Calibration Notes

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1. UK Calibration

1.1. Demographics & Preferences

1.1.1. Fraction of borrowers (external)

We could follow Ferrero, Harrison, and Nelson (2023) and set the fraction of borrowers to the fraction of mortgagors in the UK from Cloyne, Ferreira, and Surico (2020). After adjusting for the absence of renters in our economy, this results in a value of χ_b equal to 0.643. According to Figure 1 in Cloyne, Ferreira, and Surico (2020), 25% of UK households are outright homeowners, 45% mortgagors and 30% renters. Hence, the fraction of borrowers in our economy equals 0.45 / (0.25+0.45) = 0.643 as we normalized population to one and do not model renters. The same approach using US data will deliver a fraction of borrowers of 0.5 / (0.2 + 0.5) = 0.714.

This is very different from Greenwald (2018) who classifies borrower households as those with a house and a mortgage, but less than two months' income on liquid assets, yielding χ_b = 0.319. He uses data from the 1998 Survey of Consumer Finances (SCF). This definition is more consistent with the model, as borrowers are assumed to be at the LTV and PTI limits. Ideally, we would use the Wealth and Asset Survey (WAS). Since none of us has worked with this we instead follow Alex's approach and compute the share of mortgagors whose monthly savings are less than 20% of their total income. This delivers a value of χ_b = 0.2774.

What's the name of the survey we are using?

1.1.2. Income dispersion (external)

For the income shock distribution Γ_e, we follow Greenwald (2018) and choose a log-normal specification $\log e_{i,t} \sim \mathcal{N}(-\sigma_e/2, \sigma_e^2)$ which implies that

$$\int_{\bar{e}_t} e_i d\Gamma_e(e_i) = \Phi\left(\frac{\log \bar{e}_t - \sigma_e^2/2}{\sigma_e}\right)$$

where Φ is the CDF of the standard normal distribution. To capture the dispersion in which constraint is binding we set σ_e match the standard deviation of $\log(PTI_{i,t}) - \log(CLTV_{i,t})$ in the data. This term is the difference of individual borrowers' log PTI and CLVT ratios at origination, which equal to $\log e_{i,t}$ in the model, up to the offset term ω. Greenwald (2018) computes this standard deviation form purchase loans in the Fannie

Mae data for each quarter from 2000 to 2014, and set $\sigma_e = 0.411$ to be the average of this series.

Alex uses Product Sales Database (PSD) to compute the standard deviation of the individual borrowers' log DSR and CLVT ratios at origination. We set $\sigma_e = 0.53$ to be the average of this series over the period between 2005-2023.

Why did we decide to use DRS? Didn't we have data on PTI? Why are we including the post-Covid years? Can these two things explain the larger variation in the time series?

$$m_t^* \leq \frac{(\theta^{PTI} - \omega) w_t n_t e_{i,t}}{q_t^* + \alpha}$$

1.1.3. Borrower discount factor (internal)

It is chosen to match the steady state ratio of borrower house value to income ($p_t^h h_{t,b} / w_t n_{t,b}$). Greenwald (2018) reports this ratio to be 8.89 quarterly when using the 1998 SCF. This strategy yields a borrower discount factor $\beta_b = 0.965$.

In the UK, the median value of the house price to income has fluctuated around 4.5 during the period between 2005 and 2020, while the mean value has been around 5 during the same period. See Figure 11 in data-summary. Matching this lower target for the UK yields also a lower $\beta_b = 0.895$.

1.1.4. Saver discount factor (external)

It is calibrated to match the average of 10 years interest rates. Greenwald (2018) matches the average of 10-year interest rates of 6.46% during the period between 1993-1997. This result in $\beta_s = 0.987$.

For the UK, we target the 1998-2008 average of the 10 year gilt yield of 4.83%. See Figure 14 in data-summary. This obviously yields a higher savers discount factor $\beta_s = 0.9934$ as we need to generate a higher demand for safe bonds.

1.1.5. Housing utility weight (internal)

It is calibrated to match the housing expenditure share. According to ONS and for the fiscal year 2018-2019, it is equal to 20%. How much of that is utilities? As in Greenwald (2018) we should deduct them from the total share when backing out ξ. I'll assume a 4% as for the US. This gives us a target of 16%.

