Local Projections

Juan Castellanos

European University Institute

April 11, 2023

EUROPEAN UNIVERSITY INSTITUTE

Impulse Responses

- Modern dynamic macro studies the propagation of structural shocks
- Impulse response functions are the fundamental object in this type of analysis
  * Important statistics that summarize models of the economy.
  * Thus, they can be used in an indirect inference exercise to estimate the parameters of those models
- Formally, an impulse-response function describes the evolution of the variable of interest y along a specified time horizon $t + h$ after a shock of size d in a given moment t. That is,

$$IR(t, h, d) = \mathbb{E}(y_{t+h} | u_t = d, y_{t-1}, y_{t-2}, \ldots) - \mathbb{E}(y_{t+h} | u_t = 0, y_{t-1}, y_{t-2}, \ldots) \tag{1}$$

- Two fundamental questions:
  * How do we measure the shock of interest?
  * How do we estimate the impulse response function?

The Econometric Model

Estimation and Identification

- There are two approaches: (a) the structural vector autoregression (SVAR) and (b) the local projection (LP)

- In the *SVAR approach*, the estimation and identification problems are typically jointly solved. Key assumptions:

  * **Wold decomposition**: from the reduced-from to the structural representation of the VAR

$$A(L)Y_t = u_t \implies Y_t = \Phi(L)\varepsilon_t \tag{2}$$

  * **Invertibility**: from the reduced-form innovations to structural shocks

$$u_t = \Phi_0\varepsilon_t \quad \text{and} \quad \Phi_0^{-1} \text{ exists} \tag{3}$$

  * This implies that to identify the impulse responses, we need to identify Φ_0. How? Cholesky, long-run restrictions, sign restrictions, etc.

- In the *LP approach* these two problems are typically disentangled.

An AR(1) example

- Assume we have have the following AR(1) process:

$$z_t = \rho z_{t-1} + u_t \tag{4}$$

  where z_t is a scalar, $\rho \in (0,1)$ is the persistence of the process and $u_t \sim \mathcal{N}(0, \sigma_u)$

- **Wold decomposition**: since the process is stationary $\rho < 1$, we can find the $MA(\infty)$ representation:

$$\begin{aligned} z_t =& (1 - \rho L)^{-1} u_t \\ =& u_t + \rho u_{t-1} + \rho^2 u_{t-2} + \dots \end{aligned} \tag{5}$$

- We are interested in the response to structural shocks ε_t, not the responses to the reduced-from innovations u_t. That is, we are looking for $\frac{\partial z_{t+h}}{\partial \varepsilon_t}$

- **Invertibility**: structural shocks lie in the linear spaced spanned by the reduced-form innovations: $u_t = \Phi_0 \varepsilon_t$. Thus, we can write,

$$z_t = \Phi_0 \varepsilon_t + \rho \Phi_0 \varepsilon_{t-1} + \rho^2 \Phi_0 \varepsilon_{t-2} + \dots \tag{6}$$

- What's the response of variable z in period $t + h$?

$$\frac{\partial z_{t+h}}{\partial \varepsilon_t} = \rho^j \Phi_0 \quad \forall h = 0, \dots, H \tag{7}$$

Local Projections: unobserved shocks I

- Assume that the observed $n \times 1$ dimensional time series Y_t is represented by the structural VAR: $Y_t = \Phi(L)\varepsilon_t$, which for some Y_{t+h} we can re-write as follows:

$$\begin{aligned} Y_{t+h} &= \Phi_0\varepsilon_{t+h} + \Phi_1\varepsilon_{t+h-1} + \ldots + \Phi_{h-1}\varepsilon_{t+1} + \Phi_h\varepsilon_t + \Phi_{h+1}\varepsilon_{t-1} + \ldots \\ &= \Phi_h\varepsilon_t + \Phi_{h+1}\varepsilon_{t-1} + \ldots + \xi^{(h)}_{t+h} \end{aligned} \tag{8}$$

- Suppose we are interested in the impulse response associated with the first shock $\varepsilon_{1,t}$. Let $\varepsilon_{\cdot,t} = \{\varepsilon_{2,t}, \ldots, \varepsilon_{n,t}\}$ and $\Phi_{h,\cdot}$ be the $n \times (n-1)$ matrix that contains all columns of Φ except of the first one. Then,

$$Y_{t+h} = \Phi_{h,1}\varepsilon_{t,1} + \Phi_{h,\cdot}\varepsilon_{\cdot,t} + \Phi_{h+1}\varepsilon_{t-1} + \ldots + \xi^{(h)}_{t+h} \tag{9}$$

