# A Model of the Housing & Rental Markets

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## 1. Households

## 2. Production

There are three production sectors in the economy: a final good sector that produces non-durable consumption, a construction sector that produces new houses, and a rental sector that transforms owner-occupied housing into rental units.

### 2.1. Final Goods Sector

The final good sector operates a constant returns to scale technology

$$Y_c = A_c N_c \tag{1}$$

where $A_c$ is the aggregate labor productivity level and $N_c$ are the units of labor services. The equilibrium wage per unit of services is thus $w = A_c$.

### 2.2. Construction Sector

The competitive construction sector produces new housing using non-durable consumption (structures) $S$ and new available buildable land $L$ according to the technology

$$Y_h = L^{\alpha} S^{1-\alpha} \tag{2}$$

We assume that the government issues new permits equivalent to the $\bar{L}$ units of land, which are then sold at a competitive price, $p_{Land}$. Thus, each construction firm solves

$$\max_{S} \quad p_h^o L^{\alpha} S^{1-\alpha} - p_{Land} L - S \tag{3}$$

where the prices of non-durable goods is normalized to unity. The first order condition of the construction sector with respect to the structures

$$\frac{\partial \Pi_h}{\partial S} = 0 \Leftrightarrow 1 = p_h^o(1-\alpha)\left(\frac{L}{S}\right)^{\alpha} \tag{4}$$

Rearranging the first order condition we get to an expression that relates the (owner-occupied) house price to the mix of land and structures in production

$$p_h^o = \left(\frac{S}{L}\right)^{\alpha} \frac{1}{1-\alpha} \tag{5}$$

### 2.3. Rental Sector

The rental sector acts as an intermediary between the housing sector and the landlords in the economy. It operates a linear technology that allow them to transform owner occupied housing into rental units.

$$\max_{H_r} \quad p_h^r\left(\theta H_r\right) - p_h^o H_r \tag{6}$$

Profit maximization implies

$$p_h^r = \frac{1}{\theta} p_h^o \tag{7}$$

where $\theta$ controls the degree of segmentation between the two markets, and $1/\theta$ is the house price ratio between BTL and owner-occupied housing.

## 3. Equilibrium

$$\text{Housing}: \quad \bar{H} = (1-\delta)\bar{H} + Y_h \tag{8}$$

$$\text{Housing services}: \quad \bar{H} = H_o + H_r \tag{9}$$

$$\text{Housing permits}: \quad \bar{L} = L \tag{10}$$

$$\text{Resources}: \quad Y_c = C + S \tag{11}$$