_t = x_t - c_t - d_t \geq \bar{b}_t(n_t, p_t)$$
$$m_{t+1} = Ra_t + y_{t+1}$$
$$n_{t+1} = (1-\delta)d_t$$

Borrrowing Limits

1. **Ad-hoc borrowing constraints** $\implies$ **No borrowing** ✓

$$\bar{b}_t(n_t, p_t) = 0$$

2. **Collateral constraints** $\implies$ **Fixed down payment** ✓

$$\bar{b}_t(n_t, p_t) = -(1-\theta)n_t$$

3. **Endogenous borrowing constraint à la Krueger & Fernandez-Villaverde**

$$\bar{b}_t(n_t, p_t) := \{b^* \in \mathbb{R} : v_{t+1}(b^*, n_{t+1}, p_{t+1}) \geq v_{t+1}(y_{t+1}, 0, p_{t+1}) \quad \forall p_{t+1}\}$$

B. Parametrization

Calibration: standard values for now

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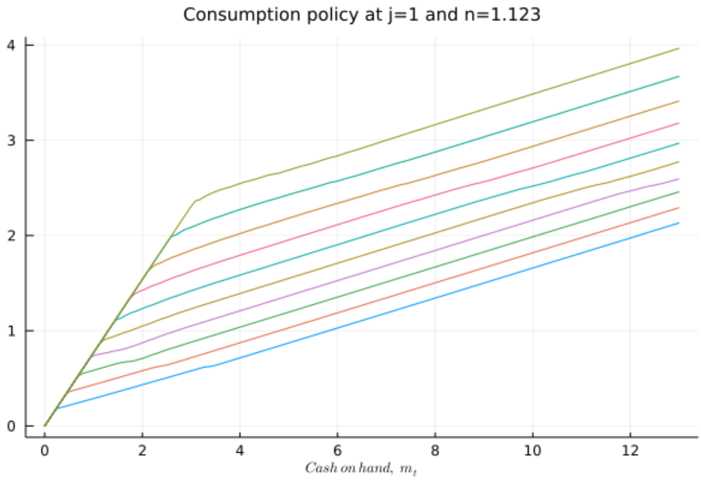
Parameter	*Value*	*Description*	*Target*
β	0.965	Discount factor	TBD
ρ	2.0	Relative risk aversion	TBD
α	0.90	Non-durable consumption share	TBD
τ	0.10	Proportional adjustment cost	TBD
δ	0.15	Durable good's depreciation rate	TBD
θ	1.00	Down payment requirement	TBD
R	1.03	Real interest rate	TBD
λ	1.00	Persistence of income	TBD
σ_ψ	0.10	SD of permanent shock	TBD
σ_ξ	0.10	SD of transitory shock	TBD

C. Household's Decision Rules

No Borrowing: Keeper Problem



No Borrowing: Adjuster Problem (36 years)

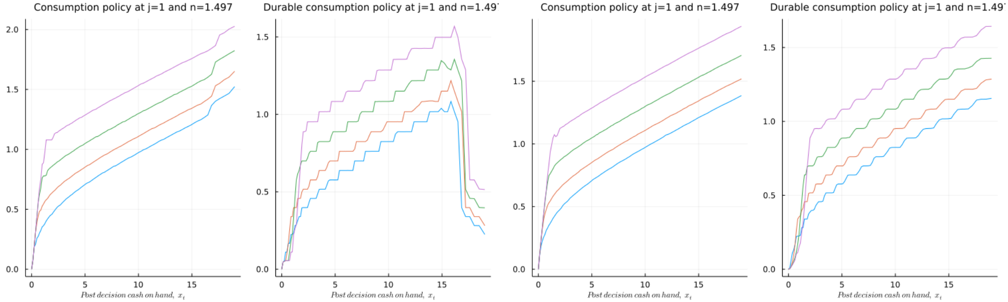


Figure: Adjuster policy functions based on **coarse** grid, J=36. LHS is nvfi, RHS vfi

The role of adjustment costs: Adjuster Problem (36 years)

