Replicating Greenwald (2018) + Extensions

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Appendix A. Model with Fixed Rate Mortgages (FRM)

This section presents the model and derives all the equilibrium conditions.

A.1. Households

A.1.1. Demographics & Preferences

The economy consist of two types of households: "*borrowers*" denoted with suscript b and "*savers*", denoted with subscript s. The relative size of each group is denoted by χ_j where $\chi_s + \chi_b = 1$.

These households differ in their preferences. Savers are more patient than borrowers, i.e. $\beta_s > \beta_b$ where β_j is the discount factor of each type of household. Moreover, the disutility from labor, η_j, is also different across types in order to guarantee that they supply the same amount of labor in steady state.

Overall, each agent of type $j \in \{b, s\}$ maximizes expected lifetime utility over non-durable consumption $c_{j,t}$, housing services $h_{j,t}$ and labor supply $n_{j,t}$

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta_j^k u\left(c_{j,t+k}, h_{j,t+k}, n_{j,t+k}\right) \tag{A1}$$

where utility takes the separable form

$$u(c, h, n) = \log(c) + \xi \log(h) - \eta_j \frac{n^{1+\varphi}}{1+\varphi}. \tag{A2}$$

Each type's stochastic discount factor, which measures how much an agent of type j values real payments at time $t+1$, is defined as

$$\Lambda_{t,t+1}^{j} \equiv \beta_j \frac{u_{j,t+1}^{c}}{u_{j,t}^{c}} \quad \text{where} \quad u_{j,t}^{c} = \frac{\partial u(c_{j,t}, h_{j,t}, n_{j,t})}{\partial c_{j,t}} \tag{A3}$$

A.1.2. Asset technology

Mortgages. Mortgages are denoted by m_t and are the only source of borrowing in the model economy. The mortgage contract is a nominal perpetuity with geometerically decaying payments.

To allow for changes in the real interest rate similar to movements in term premia or mortgage spreads, it is assumed that there is a proportional tax $\Delta_{q,t}$ on all future mortgage payments associated with a given loan, and that it follows the stochastic process

$$\Delta_{q,t} = (1 - \phi_q)\mu_q + \phi_q \Delta_{q,t-1} + \varepsilon_{q,t} \tag{A4}$$

where $\varepsilon_{q,t}$ is a white noise process refered to as term premium shock. This tax is used to introduce a time-varying wedge that moves exogenously the real cost of borrowing and it is rebated lump-sum to savers.

Under *fixed-rate mortgages*, the borrower pays a fraction ν of the remaining principal each period, so that next's period principal balance and payment both decay by a factor $(1 - \nu)$. In other words, the saver gives the borrower \$1 at origination; in exchange the saver recieves $\$(1-\nu)^k(1-\Delta_{q,t})q_t^*$ at time $t+k$, for all $k > 0$ until prepayment, where q_t^* is the equilibrium coupon rate at origination, and ν is the fraction of the principal paid each period.[1]

Mortgage debt is *pre-payable*. That is, the borrower can choose to repay the principal balance on a loan at any time, which cancels all future paments of the loan. If a borrower chooses to prepay her loan, she may choose a new loan size $m_{i,t}^*$ subject to her credit limits. Obtaining a new loan incurs a transaction cost $\kappa_{i,t} m_{i,t}^*$, where $\kappa_{i,t}$ is drawn i.i.d. across individual members of the family and across time from a distribution with c.d.f. Γ_κ. The borrower's optimal policy is to pre-pay the loan if her cost draw $\kappa_{i,t}$ is below a threshold value $\bar{\kappa}_t$ that depends only on the aggregate states and not on the individual charactersitics of the loan. Therefore, we can define the total transaction costs as

$$\Psi(\rho_t, m_t^*) = m_t^* \int^{\Gamma_k^{-1}(\rho_t)} \kappa d\Gamma_\kappa(\kappa) \tag{A5}$$

and let $\bar{\Psi}_t$ be a proportional rebate that returns these costs to the borrowers at equilibrium.

Turning to *credit limits*, a new loan for borrower i must satisfy a LTV constraint, which is given by

$$\frac{m_{i,t}^*}{p_t^h h_{i,t}^*} \leq \theta^{LTV} \tag{A6}$$

where $m_{i,t}^*$ is the balance on the new loan, p_t^h is the house price, $h_{i,t}^*$ is the quantity of

[1] Average maturity of debt is ν^{-1}.

the house purchased. The key property of this constraint is that it moves proportionally with p_t^h, so a rise in house prices loosens the constraint. Moreover, the new loan also has to satisfy a PTI constraint, defined by

$$\frac{\left(q_t^* + \alpha\right) m_{i,t}^*}{w_t n_{i,t} e_{i,t}} \leq \theta^{PTI} - \omega \tag{A7}$$

where $(q_t^* + \alpha) m_{i,t}^*$ is the borrower's initial payment with α accounting for taxes and insurance payments associated to a new mortgage, $w_t n_{i,t} e_{i,t}$ is the borrower's income, and the term ω adjust for the convention that the PTI typically applies to all recuring debt and these payments require a fix fraction of borrower's income.

Since the borrower must satisfy both constraints at origination, her overall debt limit is given by $m_{i,t}^* \leq \bar{m}_{i,t} = \min\left(\bar{m}_{i,t}^{LTV}, \bar{m}_{i,t}^{PTI}\right)$ where

$$\bar{m}_{i,t}^{LTV} = \theta^{LTV} p_t^h h_{i,t}^* \tag{A8}$$

$$\bar{m}_{i,t}^{PTI} = \frac{\left(\theta^{PTI} - \omega\right) w_t n_{t,i} e_{t,i}}{q_t^* + \alpha} \tag{A9}$$

Note that borrower's income is a function of an idiosyncratic income shock $e_{i,t}$, drawn i.i.d. across borrowers and time with c.d.f Γ_e. This perfectly insurable risk is essential here to determine the fraction of borrowers constrained by each limit. Let $\bar{e}_t$ be the minimum level of income efficiency such that

$$\bar{e}_t = \frac{\bar{m}^{LTI}}{\bar{m}^{PTI}} = \frac{\theta^{LTV} p_t^h h_{i,t}^*}{\left(\theta^{PTI} - \omega\right) w_t n_{t,i} e_{t,i} / \left(q_t^* + \alpha\right)} \tag{A10}$$

Consequently, those households with a realization of the income shock below the threshold $e_{i,t} < \bar{e}_t$ will be constrained by the PTI limit, while those with a realization above the threshold $e_{i,t} > \bar{e}_t$ will be constrained by the LTV. Thus, the aggregated debt constraint is given by

$$\begin{aligned} \bar{m}_t &= \int \min\left(\bar{m}_t^{PTI} e_i, \bar{m}_t^{LTV}\right) d\Gamma_e(e_i) \\ &= \bar{m}_t^{PTI} \int^{\bar{e}_t} e_i d\Gamma_e(e_i) + \bar{m}_t^{LTV} \left(1 - \Gamma_e(\bar{e}_t)\right) \end{aligned} \tag{A11}$$

where the first summon in (A11) represents the fraction of households constrained by the PTI while the second term is the borrowing capacity of LTV-constrained households.

Housing. Both borrowers and savers own housing, which produces a service flow each period equal to its stock, $h_{j,t}$ for $j \in \{s, b\}$. The total housing stock is fixed $\bar{H}$ as well as the saver demand $h_{s,t} = \tilde{H}_s$, so that the borrower is always the marginal buyer of housing. That is the overall house price is determined by the borrower's demand. Note that since each loan is associated to a specific house, only prepaying households can adjust their housing holdings.

Finally, as standard in the housing literature, both types of households pay maintenance costs to replace the constant fraction $\delta h_{j,t}$ which depreciates each period.

One-Period Bonds. Households can trade a one-period nominal bond, whose balances are denoted by b_t, and has a real return of R_t. It is assumed that positions in the one-period bond must be non-negative (i.e. cannot be used for borrowing) and consequently it is traded only by savers in equilibirum. It is also used by the monetary authority as a policy instrument.

A.1.3. Borrower's Problem

As shown by Greenwald (2018), the state space for the borrower's problem allows for aggregation. That is, we can focus on a single representative borrower. The endogenous variables for her problem are: total start-of-period debt balances m_{t-1}, total promised payments on existing debt, $x_{t-1} \equiv q_{t-1}m_{t-1}$, and total start of the period borrower housing $h_{b,t-1}$. If we define $\rho_t = \Gamma_\kappa(\bar{\kappa}_t)$ to be the fraction of loans prepaid, then the lows of motion for these state variables are given by

$$m_t = \rho_t m_t^* + (1-\rho_t)(1-\nu)\pi_t^{-1}m_{t-1} \tag{A12}$$

$$x_{b,t} = \rho_t q_t^* m_t^* + (1-\rho_t)(1-\nu)\pi_t^{-1}x_{b,t-1} \tag{A13}$$

$$h_{b,t} = \rho_t h_{b,t}^* + (1-\rho_t)h_{b,t-1} \tag{A14}$$

The representative consumer chooses consumption $c_{b,t}$ labor supply, $n_{b,t}$, the size of newly purchased houses, $h_{b,t}^*$, the face value of newly issued mortgages m_t^* and the fraction of loans to prepay ρ_t, in order to maximize (A1) using the aggregate utility function

$$u(c_{b,t}, h_{b,t-1}, n_{b,t}) = \log(c_{b,t}/\chi_b) + \xi \log(h_{b,t-1}/\chi_b) - \eta_b \frac{\left(n_{b,t}/\chi_b\right)^{1+\varphi}}{1+\varphi} \tag{A15}$$

subject to her budget constraint

$$\text{(A16)}\quad \begin{aligned} c_{b,t} \leq &(1-\tau_y)w_t n_{b,t} - \pi_t^{-1}\left((1-\tau_y)\, x_{b,t-1} + \nu m_{t-1}\right) + \rho_t \left(m_t^* - (1-\nu)\pi_t^{-1} m_{t-1}\right) \\ &- \delta\, p_t^h h_{b,t-1} - \rho_t\, p_t^h \left(h_{b,t}^* - h_{b,t-1}\right) - \left(\Psi(\rho_t, m_t^*) - \bar{\Psi}_t\right) + T_{b,t} \end{aligned}$$

the debt constraint

$$\text{(A17)}\quad m_t^* \leq \bar{m}_t = \bar{m}_t^{PTI} \int^{\bar{e}_t} e_i d\Gamma_e(e_i) + \bar{m}_t^{LTV}\left(1 - \Gamma_e(\bar{e}_t)\right)$$

and the lows of motion (A12) – (A14).