- Assume that $\varepsilon_{1,t} = u_{1,t} - \text{Proj}(u_{1,t}|u_{\cdot,t})$, i.e. Φ_0 is upper triangular and we ordered $\varepsilon_{1,t}$ first. Under this restriction we can write

$$\begin{aligned} Y_{t+h} &= \Phi_{h,1}\left(u_{1,t} - \text{Proj}(u_{1,t}|u_{\cdot,t})\right) + \{u_{\cdot,t-1}, u_{\cdot,t-2}, \ldots\} + \xi^{(h)}_{t+h} \\ &= \Phi_{h,1} u_{1,t} + \{u_{\cdot,t}\} + \{u_{\cdot,t-1}, u_{\cdot,t-2}, \ldots\} + \xi^{(h)}_{t+h} \\ &= \Phi_{h,1} Y_{1,t} + \{Y_{\cdot,t}, Y_{t-1}, Y_{t-2}, \ldots\} + \xi^{(h)}_{t+h} \end{aligned} \tag{10}$$

Local Projections: unobserved shocks II

- Equation (10) is now written completely in terms of observables!
- We can identify the impulse response coefficients $\Theta_{h,1}$ by regressing Y_{t+h} on $Y_{1,t}$ at a variety of horizons, controlling for the contemporaneous effects of other variables, and the lagged values of all the observed time series.
- Note that we need to control for the contemporaneous effects because we assumed that $\varepsilon_{1,t}$ is ordered first
- How would equation (10) change if we assumed that $\varepsilon_{1,t}$ is ordered last?
  * This is equivalent to assuming that $\varepsilon_{1,t} = u_{1,t}$
  * Φ_0 is lower triangular
  * Thus, there is no need to control for contemporaneous variables

$$Y_{t+h} = \Phi_{h,1} Y_{1,t} + \{Y_{\cdot,t-1}, Y_{\cdot,t-2}, \ldots\} + \xi_{t+h}^{(h)} \tag{11}$$

Local Projections: measured shocks

EUROPEAN UNIVERSITY INSTITUTE

- What if we can "observe" the shocks?
- A branch of the literature has focus on constructing measures of the shocks using the *narrative* approach
  * Monetary policy: FFR changes around small windows of an FOMC announcement
  * Fiscal policy: military news to estimate government spending changes (Ramey, 2011)
- If we have a measure of the shock, x_t, we can run the following regression to identify the impulse responses

$$y_{t+h} = \mu_h + \beta_h x_t + \gamma_h' r_t + \sum_{\ell=1}^{p} \delta_{h,\ell}' w_{t-\ell} + \xi_{h,t} \tag{12}$$

  where $w_t = (r_t', x_t, y_t, q_t')$. Here, r_t and q_t serve as controls.
- The LP impulse response of y_t with respect to x_t is given by β_h.

Beyond this class

- For this class, it is enough that you know how to estimate the IRFs with respect to the true shock since we will work with simulated data.
- In practice this is not the case. The shock of interest $\varepsilon_{1,t}$ is not observed and if it is measured, it typically has some error associated to it.
- A popular approach is to estimate the impulse response to the first shock using a two stage least square version of the LP.
- What we need? An instrumental variable (IV) that satisfies the following conditions:
  - Relevance: $\mathbb{E}[\varepsilon_{1,t} Z_t] \neq 0$
  - Contemporaneous exogeneity: $\mathbb{E}[\varepsilon_{\cdot,t} Z_t] = 0$
  - Lead-lag exogeneity: $\mathbb{E}[\varepsilon_{t+k} Z_t] = 0 \quad \forall k = \pm 1, \pm 2, \ldots$
- Advantage: no need to impose invertibility ... however, if the lag exogeneity condition not satisfied, a sufficient condition is invertibility (no free lunch)

Indirect Inference

Indirect inference: LP coefficients as moments

EUROPEAN UNIVERSITY INSTITUTE

- In an indirect inference exercise we use an econometric model to summarize key features of the data
  * Impulse response coefficients are a good candidate
  * Does it matter how we estimate them?
- As seen above, the SVAR and LP approaches are used to estimate IRFs. How do they compare?
- Plagborg-Møller and Wolf (2020, ECTA) show that these two approaches estimate the same impulse responses! So why should we care?
  * This is a population result
  * In finite samples, they only approximately agree up to horizon $h = p$, while for $h > p$ there is a bias variance trade off
- Therefore, does the choice of econometric model used to estimate the IRFs matter for the estimates of structural parameters?
- Castellanos and Cooper (2023) show that using LP coefficients is superior to using VAR coefficients since the IRFs at the estimated parameters are closer to the true/structural IRFs.