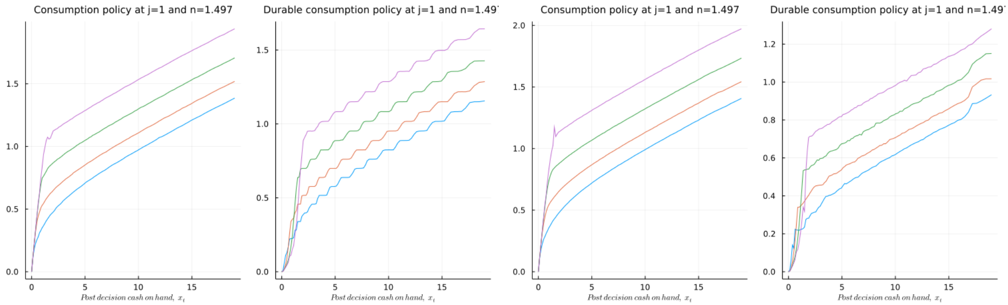


Figure: Adjuster policy functions based on **coarse** grid, J=36, no borrowing. LHS is vfi with adjustment costs, RHS vfi without adjustment costs

No Borrowing: Adjuster Problem (2 years)

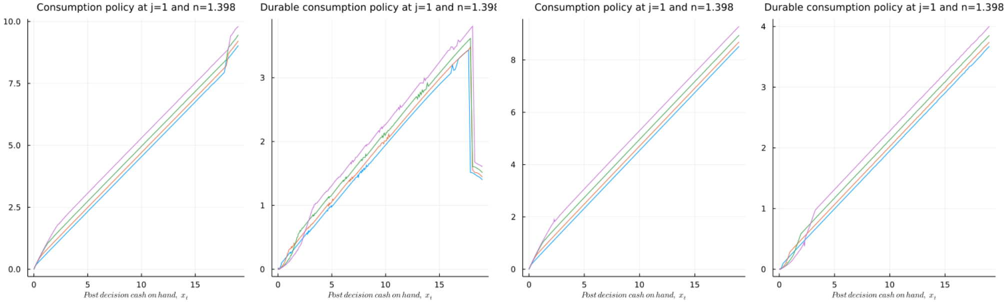
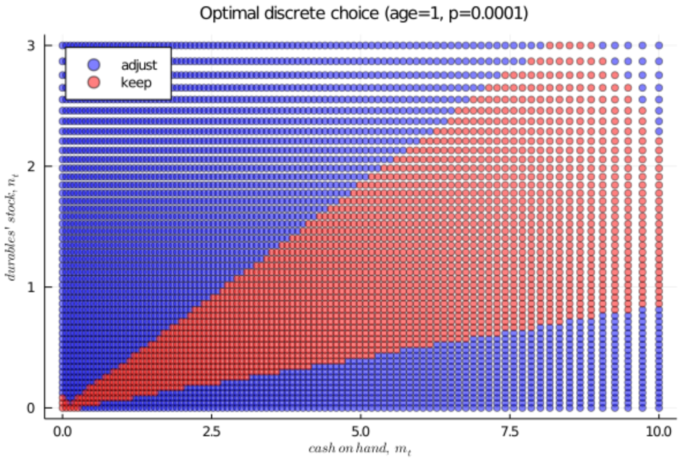
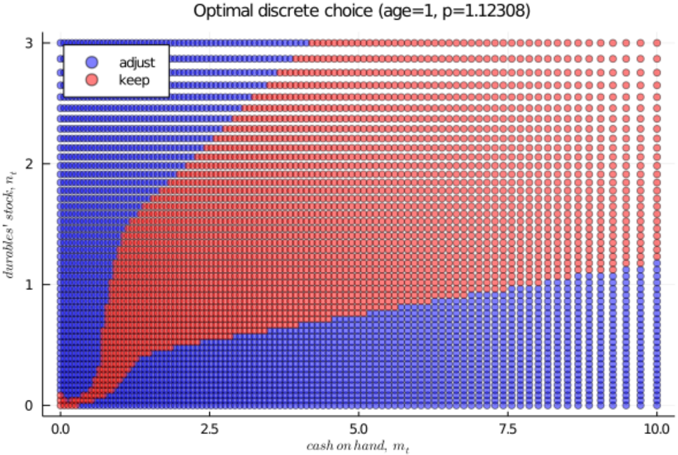


Figure: Adjuster policy functions based on **fine** grid, J=2. LHS is nvfi, RHS vfi