Recursive Formulation. The borrower's problem can be written in recursive formulation as follows

$$\begin{aligned} V_b\left(m_{t-1}, h_{b,t-1}, x_{b,t-1}\right) = &\max_{c_{b,t}, n_{b,t}, h_{b,t}^*, m_t^*, \rho_t} u\left(c_{b,t}, h_{b,t-1}, n_{b,t}\right) + \beta_b \mathbb{E}_t V_b\left(m_t, h_{b,t}, x_{b,t}\right) \\ &\text{s.t.} \quad \text{(A12)} - \text{(A17)} \end{aligned}$$

First Order Conditions. Setting up the Lagrangian

$$\begin{aligned} \mathcal{L}_{b,t} = &u\left(c_{b,t}, h_{b,t-1}, n_{b,t}\right) + \beta_b \mathbb{E}_t V_b\left(m_t, h_{b,t}, x_{b,t}\right) + \lambda_{b,t}\Big[(1-\tau_y)w_t n_{b,t} - \pi_t^{-1}\left((1-\tau_y)\, x_{b,t-1} + \nu m_{t-1}\right) \\ &+\rho_t\left(m_t^* - (1-\nu)\pi_t^{-1} m_{t-1}\right) - \delta\, p_t^h h_{b,t-1} - \rho_t\, p_t^h\left(h_{b,t}^* - h_{b,t-1}\right) - \left(\Psi(\rho_t, m_t^*) - \bar{\Psi}_t\right) + T_{b,t} - c_{b,t} \\ &+\mu_t \rho_t\left(\bar{m}_t^{PTI} \int^{\bar{e}_t} e_i d\Gamma_e(e_i) + \bar{m}_t^{LTV}\left(1 - \Gamma_e\left(\bar{e}_t\right)\right) - m_t^*\right) \\ &+\lambda_t^M\left(\rho_t m_t^* + (1-\rho_t)(1-\nu)\pi_t^{-1} m_{t-1} - m_t\right) \\ &+\lambda_t^X\left(\rho_t q_t m_t^* + (1-\rho_t)(1-\nu)\pi_t^{-1} x_{b,t-1} - x_{b,t}\right) \\ &+\lambda_t^H\left(\rho_t h_{b,t}^* + (1-\rho_t) h_{b,t-1} - h_{b,t}\right)\Big] \end{aligned}$$

The first order condition with respect to consumption and labor are

$$\begin{aligned} \frac{\partial \mathcal{L}_{b,t}}{\partial c_{b,t}} = 0 &\iff u_{s,t}^c = \lambda_{b,t} \\ \frac{\partial \mathcal{L}_{b,t}}{\partial n_{b,t}} = 0 &\iff -u_{b,t}^n = (1-\tau_y) w_t \lambda_{b,t} + \lambda_{b,t}\rho_t \mu_t \left(\frac{\left(\theta^{PTI} - \omega\right) w_t}{q_t^* + \alpha}\right) \int^{\bar{e}_t} e_i d\Gamma_e(e_i) \end{aligned}$$

which imply the following intratemporal condition

$$-\frac{u^c_{b,t}}{u^n_{b,t}} = (1-\tau_y)w_t + \rho_t\mu_t \left(\frac{\left(\theta^{PTI}-\omega\right)w_t}{q^*_t+\alpha}\right)\int^{\bar{e}_t} e_i d\Gamma_e(e_i) \tag{A18}$$

where the second term on the right hand side accounts for the borrower's incentive to relax the PTI constrant by working more hours.

The first order condition with respect to the housing choice and the housing stock will pin down the house price as borrowers are the marginal buyers in this economy. To get the equilibrium condition that determines p_h, first take the derivative with respect to $h^*_{b,t}$

$$\frac{\partial\mathcal{L}}{\partial h^*_{b,t}} = 0 \iff -\lambda_{b,t}\rho_t p^h_t + \lambda_{b,t}\rho_t p^h_t \underbrace{\mu_t\theta^{LTV}\left(1-\Gamma_e(\bar{e}_t)\right)}_{=\mathcal{C}_t} + \lambda_{b,t}\lambda^H_t\rho_t = 0$$

which implies that the lagrange multiplier associated to the housing low of motion is given by

$$\lambda^H_t = p^h_t\left(1-\mathcal{C}_t\right)$$

Then, taking the derivative of the lagrangian with respect to $h_{b,t}$ one will get

$$\frac{\partial\mathcal{L}}{\partial h_{b,t}} = 0 \iff \mathbb{E}_t\left[\beta_b u^h_{b,t+1}\right] - \lambda_t\lambda^H_t + \mathbb{E}_t\left[\beta_b\lambda_{b,t+1}p^h_{t+1}\left(\rho_{t+1}-\delta\right) + \beta_b\lambda^H_{t+1}(1-\rho_{t+1})\right] = 0$$

which after re-arranging terms results in the following recursive definition of the Lagrange multiplier associated with the housing stock low of motion

$$\lambda^H_t = \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{u^h_{b,t+1}}{u^c_{b,t+1}} + p^h_{t+1}\left(\rho_{t+1}-\delta\right) + \beta_b\lambda^H_{t+1}(1-\rho_{t+1})\right)\right]$$

Finally, substituting in λ^H_t and solving for p^h_t we get

$$p^h_t = \frac{\mathbb{E}_t\left[\Lambda^b_{t,t+1}\left(u^h_{b,t+1}/u^c_{b,t+1} + p^h_{t+1}\left(1-\delta-(1-\rho_{t+1})\mathcal{C}_{t+1}\right)\right)\right]}{1-\mathcal{C}_t} \tag{A19}$$

To get the first order condition with respect to the newly issued mortgages is useful to expand the lows of motion of the debt balances and the total promised payaments

$$m_t = \rho_t m_t^* + (1-\rho_t)(1-\nu)\pi_t^{-1}\left(\rho_{t-1}m_{t-1}^* + (1-\rho_{t-1})(1-\nu)\pi_{t-1}^{-1}m_{t-2}\right)$$
$$x_{b,t} = \rho_t q_t^* m_t^* + (1-\rho_t)(1-\nu)\pi_t^{-1}\left(\rho_{t-1}q_{t-1}^* m_{t-1}^* + (1-\rho_{t-1})(1-\nu)\pi_{t-1}^{-1}x_{b,t-2}\right)$$

and substitute these into the budget constraint instead of taking derivatives with respect to the stock and the choice as we did for housing. With these expanded expressions in mind, let's take the derivate of the lagrangian with respect to m_t^*

$$\frac{\partial \mathcal{L}_{b,t}}{\partial m_t^*} = 0 \iff \lambda_{b,t}\rho_t - \lambda_{b,t}\mu_t\rho_t = \beta_b \mathbb{E}_t\left[\lambda_{b,t+1}\pi_{t+1}^{-1}\rho_t\left(\left(1-\tau_y\right)q_t^* + \nu + \rho_{t+1}\left(1-\nu\right)\right)\right] +$$
$$+ \beta_s^2\mathbb{E}_{t+1}\left[\lambda_{b,t+2}\pi_{t+1}^{-1}\rho_t\left(\pi_{t+2}^{-1}\left(1-\rho_{t+1}\right)\left(1-\nu\right)\left(\left(1-\tau_y\right)q_t^* + \nu + \rho_{t+2}(1-\nu)\right)\right)\right] + \dots$$
$$+ \beta_s^k\mathbb{E}_{t+k}\left[\lambda_{b,t+k+1}\pi_{t+1}^{-1}\rho_t\left(\prod_{j=1}^{k}\pi_{t+j+1}^{-1}\left(1-\rho_{t+j}\right)(1-\nu)\right)\left(\left(1-\tau_y\right)q_t^* + \nu + (1-\nu)\rho_{t+k+1}\right)\right] + \dots$$

which we can write recursively as follows

$$1 = \Omega_{b,t}^m + \Omega_{b,t}^x q_t^* + \mu_t \tag{A20}$$

where μ_t is the multiplier on the borrower's aggreagate credit limit, and $\Omega_{b,t}^m$ and $\Omega_{b,t}^x$ are the marginal costs to the borrower of taking on an additional dollar of face value debt, and of promising an additional dollar of initial payments. These values are defined by

$$\Omega_{b,t}^m = \mathbb{E}_t\left[\Lambda_{t,t+1}^b\pi_{t+1}^{-1}\left(\nu + (1-\nu)\,\rho_{t+1} + (1-\nu)(1-\rho_{t+1})\Omega_{b,t+1}^m\right)\right] \tag{A21}$$
$$\Omega_{b,t}^x = \mathbb{E}_t\left[\Lambda_{t,t+1}^b\pi_{t+1}^{-1}\left(\left(1-\tau_y\right) + (1-\nu)(1-\rho_{t+1})\Omega_{b,t+1}^x\right)\right] \tag{A22}$$

where we have used the definition of the borrower's stochastic discount factor (A3).

Finally, the first order condition with respect to the prepayment choice

$$\rho_t = \Gamma_\kappa\left\{\left(1-\Omega_{b,t}^m - \Omega_{b,t}^x q_{t-1}\right)\left(1-\frac{(1-\nu)\pi_t^{-1}m_{t-1}}{m_t^*}\right) - \Omega_{b,t}^x\left(q_t^* - q_t\right)\right\} \tag{A23}$$

A.1.4. Saver's Problem

The representative saver chooses consumption $c_{s,t}$, labor supply, $n_{s,t}$ and the face value of newly issued mortgages m_t^* to maximize (A1) using the utility function

$$u(c_{s,t}, \tilde{H}_s, n_{s,t}) = \log(c_{s,t}/\chi_s) + \xi \log(\tilde{H}_s/\chi_s) - \eta_s \frac{\left(n_{s,t}/\chi_s\right)^{1+\varphi}}{1+\varphi} \tag{A24}$$

subject to his budget constraint

$$\begin{aligned} c_{s,t} \leq &\left(1-\tau_y\right) w_t n_{s,t} + \pi_t^{-1} x_{s,t-1} - \rho_t \left(m_t^* - (1-\nu)\pi_t^{-1} m_{t-1}\right) \\ &- \delta p_t^h \tilde{H}_s - \left(R_t^{-1} b_t - \pi_t^{-1} b_{t-1}\right) + \Pi_t + T_{s,t} \end{aligned} \tag{A25}$$

and low of motions for debt balances and total promised payments on existing debts

$$m_t = \rho_t m_t^* + (1-\rho_t)(1-\nu)\pi_t^{-1} m_{t-1} \tag{A26}$$

$$x_{t,s} = \rho_t \left(q_t^* - \Delta_{q,t}\right) m_t^* + (1-\rho_t)(1-\nu)\pi_t^{-1} x_{s,t-1} \tag{A27}$$

Recursive Formulation. Putting all these pieces together and setting up the problem in recursive formulation,

$$\begin{aligned} V_s(m_{t-1}, \tilde{H}_s, x_{s,t-1}) = &\max_{c_{s,t}, n_{s,t}, b_t, m_t^*} u\left(c_{s,t}, \tilde{H}_s, n_{t,s}\right) + \beta_s \mathbb{E}_t V_s\left(m_t, \tilde{H}_s, x_{s,t}\right) \\ &\text{s.t.} \quad \text{(A24)} - \text{(A27)} \end{aligned}$$

First Order Conditions. Setting up the Lagrangian

$$\begin{aligned} \mathcal{L}_{s,t} = &u\left(c_{s,t}, \tilde{H}_s, n_{t,s}\right) + \beta_s \mathbb{E}_t V_s\left(m_t, \tilde{H}_s, x_{s,t}\right) + \lambda_{s,t} \left(\left(1-\tau_y\right) w_t n_{s,t} + \pi_t^{-1} x_{s,t-1} \right. \\ &\left. -\rho_t \left(m_t^* - (1-\nu)\pi_t^{-1} m_{t-1}\right) - \delta p_t^h \tilde{H}_s - \left(R_t^{-1} b_t - \pi_t^{-1} b_{t-1}\right) + \Pi_t + T_{s,t} - c_{s,t}\right) \end{aligned}$$

The first order conditions with respect to consumption and labor are

$$\frac{\partial \mathcal{L}_{s,t}}{\partial c_{s,t}} = 0 \iff u_{s,t}^c = \lambda_{s,t}$$

$$\frac{\partial \mathcal{L}_{s,t}}{\partial n_{s,t}} = 0 \iff -u_{s,t}^n = (1+\tau_y) w_t \lambda_{s,t}$$

which imply the following equilibirum condition

$$-\frac{u^n_{s,t}}{u^c_{s,t}} = (1-\tau_y)w_t \tag{A28}$$

where the wage is expressed in real terms.