I cannot find the variables in the model that Greenwald uses to compute the housing expenditure share ...

1.1.6. Labor disutility (internal)

They are calibrated such that both the borrower and the saver provide the same amount of labor in steady state (1/3 = 8 hours per day). For the US calibration, this yields $\eta_b = 7.6983$ and $\eta_s = 5.6049$, while for the UK calibration $\eta_b = 7.6164$ and $\eta_s = 5.8031$.

1.1.7. Inverse Frisch elasticity (external)

Set to 1. Standard.

1.2. Housing & Mortgages

1.2.1. Stock of housing & savers housing demand (internal)

The housing stock ($\bar{H} = \bar{H}_s + H_b$) and savers housing demand $\bar{H}_s$ are calibrated so that the price of housing is unity at steady state, and the ratio of savers house value to income is the same as in the 1998 SCF (11.40 quarterly). For the US, this implies that $\bar{H} = 2.374$ and that $\bar{H}_s = 2.251$. Two caveats: *(i)* note that $\bar{H}_s, \bar{H}$ are different from the values reported in Greenwald' s paper: 1.867, 2.178, respectively, and *(ii)* we are effectively fixing the housing stock of the borrower.

We still need the value of the savers house to income. Notice that it will be nice to have also the borrowers house to income. For now, I will assume that *(i)* the ratio is identical to the US and hence the saver's house to income is 28% higher than the borrowers and *(ii)* the borrower value to income was the mean value of the aggregated series. Then, if we calibrate these two parameters to match a 6.4 borrowers house value to income and a steady state house price of unity, then we get $\bar{H}_s = 1.6885$ and $\bar{H} = 2.2703$

1.2.2. Housing depreciation rate (external)

Set to 0.5%. Standard.

1.2.3. Mortgage amortization (external)

The mortgage amortization parameter ν is set to match the average share of principal paid on exiting loans. For each month in 2000-2015, Greenwald computes the average loan age and interest rate for existing loans in Fannie Mae 30-Years MBS, weighted by loan balance. Given age and rate, the fraction of the loan balance paid off as principal ν_t can be computed from a standard amortization schedule. He calibrates ν so that $(1-\nu)$ is the geometric average of $(1-\nu_t)$ over all months in the sample. This results in $\nu = 0.435\%$.

Alex computes this number based on the weighted average mortgage amortization per month as a percentage of the loan. This includes capital and interest repayments. As a result, this is not entirely consistent with the model since the amortization applies only to the principal, as shown in the low of motion of mortgage debt

$$m_t = \rho m^*_{i,t} + (1-\rho)(1-\nu)\pi_t^{-1} m_{t-1}$$

This computation give a slightly larger number for the amortization parameter $\nu = 0.57 \times 3 = 1.71\%$ than for the US.

1.2.4. Refinancing rate (external)

The refinancing rate is endogenous in Greenwald (2018). However, in a follow up paper Greenwald and Guren (2021) they set this parameter to be at the steady state value of the endogenous pre-payment economy. They report a value of 0.034. When using the model with the US calibration for the baseline with endogenous pre-payement, this value equals to 0.1446. These two numbers are a lot smaller than the UK computed value of 0.302.

1.2.5. Debt limit parameters (external)

Greenwald (2018) sets $\theta^{LTV} = 0.85$ as a compromise between the mass bunching at 80%, and the masses constrained at higher institutional limits such as 90% or 95%. This result in an average LTV ratio across newly originated mortgages of 80.5% at the steady state. For the PTI limit, he chooses $\theta^{PTI} = 0.36$ to match the pre-boom underwriting standard and $\omega = 0.08$ to match the traditional PTI limit excluding other debt (28%).