No Borrowing: Adjustment vs. Inaction Regions I



No Borrowing: Adjustment vs. Inaction Regions II



Collateral Constraint: Keeper Problem

Collateral Constraint: Adjuster Problem

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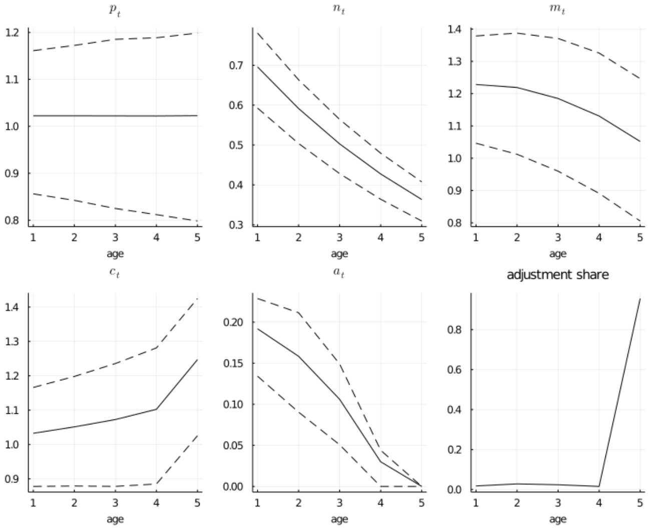
Collateral Constraint: Adjustment vs. Inaction Regions I

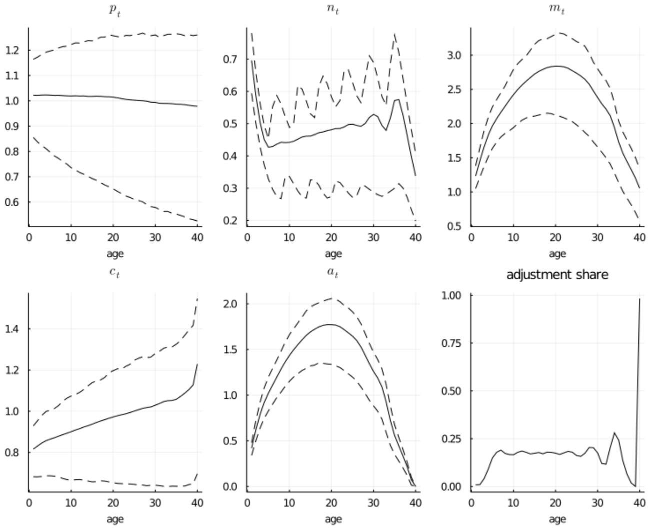
Collateral Constraint: Adjustment vs. Inaction Regions II

D. Life-Cycle Profiles

No Borrowing: Simulated Life-Cycle, J = 5



No Borrowing: Simulated Life-Cycle, J = 40



Collateral Constraint: Simulated Life-Cycle

General Equilibrium

A. The Model

Environment

- The economy is populated by a continuum of measure 1 of **households** that face an income fluctuation problem over their life-cycle.
- Labor income fluctuates because individuals differ in their **stochastic labor productivity** which is itself characterized by an (age-dependent) *Markov process*. Idiosyncratic labor productivity is also subject to a *transitory shock*.
- They can use their labor income to **consume**, either non-durable or durable goods, or **save** it into a financial asset.
- The purchase of **durable goods** can also be seen as an investment, however, their purchases are subject to *proportional adjustment costs*.
- The demand of financial assets (capital) comes from a single representative **firm** that combines it with labor to produce the single final good in the economy.

Households: Demographics

- Each household lives at most J periods. They work during the first $J_R < J$ periods of their lives and then they retire.
- In each period $j < J$ of their life they face a conditional probability of surviving and living in the next period $j+1$. We denote it by $s_j \in (0,1)$ with $s_0 = 1$ and $s_J = 1$.
- In each period a number $\lambda_1 = (1 + \sum_{j=1}^{J-1} \Pi_{i=1}^{j} s_j + g_n)$ of newborns enter the economy and the fraction of people in the economy at age j is defined recursively as:

$$\tilde{\lambda}_{j+1} = \frac{s_j \tilde{\lambda}_j}{1 + g_n}$$

- We normalize the fraction of people at each age such that the total population in this economy is equal one. That is:

$$\lambda_j = \frac{\tilde{\lambda}_j}{\sum_{j=1}^{J} \tilde{\lambda}_j}$$

Households: Endowments & Labor Productivity

- Initial **endowment** of durable consumption good, $n_0 \geq 0$.
- Initial financial position, $a_0 \geq 0$.
- One unit of time that they supply inelastically to the labor market. However, they differ in their **labor productivity** $\eta_{i,j}$, which itself depends also on age. Here, we follow two distinct approaches:

  1. *Canonical (linear) model*:

$$\eta_{i,j} = \rho \eta_{i,j-1} + \psi_{i,j},$$
$$\eta_{i,0} \overset{id}{\sim} \mathcal{N}\left(0, \sigma_{\eta_1}\right), \psi_{i,j} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{\psi}\right), \ \xi_{i,j} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{\xi}\right)$$

  2. *Non-linear model à la De Nardi et al. (2020)*:

$$\eta_{i,j} = Q_\eta\left(\nu_{i,j} \mid \eta_{i,j-1}, j\right), \nu_{i,j} \overset{iid}{\sim} U(0,1), j > 1$$
$$\eta_{i,0} \overset{id}{\sim} \mathcal{N}\left(0, \sigma_{\eta_1}\right), \ \psi_{i,j} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{\psi}\right), \ \xi_{i,j} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{\xi}\right)$$

Households: Optimization

$$v\left(a, n, \eta, j\right) = \max\left\{V_{NA}\left(a, n, \eta, j\right), V_{A}\left(a, n, \eta, j\right)\right\}$$

where

$$\begin{aligned} V_{NA}\left(a, n, \eta, j\right) = \max_{c} u\left(c, n\right) + s_j \beta \mathbb{E}\left[v\left(a', n', \eta', j+1\right)\right] \\ \text{s.t.} \quad a' &= Ra + \omega \xi_j \eta_j - c \\ n' &= (1-\delta^d)n \\ a' &\geq -(1-\theta)(1-\delta^d-\tau)n' \end{aligned}$$

and

$$\begin{aligned} V_{A}\left(a, n, \eta, j\right) = \max_{c,d} u\left(c, d\right) + s_j \beta \mathbb{E}\left[v\left(a', n', \eta', j+1\right)\right] \\ \text{s.t.} \quad a' &= Ra + \omega \xi_j \eta_j + (1-\tau)n - c - d \\ n' &= (1-\delta^d)d \\ a' &\geq -(1-\theta)(1-\delta^d-\tau)n' \end{aligned}$$

Technology: A Single Representative Firm

- There is a **single good** in this economy which is produced combining aggregate capital stock K_t and aggregate labor input L_t. That is:

$$Y_t = F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha}$$

- Since the Cobb-Douglas production function exhibits constant returns to scale, the number of firms is indeterminate at equilibrium. Thus, we can assume without loss of generality that there is a **single representative firm**.

- The final good can be either consumed or invested into physical capital or consumer durables. Thus, the **aggregate resource constraint** is given by:

$$C_t + K_{t+1} - (1-\delta)K_t + K_{t+1}^d - (1-\delta^d)K_t^d = F(K_t, L_t)$$

where C_t is aggregate consumption expenditures and K_t^d is the aggregate stock of consumer durables.

Equilibrium Definition

A stationary equilibrium is a pair of value functions $\{V^{\star}_{NA}, V^{\star}_{A}\}$; a set of policy functions for the household $\{c^{\star}_{NA}, c^{\star}_{A}, d^{\star}_{A}\}$; labor and capital demand for the representative firm $\{K, L\}$; and prices $\{\omega, r\}$ such that:

1. Given $\{\omega, r\}$, $\{V^{\star}_{NA}, v^{\star}_{A}\}$ solve the household problem and $\{c^{\star}_{NA}, c^{\star}_{A}, d^{\star}_{A}\}$ are the associated policy functions.
2. Input prices satisfy:

$$r = F_K(K, L) - \delta$$
$$\omega = F_L(K, L)$$

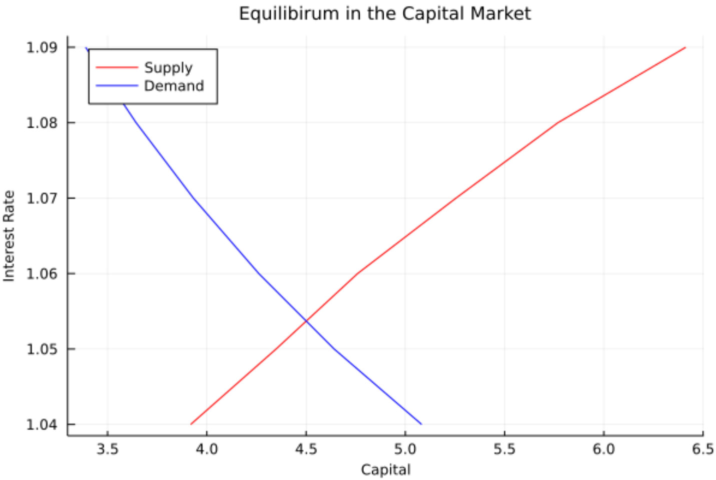
3. Markets clear:

$$\sum_{j=1}^{J}\sum_{i=1}^{N}\lambda_j a^{\star}_{ij} = K \qquad \text{(Capital Market)}$$

$$\sum_{j=1}^{J_R}\sum_{i=1}^{N}\lambda_j \xi_{ij}\eta_{ij} = L \qquad \textit{(Labor Market)}$$

B. The Equilibrium

Graphical Representation



Summary Statistics: Aggregate Variables

Variable	*Steady state value*
r	5.37%
ω	0.902
L	1.154
K	4.492
K^d	0.702
$(K + K^d)/Y$	2.293
C/Y	0.730

Summary Statistics: Life-Cycle Moments

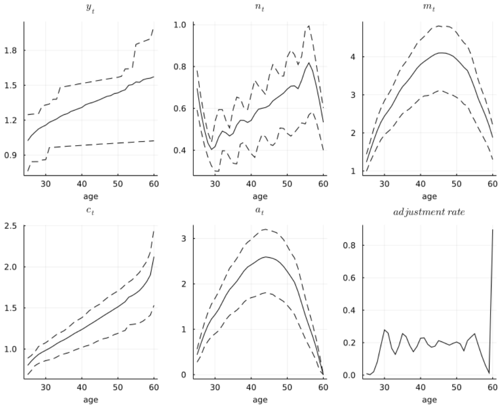
		Age Group						
Variable	*Moment*	*26-30*	*31-35*	*36-40*	*41-45*	*46-50*	*51-55*	*56-60*
y_j	*Mean*	1.213	1.444	1.614	1.723	1.796	1.865	1.817
	Std.	0.502	0.677	0.755	0.776	0.825	0.882	0.955
c_j	*Mean*	0.847	1.104	1.395	1.719	2.046	2.449	2.864
	Std.	0.209	0.341	0.472	0.613	0.747	0.932	1.213
a_j	*Mean*	1.726	3.661	5.580	6.924	7.253	5.788	1.888
	Std.	0.986	1.990	3.011	3.661	3.719	3.166	1.749
n_j	*Mean*	0.454	0.473	0.603	0.710	0.807	0.988	1.081
	Std	0.137	0.193	0.256	0.306	0.362	0.629	0.495
$(a_j + n_j)/y_j$	*Mean*	1.891	3.026	4.126	4.826	5.007	4.156	1.952
	Std.	0.755	1.429	2.177	2.667	2.889	2.781	1.791

Sensitivity Analysis

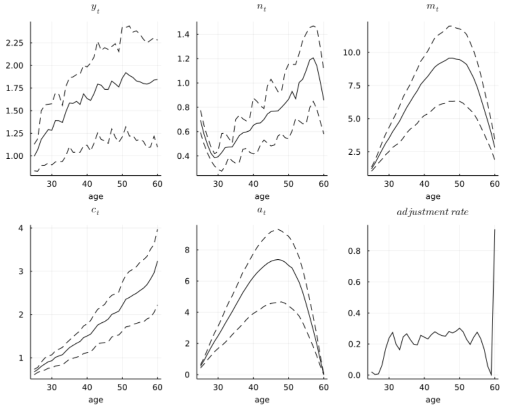
A. The Labor Productivity Process

(Linear versus Non-Linear)

PE: Simulated Life-Cycle Profiles ($R^{\star} \approx 1.05$)