Then, the first order condition with respect to the one-period bond result in

$$\frac{\partial \mathcal{L}_{s,t}}{\partial b_t} = 0 \iff \lambda_{s,t}R_t^{-1} = \beta_s \mathbb{E}_t\left[\lambda_{s,t+1}\pi_{t+1}^{-1}\right]$$

which after using the FOC with respect to consumption to substitute in the Lagrange multiplier and using the definition of the stochastic discount factor, we obtain the equilibirium condition

$$1 = R_t \mathbb{E}_t\left[\Lambda^s_{t,t+1}\pi_{t+1}^{-1}\right] \tag{A29}$$

which is an intratemporal condition that relates today's and tomorrow's marginal utilities through the real return on one-period bonds.

Finally, the first order condition with respect to the newly issued mortgages will get us the last optimality condition regarding the savers problem. For that purpose, it is useful to expand the low of motion of the of the debt balances and the total promised payments

$$m_t = \rho_t m^*_t + (1-\rho_t)(1-\nu)\pi_t^{-1}\left(\rho_{t-1}m^*_{t-1} + (1-\rho_{t-1})(1-\nu)\pi_{t-1}^{-1}m_{t-2}\right)$$
$$x_{t,s} = \rho_t\left(q^*_t - \Delta_{q,t}\right)m^*_t + (1-\rho_t)(1-\nu)\pi_t^{-1}\left(\rho_{t-1}\left(q^*_{t-1} - \Delta_{q,t-1}\right)m^*_{t-1} + (1-\rho_{t-1})(1-\nu)\pi_{t-1}^{-1}x_{s,t-2}\right)$$

With these in mind, let's take the derivative of the lagrangian with respect to the newly issued mortgages

$$\begin{aligned}\frac{\partial \mathcal{L}_{s,t}}{\partial m^*_t} = 0 \iff \lambda_{s,t}\rho_t &= \beta_s \mathbb{E}_t\left[\lambda_{s,t+1}\pi_{t+1}^{-1}\rho_t\left(\left(q^*_t - \Delta_{q,t}\right) + \rho_{t+1}\left(1-\nu\right)\right)\right] + \\ &+ \beta_s^2 \mathbb{E}_{t+1}\left[\lambda_{s,t+2}\pi_{t+1}^{-1}\rho_t\left(\pi_{t+2}^{-1}\left(1-\rho_{t+1}\right)\left(1-\nu\right)\left(\left(q^*_t - \Delta_{q,t}\right) + \rho_{t+2}(1-\nu)\right)\right)\right] + \ldots \\ &+ \beta_s^k \mathbb{E}_{t+k}\left[\lambda_{b,t+k+1}\pi_{t+1}^{-1}\rho_t\left(\prod_{j=1}^{k}\pi_{t+j+1}^{-1}\left(1-\rho_{t+j}\right)\left(1-\nu\right)\right)\left(\left(q^*_t - \Delta_{q,t}\right) + \rho_{t+k+1}(1-\nu)\right)\right] + \ldots\end{aligned}$$

which we can write recursively as follows

$$1 = \Omega^{m}_{s,t} + \Omega^{x}_{s,t}\left(q^{*}_{t} - \Delta_{q,t}\right) \tag{A30}$$

where $\Omega^{m}_{s,t}$ and $\Omega^{x}_{s,t}$ are the marginal continuation benefits to the saver of an additional unit of face value and an additional dollar of promised initial payments, respectively. These values are defined by

$$\Omega^{m}_{s,t} = \mathbb{E}_t\left[\Lambda^{s}_{t,t+1}\pi^{-1}_{t+1}\left(\rho_{t+1}(1-\nu) + (1-\rho_{t+1})(1-\nu)\Omega^{m}_{s,t+1}\right)\right] \tag{A31}$$

$$\Omega^{x}_{s,t} = \mathbb{E}_t\left[\Lambda^{s}_{t,t+1}\pi^{-1}_{t+1}\left(1 + (1-\rho_{t+1})(1-\nu)\,\Omega^{x}_{s,t+1}\right)\right] \tag{A32}$$

where we have used the definition of the saver's stochastic discount factor (A3).

A.2. Firms

The production side of the economy is populated by a competitive final good producer and a continuum of intermediate goods producers owned by the saver.

A.2.1. Final Good Producer

There is a single final good producer that bundles intermediate goods into a final good available for consumption. There is a continuum of intermediate goods indexed by $i \in [0, 1]$ which are purchased at a price $P_t(i)$. The final good is CES aggregate of these goods

$$Y_t = \left[\int_0^1 y_t(i)^{\frac{\lambda-1}{\lambda}}\,di\right]^{\frac{\lambda}{\lambda-1}}. \tag{A33}$$

Thus, their (static) profit maximization problem is given by

$$\max_{y_t(i)} P_t\left[\int_0^1 y_t(i)^{\frac{\lambda-1}{\lambda}}\,di\right]^{\frac{\lambda}{\lambda-1}} - \int_0^1 P_t(i)\,y_t(i)di$$

where P_t is the price of the final good. The first order condition of this problem results in the following optimality condition

$$P_t \left(\int_0^1 y_t(i)^{\frac{\lambda-1}{\lambda}} di \right)^{\frac{\lambda}{\lambda-1}-1} y_t(i)^{\frac{\lambda-1}{\lambda}-1} = P_t(i). \tag{A34}$$

Equation (A34) implies that the demand function for the intermediate good i is

$$y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\lambda} Y_t. \tag{A35}$$

Note that to obtain equation (A35) one only needs to solve for $y_t(i)$ in (A34) and use the definition of Y_t in (A33) to simplify the expression.

Finally, note that one can obtain a relationship between the final and intermediate goods price using the intermediate goods demand function and the zero profit condition of the perfectly competitive final good firm[2]

$$P_t = \left(\int_0^1 P_t(i)^{1-\lambda} di \right)^{\frac{1}{1-\lambda}}. \tag{A36}$$

A.2.2. Intermediate Good Producer

The intermediate of good i uses labor, $n_t(i)$, as the only input in production. The production function is linear in labor input and given by

$$y_t(i) = a_t n_t(i) \tag{A37}$$

where a_t is total factor productivity which evolves according to

$$\log a_{t+1} = (1 - \phi_a)\mu_a + \phi_a \log a_t + \varepsilon_{a,t+1} \tag{A38}$$

where $\varepsilon_{a,t+1}$ is a white noise process and refered to as productivity or TFP shock.

Given the downward sloping supply curve, firms have some market power and can set prices. It is assumed that they cannot do it freely and therefore are subject to price stickinees of the Calvo-Yun form with indexation. That is, a fraction $1-\zeta$ of firms are able to adjust their prices each period, while the remaining fraction ζ adjust thier existing

[2]The zero profit condition implies that the nominal price of the final good is equal to the sum of prices times the quantities of intermediates, i.e $P_t Y_t = \int_0^1 P_t(i) y_t(i)$.

price by the rate of steady state inflation.

Cost minimization. Firms are nevertheless able to freely choose how much labor to use each period. Hence, we can first consider the optimal labor choice for a given price and write it as a cost-minimization problem since firms will always choose labor to minimize cost regardless of the the price choosen.

$$\min_{n_t(i)} \quad W_t n_t(i)$$
$$\text{s.t.} \quad a_t n_t(i) \leq \left(\frac{P_t(i)}{P_t}\right)^{-\lambda} Y_t.$$

The problem is to minimize nominal costs subject to the restriction that the firm produces at least as much as is demanded at a given price. Set it up as a Lagrangian[3]

$$\mathcal{L} = -W_t n_t(i) + MC_t \left(a_t n_t(i) - \left(\frac{P_t(i)}{P_t}\right)^{-\lambda} Y_t \right)$$

The first order condition is

$$\frac{\partial \mathcal{L}}{\partial n_t(i)} = 0 \Leftrightarrow W_t = MC_t a_t \tag{A39}$$

so that firms hire labor up until the point where the wage equal the marginal cost times the marginal product of labor.

Optimal price setting. The pricing problem of a firm that gets to update its price in period t is dynamic since the price it picks today will, in expectation, affect its current and future profits. Thus, the firm problem consists on maximizing its expected discounted profits. We assume that the firm discounts these future profit flows by the stochastic discount factor Λ_t. In addition, it also discount these flows by ζ^t since conditional on choising its price today, there is a ζ probability of having that price (times the inflation factor) in the next period, ζ^2 for the period after that, and so on. Further, we assume that the price of those non adjusting firms is indexed to the steady state inflation rate, which implies the following indexation factor $X_{t,t+s} = \prod_{k=1}^{s} \pi_{ss}$. Finally, the optimal price choice requires that the intermediate goods demand is met. In short, intermediate firms

[3]Note that the lagrange multiplier has the interpretation of (nominal) marginal cost, i.e. how much do costs go up if the firm has to produce one more unit of output.

solve the following problem

$$
\begin{aligned}
\max_{\tilde{P}_t(i)} \ & \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \zeta^s \left(\frac{\tilde{P}_{t,t+s}(i)}{P_{t+s}} y_{t+s}(i) - mc_{t+s} y_{t+s}(i) \right) \\
\text{s.t.} \quad & \tilde{P}_{t,t+s} = X_{t,t+s} \tilde{P}_t(i) \\
& y_{t+s}(i) = \left(X_{t,t+s} \frac{\tilde{P}_t(i)}{P_{t+s}} \right)^{-\lambda} Y_{t+s} \\
& \tilde{P}_{t+s+1} = \begin{cases} P^*_{t+s+1} & \text{with prob. } 1-\zeta \\ \tilde{P}_{t+s}(i) & \text{with prob. } \zeta \end{cases}
\end{aligned}
$$

where we have expressed the profits in real terms (since this is what households care about) by dividing by the price level P_{t+s}. We can make this an unconstrained problem by substituting the constraints into the objective function[4]

$$
\max_{P^*_t} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \Lambda_{t,t+s} \zeta^s \left(X_{t,t+s} \frac{P^*_t}{P_{t+s}} - mc_{t+s} \right) \left(X_{t,t+s} \frac{\tilde{P}^*_{t+s}}{P_{t+s}} \right)^{-\lambda} Y_{t+s} \right]
$$