In the data-summary, we have plotted the distribution of LTV and LTI's over the past 15 years. Looking at the LTV distribution (Figure 6.B), we see that more than 50% of

TABLE 1. Alex's Inputs

Series	**Time series**	**Value**
Standard deviation of log(DSR) - log(CLTV)	2005-2019	0.56
Weighted average mortgage amortization per month (monthly payments in £ amounts)	2005-2023	2244 (£)
Weighted average mortgage amortization per month (% of Loan) [NB. This is the monthly payment, hence includes capital and interest repayments.]	2005-2023	0.57(%)
The refinancing rate - proportion of loans that remo every year in the stock	2018H2 – 2023H1	30.2
Share of mortgagors who would be constrained (i.e. whose monthly savings are less than 20% of their total income)	2019	27.74
Share of savers (fraction of mortgagors who are not constrained + other cash-buyers + those who own their house outright)	2019	72.25

mortgagors have LTVs that are above 75% and that most of them are between 75% and 95%, specially after the GFC. Thus, as compromise we choose an LTV limit of 85%.

Can we get a similar plot for the Payment to Income or Debt Service Ratio? For now, I am setting the $\theta_{PTI} = 0.36$ and $\omega = 0$

1.2.6. PTI offset (internal)

The term α is used to account for taxes and insurance as well to ensure that the different amortization schemes in the model and the data do not distort the tightness of the constraint. In particular, α is calibrated to ensure that there isn't too much principal repayment at the start of the loan. Consequently, Greenwald sets α such that $q_t^* + \alpha$ is equal to 10.47% annualized at steady state, which is the interest and principal payment on a loan with steady state mortgage interest rate (7.81%) under the extact amortization scheme for a fixed-rate mortgage, plus 1.75% annually for taxes and insurance.

How does he arrives to 10.47%?

1.2.7. Term premium (internal / external)

The *average* term premium μ_q is chosen to match the 1993-1997 average mortgage rates of 7.81%. For the US calibration, this requires a $\mu_q = 0.320\%$. For the UK calibration we try to match a mortgage rate of 6.83% which implies a 2 p.p spread over the 10-year gilt yields. Figure 9 in the data-summary shows that spreads were around 1% before jumping to 3% after the Great Recession. We choose a 2 p.p. as a compromise between the two. This yields an average term premium $\mu_q = 0.21\%$

Greenwald sets the *persistence* of the shock $\phi_q = 0.852$ to match the average quarterly autocorrelation of the spread between mortgage rates and two-year treasuries. We can keep this parameter fixed for the UK.

1.2.8. Income tax rate (external)

The tax rate τ_y was set to the national average prior to mortgage interest deductions. For the US, this result in 0.204. For UK, Steve come up with that number using UK data (not sure which one) and set the value to 0.212.

1.3. Productive technology

1.3.1. TFP shock (external / internal)

Greenwald (2018) follows Garriga, Kydland, and Šustek (2021) to calibrate the exogenous process for productivity a_t. The *persistence* ρ_a and *standard deviation* σ_a are set to 0.965 and 0.082, respectively. For the UK, we take these from COMPASS the workhorse DSGE model used at the Bank of England for policy analysis and prediction (Burgess, Fernandez-Corugedo, Groth, Harrison, Monti, Theodoridis, and Waldron 2013). These equal to 0.9 and 0.05, respectively.

The *mean* of the TFP shock is chosen internally such that output in steady state equal one, $y_{ss} = 1$. For US calibration, this requires setting $\mu_a = 1.0$, while for the UK $\mu_a = 1.02$.

Concerns: In COMPASS the TFP shock is expressed in levels, i.e. $\varepsilon_{t,a} = \rho_a \varepsilon_{t-1,a} + \sqrt{1-\rho_a}\sigma_a \nu_a$, while here and in the code is in logs: $\log \varepsilon_{t,a} = (1-\rho_a)\mu_a + \rho_a \log \varepsilon_{t-1,a} + \sigma_a \nu_a$. I don't know if COMPASS numbers really map into this ... If we have a series for TFP shocks it is easy to back out these numbers!

1.3.2. Calvo adjustment (external)

ζ is set to standard value in the literature, 0.75, which implies an average price duration of 4 quarters. Recall that the average price duration is $(1 - \zeta)^{-1}$.

1.3.3. Elasticity of substitution among varieties (external)

λ is also set to a standard value in the literature of 6.0, which implies a steady state mark-up of 20%. Recall that the steady state mark-up in a NK model is defined as $\frac{\lambda}{\lambda - 1}$.