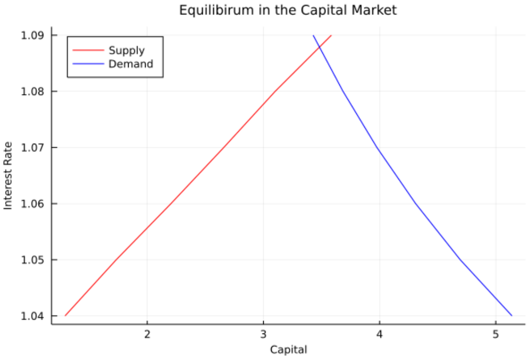


(a) Linear Process

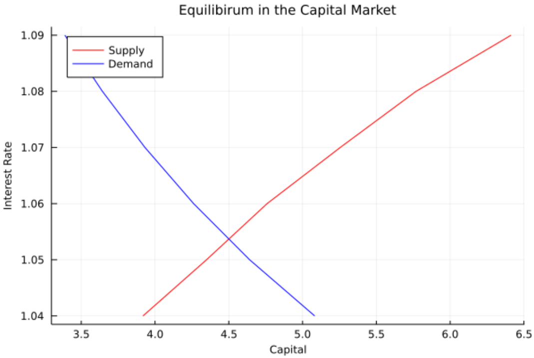


(b) Non-Linear Process

GE: Equilibrium in the Capital Market



(a) Linear Process



(b) Non-Linear Process

GE: Aggregate Results

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Variable	*Non-Linear Process*	*Linear Process*
r	5.37%	8.80%
ω	0.902	0.832
L	1.154	1.669
K	4.492	3.476
K^d	0.702	0.547
$(K + K^d) / Y$	2.293	2.175
C / Y	0.730	0.744

B. Adjustment Costs

PE: Simulated Life-Cycle Profiles ($R^{\star} \approx 1.05$)

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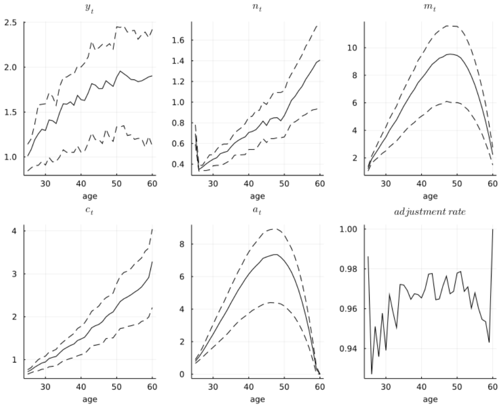
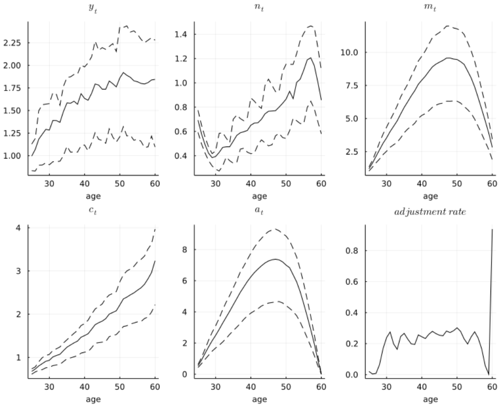
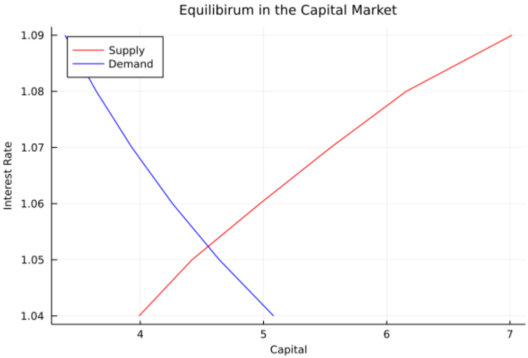


(a) Zero Adjustment Costs

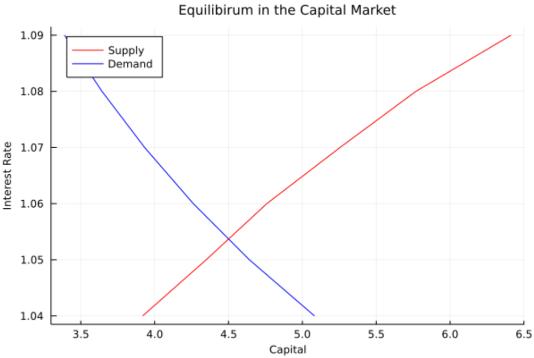


(b) Proportional Adjustment Costs

GE: Capital Market Clearing



(a) Zero Adjustment Costs



(b) Proportional Adjustment Costs

GE: Aggregate Results

Variable	*Zero Adjustment Costs*	*Proportional Adjustment Cost*
r	5.24%	5.37%
ω	0.904	0.902
L	1.154	1.154
K	4.513	4.492
K^d	0.742	0.702
$(K + K^d)/Y$	2.308	2.293
C/Y	0.728	0.730