Then, taking the derivative with respect to P^*_t we obtain

$$
\begin{aligned}
\mathbb{E}_t \left[\sum_{s=0}^{\infty} \Lambda_{t,t+s} \zeta^s \left(y_{t+s}(i) \frac{X_{t,t+s}}{P_{t+s}} - \frac{P^*_t}{P_{t+s}} X_{t,t+s} \lambda \frac{y_{t+s}(i)}{P^*_t} + mc_{t+s} \frac{y_{t+s}(i)}{P^*_t} \lambda \right) \right] &= 0 \\
\mathbb{E}_t \left[\sum_{s=0}^{\infty} \Lambda_{t,t+s} \zeta^s \left(y_{t+s}(i) X_{t,t+s} (1-\lambda) \frac{P^*_t}{P_{t+s}} + mc_{t+s} \lambda y_{t+s}(i) \right) \right] &= 0 \\
\mathbb{E}_t \left[\sum_{s=0}^{\infty} \Lambda_{t,t+s} \zeta^s \left(X_{t,t+s} \frac{P^*_t}{P_{t+s}} + \frac{\lambda}{1-\lambda} mc_{t+s} \right) y_{t+s}(i) \right] &= 0 \\
\mathbb{E}_t \left[\sum_{s=0}^{\infty} \Lambda_{t,t+s} \zeta^s \left(X_{t,t+s} \frac{P^*_t}{P_{t+s}} + \frac{\lambda}{1-\lambda} mc_{t+s} \right) \left(X_{t,t+s} \frac{P^*_t}{P_{t+s}} \right)^{-\lambda} Y_{t+s} \right] &= 0
\end{aligned}
$$

Rearrange terms,

$$
\underbrace{\frac{\lambda}{\lambda-1} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \Lambda_{t,t+s} \zeta^s mc_{t+s} \left(X_{t,t+s} \frac{P^*_t}{P_{t+s}} \right)^{-\lambda} Y_{t+s} \right]}_{=z_{1,t}} = \underbrace{\mathbb{E}_t \left[\sum_{s=0}^{\infty} \Lambda_{t,t+s} \zeta^s \left(X_{t,t+s} \frac{P^*_t}{P_{t+s}} \right)^{1-\lambda} Y_{t+s} \right]}_{=z_{2,t}}
$$

[4]Note that we can drop the index i because all firms updating their price will make the same decision. This follows from equation (A39) which states the marginal costs are ratio of the aggregate wage and TFP and consequently equivalent across firms.

and find a recursive expression for each side of the first order condition. Start from the left hand side (LHS) by excluding the first summon

$$z_{1,t} = \frac{\lambda}{\lambda-1} mc_t \left(\frac{P_t^*}{P_t}\right)^{-\lambda} Y_t + \frac{\lambda}{\lambda-1} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \Lambda_{t,t+s+1} \zeta^{s+1} mc_{t+s+1} \left(X_{t+1,t+s+1} \frac{P_t^*}{P_{t+s+1}}\right)^{-\lambda} Y_{t+s+1}\right]$$

$$z_{1,t} = \frac{\lambda}{\lambda-1} mc_t \left(\frac{P_t^*}{P_t}\right)^{-\lambda} Y_t + \frac{\lambda}{\lambda-1} \zeta \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\pi_{ss} \frac{P_t^*}{P_{t+1}^*}\right)^{-\lambda} \sum_{s=0}^{\infty} \lambda_{t+1,t+s+1} \zeta^s mc_{t+s+1} \left(X_{t,t+s} \frac{P_t^*}{P_{t+s+1}}\right)^{-\lambda} Y_{t+s+1}\right]$$

$$z_{1,t} = \frac{\lambda}{\lambda-1} mc_t \left(\frac{P_t^*}{P_t}\right)^{-\lambda} Y_t + \frac{\lambda}{\lambda-1} \zeta \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\pi_{ss} \frac{P_t^*}{P_{t+1}^*}\right)^{-\lambda} z_{1,t+1}\right]$$

where we have already expressed the LHS in recursive form. Simplifying that expression furhter we get

$$z_{1,t} = \frac{\lambda}{\lambda-1} mc_t \left(\tilde{p}_t\right)^{-\lambda} Y_t + \frac{\lambda}{\lambda-1} \zeta \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\pi_{ss} \frac{P_t}{P_{t+1}} \frac{\frac{P_t^*}{P_t}}{\frac{P_{t+1}^*}{P_{t+1}}}\right)^{-\lambda} z_{1,t+1}\right]$$

$$z_{1,t} = \frac{\lambda}{\lambda-1} mc_t \left(\tilde{p}_t\right)^{-\lambda} Y_t + \frac{\lambda}{\lambda-1} \zeta \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{\pi_{t+1}}{\pi_{ss}} \frac{\tilde{p}_{t+1}}{\tilde{p}_t}\right)^{\lambda} z_{1,t+1}\right]$$

Now turning to the right hand side (RHS) and working out that expression in a similar manner

$$z_{2,t} = Y_t \left(\tilde{p}_t\right)^{1-\lambda} + \mathbb{E}_t \left[\sum_{s=0}^{\infty} \Lambda_{t,t+s+1} \zeta^s \left(X_{t,t+s+1} \frac{P_t^*}{P_{t+s+1}}\right)^{1-\lambda} Y_{t+s+1}\right]$$

$$z_{2,t} = Y_t \left(\tilde{p}_t\right)^{1-\lambda} + \zeta \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\pi_{ss} \frac{P_t^*}{P_{t+1}}\right)^{1-\lambda} \sum_{s=0}^{\infty} \Lambda_{t+1,t+s+1} \zeta^s \left(X_{t+1,t+s+1} \frac{P_{t+1}^*}{P_{t+s+1}}\right)^{1-\lambda} Y_{t+s+1}\right]$$

$$z_{2,t} = Y_t \left(\tilde{p}_t\right)^{1-\lambda} + \zeta \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\pi_{ss} \frac{P_t^*}{P_{t+1}}\right)^{1-\lambda} z_{2,t+1}\right]$$

$$z_{2,t} = Y_t \left(\tilde{p}_t\right)^{1-\lambda} + \zeta \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{\pi_{t+1}}{\pi_{ss}} \frac{\tilde{p}_{t+1}}{\tilde{p}_t}\right)^{\lambda-1} z_{2,t+1}\right]$$

In short, the optimal price setting of the intermediate firm results in the following equilibrium conditions:

$$1 = \frac{z_{1,t}}{z_{2,t}} \tag{A40}$$

(A41) $$z_{1,t} = \frac{\lambda}{\lambda-1} mc_t \left(\tilde{p}_t\right)^{-\lambda} Y_t + \frac{\lambda}{\lambda-1} \zeta \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{\pi_{t+1}}{\pi_{ss}} \frac{\tilde{p}_{t+1}}{\tilde{p}_t} \right)^{\lambda} z_{1,t+1} \right]$$

(A42) $$z_{2,t} = Y_t \left(\tilde{p}_t\right)^{1-\lambda} + \zeta \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{\pi_{t+1}}{\pi_{ss}} \frac{\tilde{p}_{t+1}}{\tilde{p}_t} \right)^{\lambda-1} z_{2,t+1} \right]$$

A.2.3. Inflation Dynamics, Price Dispersion and Aggregation

Inflation dynamics. The low of motion for inflation can be derived from the price index in equation (A36) as follows

$$P_t^{1-\lambda} = \int_0^1 P_t(i)^{1-\lambda} di$$
$$P_t^{1-\lambda} = \int_0^{\zeta} \left(X_{t,t+1} P_{t-1}(i) \right)^{1-\lambda} di + \int_{\zeta}^1 \left(P_t^* \right)^{1-\lambda} di$$

Then, applying the law of large numbers and dividing by the price level in period $t-1$ we get

$$P_t^{1-\lambda} = \zeta (\pi_{ss} P_{t-1})^{1-\lambda} + (1-\zeta) \left(P_t^* \right)^{1-\lambda}$$
$$(\pi_t)^{1-\lambda} = \zeta \left(\pi_{ss}\right)^{1-\lambda} + (1-\zeta) \left(\frac{P_t^*}{P_{t-1}} \frac{P_t}{P_t} \right)^{1-\lambda}$$
$$(\pi_t)^{1-\lambda} = \zeta \left(\pi_{ss}\right)^{1-\lambda} + (1-\zeta) \left(\frac{P_t^*}{P_{t-1}} \frac{P_t}{P_t} \right)^{1-\lambda}$$
$$(\pi_t)^{1-\lambda} = \zeta \left(\pi_{ss}\right)^{1-\lambda} + (1-\zeta) \left(\frac{P_t^*}{P_t} \pi_t \right)^{1-\lambda}$$

Finally, rearraging terms and solving for π_t we obtain the low of motion of inflation

(A43) $$\pi_t = \pi_{ss} \left[\frac{1 - (1-\zeta)\, \tilde{p}_t^{1-\lambda}}{\zeta} \right]^{\frac{1}{\lambda-1}}$$

where $\tilde{p}_t$ is the ratio of the optimal price for reseting firms relative to the average price.

Price dispersion. Let's define price dispersion as

(A44) $$\mathcal{D}_t = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\lambda} di$$

Then,

$$\mathcal{D}_t = (1-\zeta)\int_0^1 \left(\frac{P_t^*}{P_t}\right)^{-\lambda} di + \zeta \int_0^1 \left(\frac{\pi_{ss} P_{t-1}(i)}{P_t}\right)^{-\lambda} di$$
$$\mathcal{D}_t = (1-\zeta)\left(\tilde{p}_t\right)^{-\lambda} + \zeta \int_0^1 \left(\frac{\pi_{ss} P_{t-1}(i)}{P_t}\frac{P_{t-1}}{P_{t-1}}\right)^{-\lambda} di$$
$$\mathcal{D}_t = (1-\zeta)\left(\tilde{p}_t\right)^{-\lambda} + \zeta \left(\frac{\pi_{ss}}{\pi_t}\right)^{-\lambda} \int_0^1 \left(\frac{P_{t-1}(i)}{P_{t-1}}\right)^{-\lambda} di$$

Finally, using (A44) we get a recursive expression for the price dispersion index

(A45) $$\mathcal{D}_t = (1-\zeta)\left(\tilde{p}_t\right)^{-\lambda} + \zeta \left(\frac{\pi_{ss}}{\pi_t}\right)^{-\lambda} \mathcal{D}_{t-1}$$

Aggregation. Marginal costs are positive in equilibrium. Hence, the constraint in the cost minimization problem holds with equality, i.e. intermediate good firms meet demand. As a result, we can derive an expression that relates total output to the aggregate amount of hours worked as follows

$$a_t n_t(i) - \left(\frac{P_t(i)}{P_t}\right)^{-\lambda} Y_t = 0$$

which after aggregating over all different varieties reads

$$\int_0^1 \left(a_t n_t(i) - \left(\frac{P_t(i)}{P_t}\right)^{-\lambda} Y_t\right) di = 0$$

and solving for Y_t yields

(A46) $$Y_t = \frac{n_t a_t}{\mathcal{D}_t}$$

where we have used the definition of price dispersion (A44) and the fact that hour worked is independent of i since all firms face the same marginal costs, i.e. $n_t(i) = n_t$.