1.4. Monetary policy

1.4.1. Interest rate smoothing (external)

We set it to the value estimated in COMPAS $\phi_r = 0.8336$

> **Concern**: Taylor rule specification is different across models.

1.4.2. Taylor rule weight on the inflation (external)

Also set to the value estimated in COMPAS $\varphi_\pi = 1.479$.

> **Concern**: Taylor rule specification is different across models.

1.4.3. Steady state inflation (internal)

Greenwald (2018) targets 10 year inflation expectations (3.25%) to pin down the average inflation $\pi_{ss} = 1.008$. We instead assume that inflation expectations are anchored to the 2% target, which results in $\pi_{ss} = 1.0055$.

1.4.4. Inflation target and interest rate shock

Values are taken from Garriga, Kydland, and Šustek (2021)

2. Model fit: how does UK compare to US?

TABLE 2. Parameter values

Parameter	Interpretation	UK Value	US Value	Internal
Demographics & Preferences				
χ_b	Fraction of borrowers	0.2774	0.319	N
σ_e	Income dispersion	0.53	0.411	N
β_b	Borr. discount factor	0.869	0.965	Y
β_s	Saver discount factor	0.993	0.987	N
ξ	Housing utility weight	0.25	0.25	N
η_b	Borr. labor disutility	7.6164	8.189	Y
η_s	Saver labor disutility	5.8031	5.622	Y
φ	Inv. Frisch elasticity	1.0	1.0	N
Housing & Mortgages				
$\log \bar{H}$	Log housing stock	1.6885	2.178	Y
$\log \bar{H}_s$	Log saver housing stock	2.2703	2.251	Y
δ_h	Housing depreciation	0.005	0.005	N
ν	Mortgage amortization	1.71%	0.435%	N
ρ_b	Refinancing rate	0.1428	0.1446	N
θ^{PTI}	Max PTI ratio	0.36	0.36	N
θ^{LTV}	Max LTV ratio	0.85	0.85	N
α	PTI offset (taxes, etc.)	0.285%	0.285%	Y
ω	PTI offset (other debt)	0	0.08	N
μ_q	Term premium (mean)	0.21%	0.320%	Y
ϕ_q	Term premium (pers.)	0.852	0.852	N
τ_y	Income tax rate	0.212	0.204	N
Productive technology				
μ_a	Mean (TFP shock)	1.02	1.0	Y
ϕ_a	Persistence (TFP shock)	0.9	0.964	N
σ_a	Standard deviation (TFP shock)	0.05	0.008	N
λ	Variety elasticity	6.0	6.0	N
ζ	Price stickiness	0.75	0.75	N
Monetary policy				
ϕ_r	Interest rate smoothing	0.8336	0.89	N
φ_π	Taylor rule weight on inflation	1.497	1.5	N
π_{ss}	Steady state inflation	1.005	1.008	N
$\phi_{\bar{\pi}}$	Persistence (infl. target shock)	0.994	0.994	N
ϕ_η	Persistence (interest rate shock)	0.15	0.15	N
$\sigma_{\bar{\pi}}$	Standard deviation (infl. target shock)	0.0015	0.0015	N
σ_η	Standard deviation (interest rate shock)	0.0041	0.0041	N

TABLE 3. Model Fit: Targeted Moments

	UK Value	**HRM Model (2 yr)**	**US Value**	**FRM Model**
House Value to Income (Borrower)	5.0	5.0747	8.89	8.9423
House Value to Income (Saver)	6.4	6.4519	11.4	11.386
Housing Expenditure Share	16%	?	20%	?
Average Loan to Value	0.6512	0.68	0.61251	0.805
House Price	1.0212	1	0.95112	1
10 Year Rates	4.82%	4.83%	6.46%	6.46%
Mortgage Rates	6.80%	6.83%	7.81%	7.81%
$q_t^* + \alpha$	?	10.45%	10.47%	10.47%
Output	1	1.0076	1	0.99894
Inflation	2.0%	2.24 %	3.25%	3.22%
Hours Worked (Borrower)	1/3	0.33018	1/3	0.33138
Hours Worked (Saver)	1/3	0.32901	1/3	0.33373

References

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