C. Borrowing Constraints

D. Are There Some Interactions?

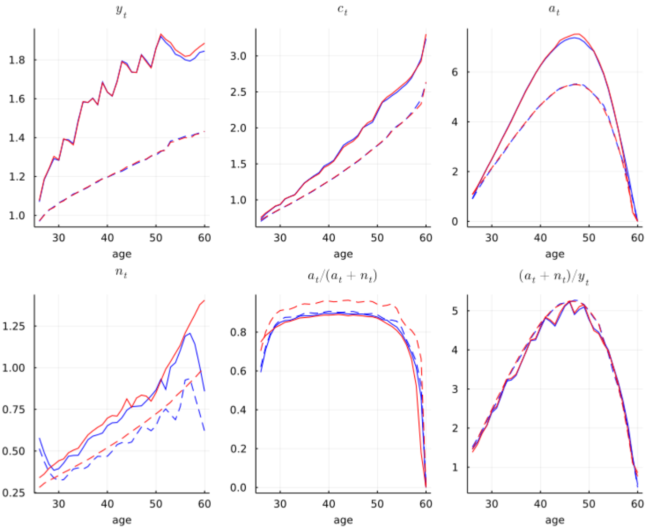
Interactions: labor productivity & adjustment costs I

	Non-linear Income		Linear Income	
	Zero Costs	*Proportional Costs*	*Zero Costs*	*Proportional Costs*
r	5.24%	5.37%	8.73%	8.80%
ω	0.904	0.902	0.833	0.832
L	1.154	1.154	1.669	1.669
K	4.513	4.492	3.482	3.476
K^d	0.742	0.702	0.578	0.547
$(K + K^d) / Y$	2.308	2.293	2.188	2.175
C / Y	0.728	0.730	0.742	0.744

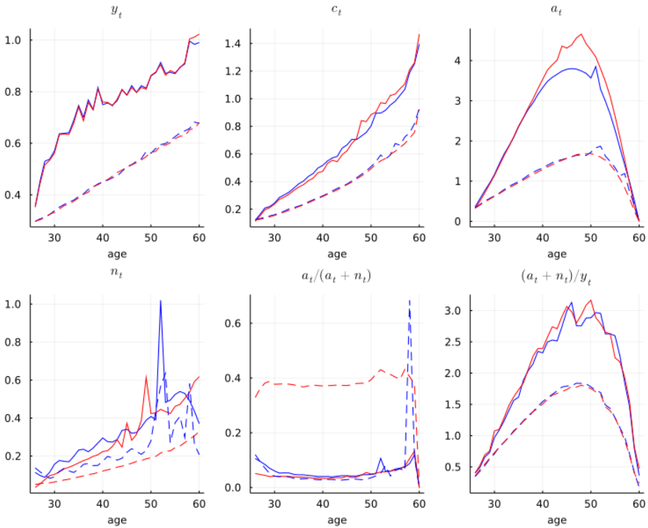
Interactions: labor productivity & adjustment costs II

Cross Sectional Mean

1. Non-Linear + Adj. Costs (blue solid)
2. Non-Linear + Zero Costs (red solid)
3. Linear + Adj. Costs (blue dash)
4. Linear + Zero Costs (red dash)



Interactions: labor productivity & adjustment costs III



Cross Sectional Std.

1. Non-Linear + Adj. Costs (blue solid)
2. Non-Linear + Zero Costs (red solid)
3. Linear + Adj. Costs (blue dash)
4. Linear + Zero Costs (red dash)

Appendix

A. Solution Method

Normalizing the Collateral Constraint

- Define $\hat{a}' \equiv a' + (1-\theta)(1-\tau)n'$
- The non-adjuster problem now reads:

$$V_{NA}(\hat{a}, n, \eta, j) = \max_{c} u(c, n) + s_j \beta \mathbb{E}\left[v\left(a', n', \eta', j+1\right)\right]$$
$$\text{s.t.} \quad \hat{a}' = R\hat{a} + \omega \xi_j \eta_j + (1-\delta-R)(1-\theta)(1-\tau)n - c$$
$$n' = (1-\delta^d)n$$
$$\hat{a}' \geq 0$$