A.3. Monetary Authority

The monetary authority follows a Taylor rule of the form:

$$\log R_t = \log \bar{\pi}_t + \phi_r (\log R_{t-1} - \log \bar{\pi}_{t-1}) + (1 - \phi_r)\left[(\log R_{ss} - \log \pi_{ss}) + \psi_\pi (\log \pi_t - \log \bar{\pi}_t)\right] \tag{A47}$$

where $\bar{\pi}$ is a time-varying inflation target defined by

$$\log \bar{\pi}_t = (1 - \psi_{\bar{\pi}}) \log \pi_{ss} + \psi_{\bar{\pi}} \log \bar{\pi}_{t-1} + \varepsilon_{\pi,t} \tag{A48}$$

where $\varepsilon_{\bar{\pi},t}$ is a white noise process that is refered to as inflation target shock.

A.4. Equilibrium

A competitive equilibrium in this economy is a sequence of endogenous states (m_{t-1}, x_{t-1}), allocations (c_j, t, n_j, t), mortgage and housing market quantitites $(h^*_{b,t}, m^*_t, \rho_t)$ and prices $(\pi_t, w_t, p^h_t, R_t, q^*_t)$ such that borrower, saver and firm optimality conditions are satisfied, as well as labor, housing, bonds and goods markets clear.

A.4.1. Optimality conditions

Borrower.

$$-\frac{u^c_{b,t}}{u^n_{b,t}} = (1 - \tau_y) w_t + \rho_t \mu_t \left(\frac{\left(\theta^{PTI} - \omega\right) w_t}{q^*_t + \alpha} \right) \int^{\bar{e}_t} e_i d\Gamma_e(e_i) \tag{A49}$$

$$p^h_t = \frac{\mathbb{E}_t \left[\Lambda^b_{t,t+1} \left(u^h_{b,t+1}/u^c_{b,t+1} + p^h_{t+1} (1 - \delta - (1 - \rho_{t+1}) \mathcal{C}_{t+1}) \right) \right]}{1 - \mathcal{C}_t} \tag{A50}$$

$$1 = \Omega^m_{b,t} + \Omega^x_{b,t} q^*_t + \mu_t \tag{A51}$$

$$\Omega^m_{b,t} = \mathbb{E}_t \left[\Lambda^b_{t,t+1} \pi^{-1}_{t+1} \left(\nu \tau_y + (1 - \nu) \rho_{t+1} + (1 - \nu)(1 - \rho_{t+1}) \Omega^m_{b,t+1} \right) \right] \tag{A52}$$

$$\Omega^x_{b,t} = \mathbb{E}_t \left[\Lambda^b_{t,t+1} \pi^{-1}_{t+1} \left((1 - \tau_y) + (1 - \nu)(1 - \rho_{t+1}) \Omega^x_{b,t+1} \right) \right] \tag{A53}$$

$$\rho_t = \Gamma_\kappa \left\{ \left(1 - \Omega^m_{b,t} - \Omega^x_{b,t} q_{t-1} \right) \left(1 - \frac{(1 - \nu) \pi^{-1}_t m_{t-1}}{m^*_t} \right) - \Omega^x_{b,t} \left(q^*_t - q_t \right) \right\} \tag{A54}$$

Saver.

$$-\frac{u^n_{s,t}}{u^c_{s,t}} = (1-\tau_y)w_t \tag{A55}$$

$$1 = R_t \mathbb{E}_t \left[\Lambda^s_{t,t+1}\pi^{-1}_{t+1}\right] \tag{A56}$$

$$1 = \Omega^m_{s,t} + \Omega^x_{s,t}\left(q^*_t - \Delta_{q,t}\right) \tag{A57}$$

$$\Omega^m_{s,t} = \mathbb{E}_t \left[\Lambda^s_{t,t+1}\pi^{-1}_{t+1}\left(\rho_{t+1}(1-\nu) + (1-\rho_{t+1})(1-\nu)\Omega^m_{s,t+1}\right)\right] \tag{A58}$$

$$\Omega^x_{s,t} = \mathbb{E}\left[\Lambda^s_{t,t+1}\pi^{-1}_{t+1}\left(1 + (1-\rho_{t+1})(1-\nu)\,\Omega^x_{s,t+1}\right)\right] \tag{A59}$$

Firm.

$$1 = \frac{z_{1,t}}{z_{2,t}} \tag{A60}$$

$$z_{1,t} = \frac{\lambda}{\lambda-1} mc_t \left(\tilde{p}_t\right)^{-\lambda} Y_t + \frac{\lambda}{\lambda-1}\zeta \mathbb{E}_t \left[\Lambda_{t,t+1}\left(\frac{\pi_{t+1}}{\pi_{ss}}\frac{\tilde{p}_{t+1}}{\tilde{p}_t}\right)^{\lambda} z_{1,t+1}\right] \tag{A61}$$

$$z_{2,t} = Y_t\left(\tilde{p}_t\right)^{1-\lambda} + \zeta\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{\pi_{t+1}}{\pi_{ss}}\frac{\tilde{p}_{t+1}}{\tilde{p}_t}\right)^{\lambda-1} z_{2,t+1}\right] \tag{A62}$$

$$\pi_t = \pi_{ss}\left[\frac{1-(1-\zeta)\,\tilde{p}^{1-\lambda}_t}{\zeta}\right]^{\frac{1}{\lambda-1}} \tag{A63}$$

$$\mathcal{D}_t = (1-\zeta)\left(\tilde{p}_t\right)^{-\lambda} + \zeta\left(\frac{\pi_{ss}}{\pi_t}\right)^{-\lambda}\mathcal{D}_{t-1} \tag{A64}$$

$$Y_t = \frac{a_t n_t}{\mathcal{D}_t} \tag{A65}$$

A.4.2. Market clearing

Goods Market.

$$c_{b,t} + c_{s,t} + \delta\, p^h_t \tilde{H} = Y_t \tag{A66}$$

Bond Market.

$$b_{s,t} = 0 \tag{A67}$$

Housing Market.

(A68) $$h_{b,t} + \tilde{H}_s = \tilde{H}$$

Labor Market.

(A69) $$n_{b,t} + n_{s,t} = n_t$$

Appendix B. Model with Adjustable Rate Mortgages (ARM)

Under adjustable-rate mortgages (ARMs), the saver gives the borrower \$1 at origination. In exchange, the saver recives $\$(1-\nu)^k q^*_{t+k-1}$ at time $t+k$, for all $k>0$ until pre-payment, where $q^*_{t+k-1} = \left(R_{t+k-1}-1\right)+\nu$. This coupon rate is obtained from arbitrage considerations, since a saver must be indiferent between holding an adjustable rate mortgage for one period and the one-period bond, since both are short term risk-free assets.

Under ARM contracts, promised payment is no longer an endogenous state variable, but is instead defined period-by-period using

$$x_{b,t} = q^*_t m_t \tag{A70}$$

$$x_{s,t} = \left(q^*_t - \Delta_{q,t}\right) m_t \tag{A71}$$

which implies that the borrower's and saver's problem slightly change. I derive the new optimality conditions below.

B.1. Borrower's problem

The Lagrangian now is given by

$$\begin{aligned}\mathcal{L}^{ARM}_{b,t} =& u\left(c_{b,t}, h_{b,t-1}, n_{b,t}\right) + \beta_b \mathbb{E}_t V_b\left(m_t, h_{b,t}\right) + \lambda_{b,t}\Big((1-\tau_y)w_t n_{b,t} - \pi_t^{-1}\left(\left(1-\tau_y\right) q^*_t m_{t-1} + \nu m_{t-1}\right) \\ &+\rho_t\left(m^*_t - (1-\nu)\pi_t^{-1} m_{t-1}\right) - \delta\, p^h_t h_{b,t-1} - \rho_t\, p^h_t\left(h^*_{b,t} - h_{b,t-1}\right) - \left(\Psi(\rho_t, m^*_t) - \bar{\Psi}_t\right) + T_{b,t} - c_{b,t} \\ &+\mu_t\rho_t\left(\bar{m}^{PTI}_t \int^{\bar{e}_t} e_i d\Gamma_e(e_i) + \bar{m}^{LTV}_t\left(1-\Gamma_e\left(\bar{e}_t\right)\right) - m^*_t\right)\Big)\end{aligned}$$

As a result, the derivative with respect to the face value of new debt simplifies to

$$\begin{aligned}\frac{\partial \mathcal{L}^{AMR}_{b,t}}{\partial m^*_t} = 0 \iff & \lambda_{b,t}\rho_t - \lambda_{b,t}\mu_t\rho_t = \beta_s \mathbb{E}_t\left[\lambda_{b,t+1}\pi^{-1}_{t+1}\rho_t\left(\left(1-\tau_y\right) q^*_t + \nu + (1-\nu)\,\rho_{t+1}\right)\right] + \\ &+ \beta_s^2 \mathbb{E}_{t+1}\left[\lambda_{b,t+2}\pi^{-1}_{t+1}\rho_t\left(\pi^{-1}_{t+2}\left(1-\rho_{t+1}\right)(1-\nu)\left(\nu + (1-\nu)\,\rho_{t+2}\right)\right)\right] + \dots \\ &+ \beta_s^k \mathbb{E}_{t+k}\left[\lambda_{b,t+k+1}\pi^{-1}_{t+1}\rho_t\left(\prod_{j=1}^{k}\pi^{-1}_{t+j+1}\left(1-\rho_{t+j}\right)(1-\nu)\right)\left(\nu + (1-\nu)\,\rho_{t+k+1}\right)\right] + \dots\end{aligned}$$

which in recursive from can be rewritten as follows

$$\Omega_{b,t} = 1 - \mu_t \tag{A72}$$

where $\Omega_{b,t}$ is the total continuation cost of an additional unit of debt and is now defined as

$$\Omega_{b,t} = \mathbb{E}_t \left[\Lambda^b_{t,t+1} \pi^{-1}_{t+1} \left(\left(1-\tau_y\right) q^*_t + \nu + (1-\nu)\,\rho_{t+1} + (1-\nu)\,(1-\rho_{t+1})\,\Omega_{b,t+1} \right) \right] \tag{A73}$$

Moreover, the optimality condition with respect to the prepayment choice is also affected by this change. It simplifies to the following condition

$$\frac{\partial \mathcal{L}^{ARM}_{b,t}}{\partial \rho_t} = 0 \iff$$

which can be rewritten as

$$\rho_t = \Gamma_k \left\{ \left(1-\Omega_{b,t}\right) \left(1 - \frac{(1-\nu)\pi^{-1}_t m_{t-1}}{m^*_t}\right) \right\} \tag{A74}$$

