Linking Rental Elasticities to the Distance-Regression Coefficient

Objective. Provide a back-of-the-envelope mapping between (i) rental supply/demand elasticities and (ii) the cross-sectional regression coefficient in a specification like

$$\Delta \ln p_{r,i} = \gamma_0 + \gamma_1 \,\text{Distance}_i + \nu_i, \tag{1}$$

where Distance_i measures how *less binding* the reform is in region i (higher distance $\Rightarrow$ less binding; lower distance $\Rightarrow$ more binding).

1. Rental market equilibrium. In each region i, the rental market clears:

$$Q_i^S(p_{r,i}) = Q_i^D(p_{r,i}; Z_i), \tag{2}$$

where $p_{r,i}$ is rent and Z_i is an exogenous shifter of rental demand induced by the reform (e.g., the mass of households pushed from owning into renting). The key empirical idea is that Z_i varies across regions with how binding the reform is.

2. Log-linearize supply and demand. Log-linearize around a pre-reform point. Define elasticities:

$$\varepsilon^S \equiv \frac{\partial \ln Q_i^S}{\partial \ln p_{r,i}} > 0, \qquad \varepsilon^D \equiv -\frac{\partial \ln Q_i^D}{\partial \ln p_{r,i}} > 0, \tag{3}$$

so that ε^D is the *absolute value* of the demand elasticity.

Let $\Delta \ln Z_i$ denote the reform-induced outward shift in rental demand at fixed rent. Then the log changes in quantities satisfy:

$$\Delta \ln Q_i^S = \varepsilon^S \,\Delta \ln p_{r,i}, \tag{4}$$

$$\Delta \ln Q_i^D = -\varepsilon^D \,\Delta \ln p_{r,i} + \Delta \ln Z_i. \tag{5}$$

3. Use market clearing to solve for rent changes. Impose $\Delta \ln Q_i^S = \Delta \ln Q_i^D$ from (2):

$$\varepsilon^S \,\Delta \ln p_{r,i} = -\varepsilon^D \,\Delta \ln p_{r,i} + \Delta \ln Z_i. \tag{6}$$

Collect terms:

$$(\varepsilon^S + \varepsilon^D)\Delta \ln p_{r,i} = \Delta \ln Z_i, \tag{7}$$

so the equilibrium pass-through from the demand shifter to rents is:

$$\boxed{\Delta \ln p_{r,i} \approx \frac{1}{\varepsilon^S + \varepsilon^D}\,\Delta \ln Z_i.} \tag{8}$$

Hence, rents respond more to a given demand shift when the market is less elastic overall (smaller $\varepsilon^S + \varepsilon^D$).

4. Connect the demand shifter to the distance measure. Assume the reform-induced demand shift is decreasing in distance (more binding $\Rightarrow$ larger outward shift):

$$\Delta \ln Z_i \approx a - b\,\text{Distance}_i, \qquad b > 0. \tag{9}$$

Substitute (9) into (8):

$$\Delta \ln p_{r,i} \approx \frac{a}{\varepsilon^S + \varepsilon^D} - \frac{b}{\varepsilon^S + \varepsilon^D}\,\text{Distance}_i. \tag{10}$$

5. Read off the regression coefficient. Comparing (10) to the regression (1) yields the (hand-wavey) mapping:

$$\boxed{\gamma_1 \approx -\frac{b}{\varepsilon^S + \varepsilon^D}.} \tag{11}$$

Interpretation: b captures how strongly reform bindingness translates into a rental-demand shifter; the denominator captures how elastically the rental market adjusts via both supply and demand.

Supply-only special case (optional). If rental demand as relatively inelastic(as it is in our model) then, set $\varepsilon^D \approx 0$. Then:

$$\gamma_1 \approx -\frac{b}{\varepsilon^S}. \tag{12}$$

So a larger rental supply elasticity mechanically implies a smaller magnitude $|\gamma_1|$.