- While the adjuster problem is now given by:

$$V_{A}(\hat{a}, n, \eta, j) = \max_{c,d} u(c, d) + s_j \beta \mathbb{E}\left[v\left(a', n', \eta', j+1\right)\right]$$
$$\text{s.t.} \quad \hat{a}' = R\hat{a} + \omega \xi_j \eta_j + (1-\tau)n + (1-\delta-R)(1-\theta)(1-\tau)n - c - d$$
$$n' = (1-\delta^d)d$$
$$\hat{a}' \geq 0$$

Re-writing the problem in terms of "cash-on-hand"

- Define $m \equiv R\hat{a} + \omega\xi_j\eta_j + (1-\delta-R)(1-\theta)(1-\tau)n$
- Then, we could rewrite the non-adjuster problem as follows:

$$
\begin{aligned}
V_{NA}(m,n,\eta,j) = \max_{c}\, & u(c,n) + s_j\beta\mathbb{E}\left[v(m',n',\eta',j+1)\right] \\
\text{s.t.} \quad \hat{a}' &= m - c \geq 0 \\
n' &= (1-\delta^d)n \\
m' &= R\hat{a}' + \omega\xi_j\eta_j' + (1-\delta-R)(1-\theta)(1-\tau)n'
\end{aligned}
$$

- And the adjuster problem as:

$$
\begin{aligned}
V_{A}(m,n,\eta,j) = \max_{c,d}\, & u(c,d) + s_j\beta\mathbb{E}\left[v(m',n',\eta',j+1)\right] \\
\text{s.t.} \quad \hat{a}' &= m + (1-\tau)n - c - d \geq 0 \\
n' &= (1-\delta^d)d \\
m' &= R\hat{a}' + \omega\xi_j\eta_j' + (1-\delta-R)(1-\theta)(1-\tau)n'
\end{aligned}
$$

Nested Value Function Iteration

- *Post-decision value function:*

$$W\left(\hat{a}', d, \eta, j\right) = s_j \beta \mathbb{E}\left[v\left(m', n', \eta', j+1\right)\right]$$

- *Non-adjuster problem:*

$$\begin{aligned} V_{NA}\left(m, n, \eta, j\right) &= \max_c u\left(c, n\right) + W\left(\hat{a}', d, \eta, j\right) \\ \text{s.t.} \quad \hat{a}' &= m - c \geq 0 \\ d &= n \end{aligned}$$

- *Adjuster problem:*

$$\begin{aligned} V_{A}\left(x, \eta, j\right) &= \max_d V_{NA}\left(m, d, \eta, j\right) \\ \text{s.t.} \quad m &= x - d \\ d &\in [0, x] \end{aligned}$$

Nested Endogenous Grid Method

B. Speed & Accuracy

Speech: Serial Computation

	Method	Age				
		$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
Last period	NVFI+					0.592
	NEGM+					0.588
Post-decision	NVFI+	77.621	75.968	76.696	76.919	
	NEGM+	161.144	146.7429	151.412	151.853	
Keeper	NVFI+	6.651	6.597	6.267	7.524	
	NEGM+	0.570	0.485	0.477	0.771	
Adjuster	NVFI+	4.572	4.860	4.784	4.881	
	NEGM+	4.496	4.392	4.507	4.703	
TOTAL	NVFI+					$5min.\ 55sec.$
	NEGM+					$10min.\ 33sec.$

Speech: Parallel Computation (6 cores)

	Method	Age				
		$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
Last period	NVFI+					2.453
	NEGM+					2.605
Post-decision	NVFI+	17.952	18.193	17.499	19.234	
	NEGM+	40.537	38.345	38.387	38.182	
Keeper	NVFI+	17.741	17.538	17.304	17.445	
	NEGM+	0.614	0.641	0.602	1.038	
Adjuster	NVFI+	0.968	0.955	1.049	1.354	
	NEGM+	1.019	1.194	1.013	1.537	
TOTAL	NVFI+					$2min.\ 30sec.$
	NEGM+					$2min.\ 46sec.$

Speed: Parallel Computation (HPC with up to 24 cores)

Maximum speed times:

- NVFI+: 2min09sec
- NEGM+: 1min30sec