B.2. Saver's problem

The Lagrangian for the saver's problem with ARM is given by

$$\begin{aligned} \mathcal{L}^{ARM}_{s,t} = & u\left(c_{s,t}, \tilde{H}_s, n_{t,s}\right) + \beta_s \mathbb{E}_t V_s\left(m_t, \tilde{H}_s, x_{s,t}\right) + \lambda_{s,t} \Big(\left(1-\tau_y\right) w_t n_{s,t} + \pi^{-1}_t (q^*_t - \Delta_{q,t}) m_{t-1} \\ & - \rho_t \left(m^*_t - (1-\nu)\pi^{-1}_t m_{t-1}\right) - \delta\, p^h_t \tilde{H}_s - \left(R^{-1}_t b_t - b_{t-1}\right) + \Pi_t + T_{s,t} - c_{s,t} \Big) \end{aligned}$$

and consequently its derivative with respect to m^*_t simplifies to

$$\begin{aligned} \frac{\partial \mathcal{L}_{s,t}}{\partial m^*_t} = 0 \iff \lambda_{s,t}\rho_t = & \beta_s \mathbb{E}_t \left[\lambda_{s,t+1} \pi^{-1}_{t+1} \rho_t \left(\left(q^*_t - \Delta_{q,t}\right) + (1-\nu)\,\rho_{t+1} \right) \right] + \\ & + \beta^2_s \mathbb{E}_{t+1} \left[\lambda_{s,t+2} \pi^{-1}_{t+1} \rho_t \left(\pi^{-1}_{t+2} (1-\rho_{t+1})\,(1-\nu)\,(1 + (1-\nu)\rho_{t+2}) \right) \right] + \dots \\ & + \beta^k_s \mathbb{E}_{t+k} \left[\lambda_{b,t+k+1} \pi^{-1}_{t+1} \rho_t \left(\prod_{j=1}^{k} \pi^{-1}_{t+j+1} \left(1-\rho_{t+j}\right)(1-\nu) \right) \left(1 + (1-\nu)\rho_{t+k+1}\right) \right] + \dots \end{aligned}$$

which can be rewritten in recursive form

$$\Omega_{s,t} = 1 \tag{A75}$$

where $\Omega_{s,t}$ is again the total continuation benefit of an additional unit of debt and it is given by

$$\Omega_{s,t} = \mathbb{E}_t \left[\Lambda^s_{t,t+1} \pi^{-1}_{t+1} \left(\left(q^*_t - \Delta_{q,t} \right) + (1-\nu)\, \rho_{t+1} + (1-\nu)(1-\rho_{t+1}) \Omega_{s,t+1} \right) \right] \tag{A76}$$

Appendix C. Model with Fixed Period Adjustable Mortgages (FPAM)

This type of mortgage contract has a fixed period followed by an adjustable rate period. The most common configuration of these type of contracts is a fixed period of 5 years followed by adjustable rates that reset every 12 months. In fact, they are a hybrid combination of the FRM and ARM contracts.

C.1. An example: a one period adjustable mortgage

Let us assume for simplicity that borrowers are in the fixed rate contract for one period and then they move to the adjustable rate mortgage. This new assumption will influence the promised payments made by the borrower and received by the saver. In particular, its low of motion in the borrower's problem is now given by

(A77)
$$x_{b,t}^{FPMA} = \rho_t q_t^* m_t^* + (1-\rho_t)(1-\nu)\pi_t^{-1} q_{t-1}^* m_{t-1}$$

while similarly for the saver is given by

(A78)
$$x_{s,t}^{FPMA} = \rho_t (q_t^* - \Delta_{q,t}) m_t^* + (1-\rho_t)(1-\nu)\pi_t^{-1} \left(q_{t-1}^* - \Delta_{q,t-1}\right) m_{t-1}$$

which again implies that both borrower and saver's problem will slightly change as shown below.

C.1.1. Borrower's problem

Under FPAM, the borrower's problem is characterized by the following Lagrangian

$$\begin{aligned}
\mathcal{L}_{b,t}^{FPAM} =& u\left(c_{b,t}, h_{b,t-1}, n_{b,t}\right) + \beta_b \mathbb{E}_t V_b\left(m_t, h_{b,t}, x_{b,t}\right) + \lambda_{b,t}\Big((1-\tau_y) w_t n_{b,t} + \\
& -\pi_t^{-1}\left((1-\tau_y)\, x_{b,t-1}^{FPAM} + \nu m_{t-1}\right) + \rho_t\left(m_t^* - (1-\nu)\pi_t^{-1} m_{t-1}\right) - \delta\, p_t^h h_{b,t-1} + \\
& -\rho_t\, p_t^h \left(h_{b,t}^* - h_{b,t-1}\right) + \left(\Psi(\rho_t, m_t^*) - \bar{\Psi}_t\right) + T_{b,t} - c_{b,t} + \\
& +\mu_t \rho_t \left(\bar{m}_t^{PTI} \int^{\bar{e}_t} e_i d\Gamma_e(e_i) + \bar{m}_t^{LTV}\left(1 - \Gamma_e\left(\bar{e}_t\right)\right) - m_t^*\right)\Big)
\end{aligned}$$

and consequently, its derivative with respect to m_t^* is now given by

$$\frac{\partial \mathcal{L}^{FPAM}_{b,t}}{\partial m^*_t} = 0 \iff \lambda_{b,t}\rho_t - \lambda_{b,t}\mu_t\rho_t = \beta_s \mathbb{E}_t\left[\lambda_{b,t+1}\pi^{-1}_{t+1}\rho_t\left(\left(1-\tau_y\right)q^*_t + \nu + \rho_{t+1}\left(1-\nu\right)\right)\right] +$$

$$+\beta_s^2\mathbb{E}_{t+1}\left[\lambda_{b,t+2}\pi^{-1}_{t+1}\rho_t\left(\pi^{-1}_{t+2}\left(1-\rho_{t+1}\right)\left(1-\nu\right)\left(\left(1-\tau_y\right)q^*_t + \nu + \rho_{t+2}(1-\nu)\right)\right)\right] + \dots$$

$$+\beta_s^3\mathbb{E}_{t+2}\left[\lambda_{b,t+3}\pi^{-1}_{t+1}\rho_t\left(\pi^{-1}_{t+2}\pi^{-1}_{t+3}\left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right)\left(1-\nu\right)^2\right)\left(\nu + \rho_{t+3}(1-\nu)\right)\right] + \dots$$

$$+\beta_s^k\mathbb{E}_{t+k}\left[\lambda_{b,t+k+1}\pi^{-1}_{t+1}\rho_t\left(\prod_{j=1}^{k}\pi^{-1}_{t+j+1}\left(1-\rho_{t+j}\right)\left(1-\nu\right)\right)\left(\nu + \rho_{t+k+1}(1-\nu)\right)\right] + \dots$$

which we can rewrite as follows

$$1 = \Omega^m_{b,t} + \mho^x_{b,t}q^*_t + \mu_t \tag{A79}$$

where μ_t is the multiplier on borrower's aggregate credit limit, $\Omega^m_{b,t}$ is the marginal cost to the borrower of taking an additional dollar of face value debt ans is defined as in the model with FRM (equation (A22)), and $\mho^x_{b,t}$ is the marginal cost to the borrower of promising an additional dollar of initial payments and it is given by:

$$\begin{aligned}\mho^x_{b,t} =& \mathbb{E}_t\left[\Lambda^b_{t,t+1}\pi^{-1}_{t+1}(1-\tau_y)\right] + \\ &+ \mathbb{E}_{t+1}\left[\Lambda^b_{t,t+1}\Lambda^b_{t+1,t+2}\pi^{-1}_{t+1}\pi^{-1}_{t+2}(1-\rho_{t+1})(1-\nu)(1-\tau_y)\right]\end{aligned} \tag{A80}$$

Intutitvely, since the contract switches to adjustable rates after the first fixed term period, the marginal continuation cost to the borrower of an additional dollar of payments is not anymore the infinte sum of discounted flow of all payments but it is just truncated at the end of the fixed period. In oder words, $\mho^x_{b,t}$ is the finite counterpart of the recursive $\Omega^x_{b,t}$ in the FRM problem.

Note that the first order condition with respect to the pre-payment choice also changes. However, we will start by considering the model with exogenous pre-payments.

C.1.2. Saver's problem

The Lagrangian for the saver's problem under FPAM is given by

$$\begin{aligned}\mathcal{L}^{FPAM}_{s,t} =& u\left(c_{s,t}, \tilde{H}_s, n_{t,s}\right) + \beta_s\mathbb{E}_t V_s\left(m_t, \tilde{H}_s, x_{s,t}\right) + \lambda_{s,t}\left(\left(1-\tau_y\right)w_t n_{s,t} + \pi^{-1}_t x^{FPAM}_{b,t-1}\right. \\ &\left. -\rho_t\left(m^*_t - (1-\nu)\pi^{-1}_t m_{t-1}\right) - \delta\, p^h_t\tilde{H}_s - \left(R^{-1}_t b_t - b_{t-1}\right) + \Pi_t + T_{s,t} - c_{s,t}\right)\end{aligned}$$

and consequently its derivative with respect to m_t^* simplifies to

$$\frac{\partial \mathcal{L}_{s,t}}{\partial m_t^*} = 0 \iff \lambda_{s,t}\rho_t = \beta_s \mathbb{E}_t \left[\lambda_{s,t+1}\pi_{t+1}^{-1}\rho_t \left(\left(q_t^* - \Delta_{q,t}\right) + \rho_{t+1}\left(1-\nu\right)\right)\right] +$$
$$+ \beta_s^2 \mathbb{E}_{t+1}\left[\lambda_{s,t+2}\pi_{t+1}^{-1}\rho_t \left(\pi_{t+2}^{-1}\left(1-\rho_{t+1}\right)\left(1-\nu\right)\left(\left(q_t^* - \Delta_{q,t}\right) + \rho_{t+2}(1-\nu)\right)\right)\right] + \dots$$
$$+ \beta_s^2 \mathbb{E}_{t+1}\left[\lambda_{s,t+3}\pi_{t+1}^{-1}\rho_t \left(\pi_{t+2}^{-1}\pi_{t+3}^{-1}\left(1-\rho_{t+1}\right)\left(1-\rho_{t+2}\right)\left(1-\nu\right)^2\left(1+\rho_{t+2}(1-\nu)\right)\right)\right] + \dots$$
$$+ \beta_s^k \mathbb{E}_{t+k}\left[\lambda_{b,t+k+1}\pi_{t+1}^{-1}\rho_t \left(\prod_{j=1}^{k}\pi_{t+j+1}^{-1}\left(1-\rho_{t+j}\right)\left(1-\nu\right)\right)\left(1+\rho_{t+k+1}(1-\nu)\right)\right] + \dots$$

which can be rewriten as

$$1 = \Omega_{s,t}^m + \mho_{s,t}^x (q_t^* - \Delta_{q,t}) \tag{A81}$$

where $\Omega_{s,t}^m$ is the marginal continuation benefit of an additional unit of face value debt and again defined as in the problem with FRM (equation (A31)), and $\mho_{s,t}^x$ is the marginal benefit to the saver of promising an additional dollar of inital payments and it is defined as

$$\mho_{s,t}^x = \mathbb{E}_t\left[\Lambda_{t,t+1}^s \pi_{t+1}^{-1}\right] + \mathbb{E}_{t+1}\left[\Lambda_{t,t+1}^s \Lambda_{t+1,t+2}^s \pi_{t+1}^{-1}\pi_{t+2}^{-1}(1-\rho_{t+1})(1-\nu)\right] \tag{A82}$$

C.2. The general case: a T periods adjustable mortgage

In a $T > 1$ period fixed adjustable mortgage, the promised payments low of motion is

$$\begin{aligned} x_{b,t}^{FPAM} =& \rho_t q_t^* m_t^* + \left[\sum_{\tau=1}^{T-1}\left(\prod_{i=0}^{\tau-1}(1-\rho_{t-i})(1-\nu)\pi_{t-i}^{-1}\right)\rho_{t-\tau}(q_{t-\tau}^* - \Delta_{q,t})m_{t-\tau}^*\right] + \\ &+ \left(\prod_{i=0}^{T-1}(1-\rho_{t-i})(1-\nu)\pi_{t-i}^{-1}\right) q_{t-T}^* m_{t-T} \end{aligned} \tag{A83}$$

for the borrower, while similarly for the saver is given by

$$\begin{aligned} x_{s,t}^{FPAM} =& \rho_t (q_t^* - \Delta_{q,t}) m_t^* + \left[\sum_{\tau=1}^{T-1}\left(\prod_{i=0}^{\tau-1}(1-\rho_{t-i})(1-\nu)\pi_{t-i}^{-1}\right)\rho_{t-\tau}(q_{t-\tau}^* - \Delta_{q,t})m_{t-\tau}^*\right] + \\ &+ \left(\prod_{i=0}^{T-1}(1-\rho_{t-i})(1-\nu)\pi_{t-i}^{-1}\right)(q_{t-T}^* - \Delta_{q,t-T}) m_{t-T} \end{aligned} \tag{A84}$$

If one uses these low of motions in the borrower and saver's budget constraint, the solution of the optimization problem faced by each agent will be almost identical to the one for the simplified example above, with the expection that the marginal cost for the borrower of promising one additional dollar of payments, $\mathcal{O}^x_{b,t}$, will be now defined as

$$
\begin{aligned}
\mathcal{O}^x_{b,t} =& \mathbb{E}_t\left[\Lambda^b_{t,t+1}\pi^{-1}_{t+1}(1-\tau_y)\right]+ \\
&+\sum_{\tau=1}^{T-1}\mathbb{E}_{t+\tau}\left[\Lambda^b_{t,t+1}\pi^{-1}_{t+1}\left(\prod_{j=1}^{\tau}\Lambda^b_{t+1+j,t+j}\pi^{-1}_{t+1+j}(1-\rho_{t+j})\right)(1-\tau_y)\right]
\end{aligned}
\tag{A85}
$$

while the marginal benefit for the saver of an additional dollar of payments, $\mathcal{O}^x_{s,t}$, is

$$
\begin{aligned}
\mathcal{O}^x_{s,t} =& \mathbb{E}_t\left[\Lambda^b_{t,t+1}\pi^{-1}_{t+1}\right]+ \\
&+\sum_{\tau=1}^{T-1}\mathbb{E}_{t+\tau}\left[\Lambda^b_{t,t+1}\pi^{-1}_{t+1}\left(\prod_{j=1}^{\tau}\Lambda^b_{t+1+j,t+j}\pi^{-1}_{t+1+j}(1-\rho_{t+j})(1-\nu)\right)\right]
\end{aligned}
\tag{A86}
$$

Appendix D. Results: Interest Rate Transmission

As in the paper, this section illustrates how the novel features of the model amplify transmission from nominal interest rates into debt, house prices, and economic activity, and demostrates the implications from monetary policy.

D.1. The Constraint Switching Effect

To isolate the effects of the credit limit structure, we compare three model economies:

- *Benchmark economy*: both credit limits apply, i.e. $\bar{m}_t = \bar{m}_t^{PTI} \int^{\bar{e}_t} e_i d\Gamma_e(e_i) + \bar{m}_t^{LTV} (1 - \Gamma_e(\bar{e}_t))$.
- *PTI economy*: imposes only the PTI constraint, i.e. $\bar{m}_t = \bar{m}_t^{PTI}$.
- *LTV economy*: imposes only the LTV constraint, i.e. $\bar{m}_t = \bar{m}_t^{LTV}$.

These economies are otherwise identical in their specification and parameter values, with one exception: the credit parameters θ^{LTV} and θ^{PTI} are recalibrated in the PTI and LTV economies so that their steady state debt limits match those of the Benchmark economy. This requires that $\theta^{LTV} = 0.729$ and $\theta^{PTI} = 0.272$.

To demostrate how this channel can work through movements in nominal rates only, Figure A1 displays the response to a near-permanent –1% shock to the inflation target. This shock induces a near 0.5% fall in nominal mortgage rates. As credit conditions become more favourable, the fraction of borrowers that prepay their mortgages increases to secure lower fixed rates. As a result, new debt starts to be issued, which combined with larger loans (2% increase), it leads to an increase in the stock of debt.

This channel is present in all three economies but it is much larger in the PTI economy than in the LTV economy. In fact, the increse in debt after 20Q is 2.5 times larger (4.8% vs. 1.9%). This occurs because PTI limits are strongly affected by interest rates, which directly shift PTI constraints, potentially increasing the size of new loans in the PTI economy. In contrast, debt limits in the LTV economy are only indirectly affected by interest rates through house prices, which remain largely unchanged. As a result, the modest debt response is driven by a combination of lower inflation and an increase in the share of borrowers prepaying to lock in lower fixed rates on their mortgages.

Turning to the Benchmark economy, we also observe a substantial increase in debt (3.2% after 20Q) that is closer to that of the PTI economy than that of the LTV economy. This occurs dispite the fact that in the model, the majority of borrowers are constrained by the LTV (85% at steady state). The fact that the Benchmark economy is not the convex

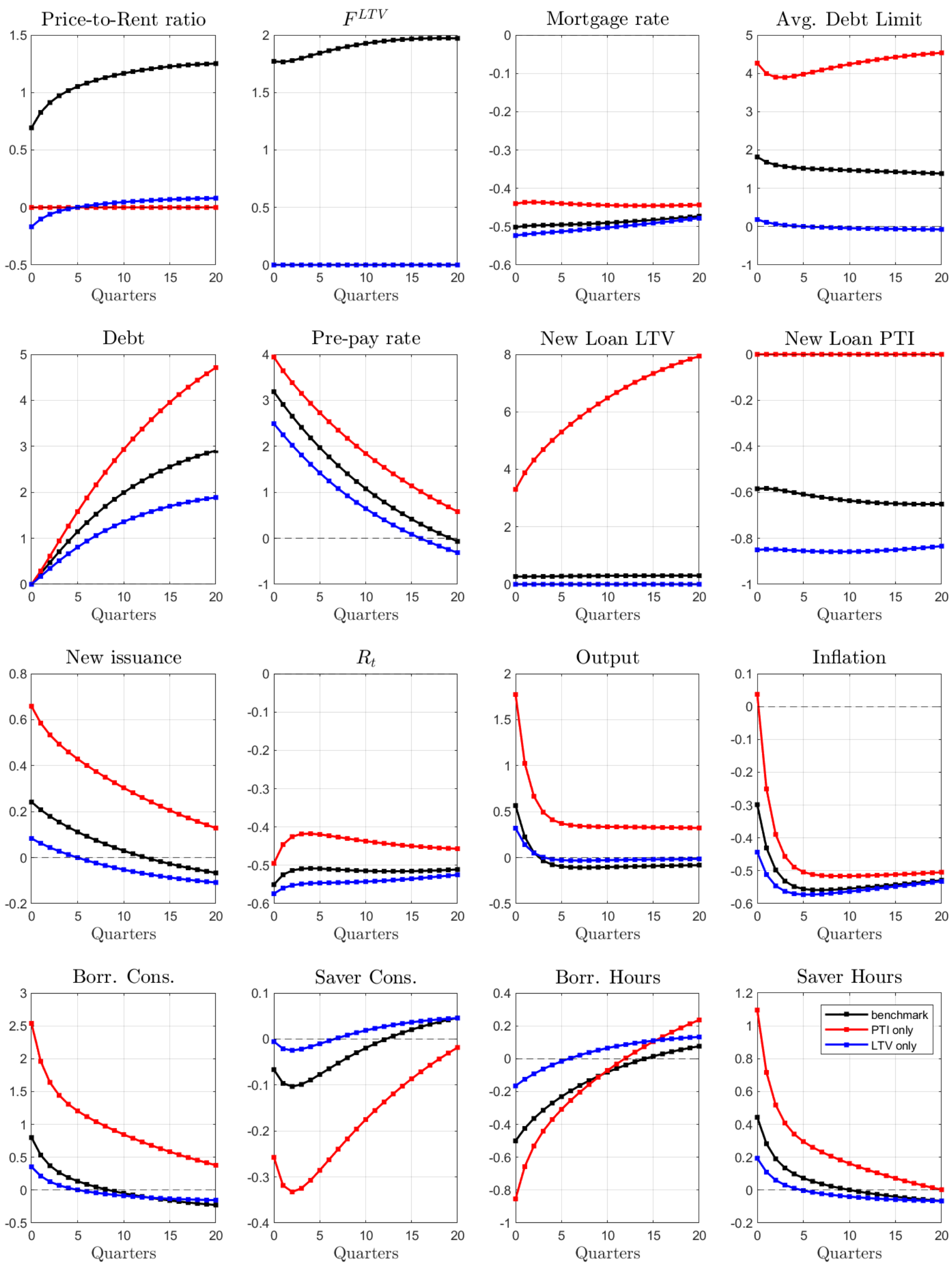

FIGURE A1. Response to –1% Inflation Target Shock.

Variable definitions are as follows. Price-Rent Ratio: $p_t^h/(u_t^h/u_t^c)$. Mortgage rate: $q_t^* - \nu$. Avg. Debt Limit $\bar{m}_t$. Debt: m_t. Prepay Rate: ρ_t. New issuance $\rho_t(m_t^* - (1-\nu)\pi_t^{-1}m_{t-1})$. New Loan LTV: $m_t^*/p_t^h h_{b,t}^*$. New Loan PTI: $(q_t^* + \alpha)m_t^*/w_t n_{b,t}$.

combination of the PTI and LTV economies is due to the constraint switching effect: as PTI limits loosen in response to the shock, many borrowers previously constrained by the PTI now find the LTV to be more restrictive, driving F^{LTV} up by nearly 2 percentage points. As a result, the implied price-to-rent ratio rises 1.7% because these new LTV-constrained borrowers increase thier borrowing limit with additional housing collateral boosting housing demand and house prices. Note that this price to rent ratio response is in stark contrast with the response in the LTV and PTI economies in which the house price to rent ratio has a small or zero change.

This stronger effect on house prices is what explains the amplified debt response in the Benchmark economy (in comparison with the LTV economy). While it is true that the debt limit is relaxed for the PTI-constraint households, there are too few of these households (15% at steady state) to drive the differential response in mortgage debt. However, the new LTV-constrained households, which represent the majority of the population, can now increase their debt limits to a greater extend due to the larger response of house prices.

D.2. Frontloading Effect

To see this mechanism in action, we compare three alternative economies:

- Benchmark economy where prepayment rates are *endogenously* determined.
- Benchmark economy with *exogenous* prepayment.
- LTV economy with *exogenous* prepayment.

In the two economies with exogenous prepayment, the prepayment rate is fixed to its steady state value at all times, i.e. $\rho_t = \rho_{ss}\ \forall t$.

To demonstrate how the frontloading effect operates, we depict in Figure A2 the response to a –1% term premium shock. This induces a decline in the real mortgage rate that is close to 0.3%, before gradually decaying. Due to the constraint switching effect, this fall in rates generates a much larger increase in debt limits in both version of the Benchmark economy relative to the LTV economy. But dispite the similar rise in debt limits, the paths of credit issuance across the variations of the Benchmark economy are very different. The endogenous prepayment version delivers a much more frontloaded path of issuance that begins far above, and eventually falls below, the smaller but more persistent issuance of the exogenous prepayment economies.

This pattern leads to highly disparte effects on output, whose response is two and half times larger on impact in the endogenous prepayment Benchmark economy (0.15%) rel-

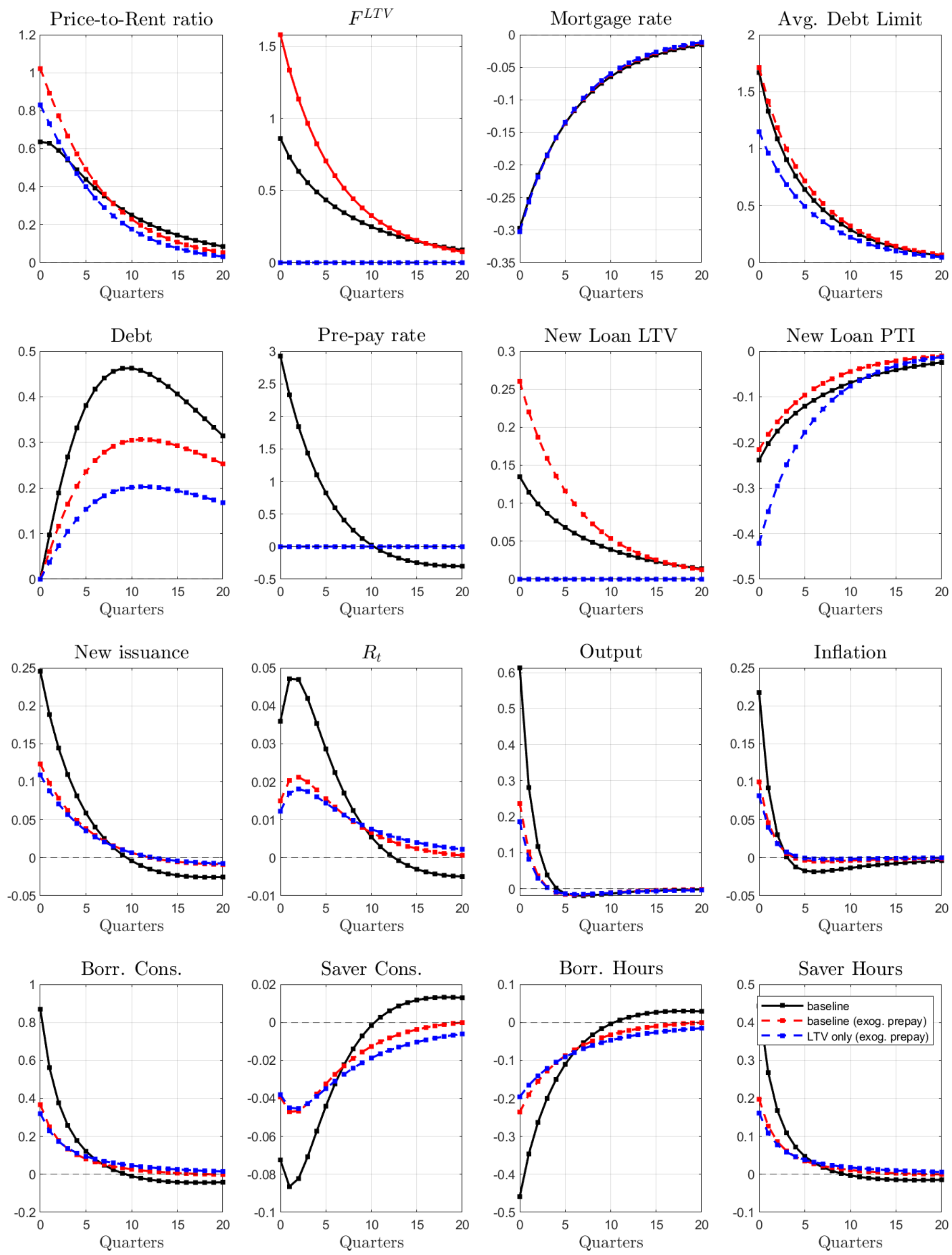

FIGURE A2. Response to –1% Term Premium Shock.

Variable definitions are as follows. Price-Rent Ratio: $p_t^h/(u_t^h/u_t^c)$. Mortgage rate: $q_t^* - \nu$. Avg. Debt Limit $\bar{m}_t$. Debt: m_t. Prepay Rate: ρ_t. New issuance $\rho_t(m_t^* - (1-\nu)\pi_t^{-1}m_{t-1})$. New Loan LTV: $m_t^*/p_t^h h_{b,t}^*$. New Loan PTI: $(q_t^* + \alpha)m_t^*/w_t n_{b,t}$.

ative to its exogenous counterpart (0.06%) which is instead close to that of the exogenous prepayment LTV economy (0.04%).

D.3. Monetary Policy

To understand the implications of the mortgage credit channel for conventional monetary policy transmission we study two alternative economies:

- Benchmark economy
- LTV economy with exogenous prepayment

This exercise is carried out under a policy rule that guarantees *perfect inflation stabilization*. That is, the monetary authority moves the policy rate as much as needed to perfectly stabilize inflation. This rule allows us to evaluate the strenght of monetary policy under the two scenarios since smaller movements in the policy rule to keep inflation at target following a shock, would imply a more effective monetary policy.

Figrue A3 compares the response to a 1% productivity shock in the afromentioned economies to ilustrate the implications of the constraint switching and frontloading effects for monetary policy. The productivity shock is deflationary and persistent, so the monetary authority must persistently cut rates to return inflation to target. However, the initial required fall in the policy rate is more than 15% larger in the economy without PTI credit limits and exogenous prepayment (10 bsp vs 8 bsp). Overall, these results indicate that monetary policy is stronger due to the mortgage credit channel, requiring smaller movements in the policy rate to stabilize inflation.

The weaker response of rates in the Benchmark economy is explained by a wave of new borrowing. It increases demand putting upward pressure on prices and consequently requires less monetary stimulus to correct the deflationary shock. As a result, it creates a dilemma for policymakers since the smaller changes in the policy rate are associated with a larger shift in mortgage issuance, with debt rising by more than 50% in the Benchmark economy after 20Q (0.57% vs. 0.37%).

The evolution of the house price to rent ratio is also substantially different across the two economies. In the model without PTI limits, the price to rent ratio falls because the user cost of owning ($u^h_{b,t}/u^c_{b,t}$) increases as the marginal utility of consumption falls due to higher borrower consumption. This effect is muted in the Benchmark economy because the new LTV-constrained households increase their debt limit using housing as collateral which in turn boost housing demand and prices.

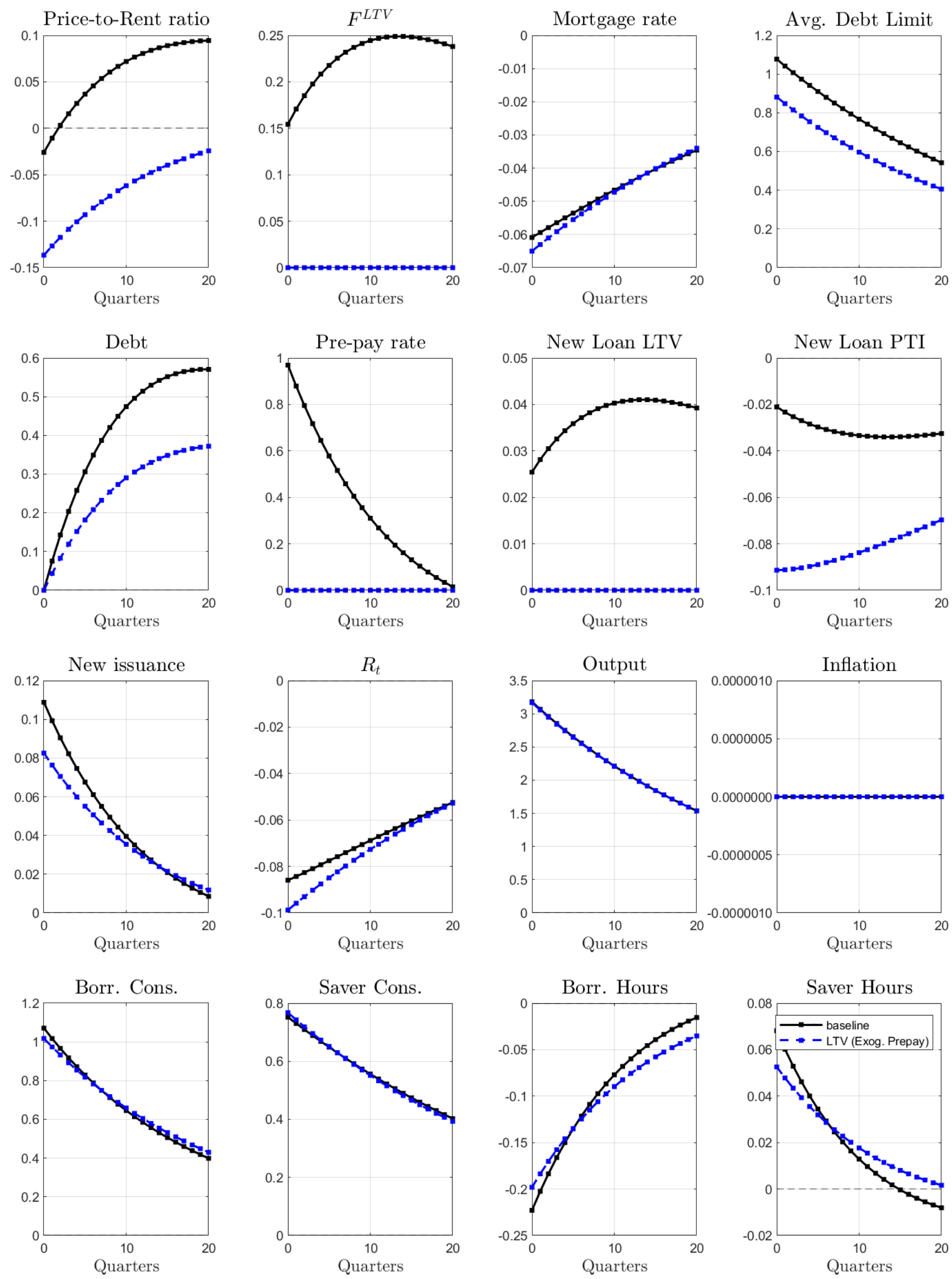

FIGURE A3. Response to 1% Productivity Shock.

Variable definitions are as follows. Price-Rent Ratio: $p_t^h/(u_t^h/u_t^c)$. Mortgage rate: $q_t^* - \nu$. Avg. Debt Limit $\bar{m}_t$. Debt: m_t. Prepay Rate: ρ_t. New issuance $\rho_t(m_t^* - (1-\nu)\pi_t^{-1}m_{t-1})$. New Loan LTV: $m_t^*/p_t^h h_{b,t}^*$. New Loan PTI: $(q_t^* + \alpha)m_t^*/w_t n_{b,t}$.

D.4. Adjustable Rate Mortgages

To see the impact of the type of mortgage contract on the dynamics, we can compare the *Benchmark* economy with the *ARM economy* presented in Section B. The difference between the responses across economies depends substantially on the type of shock. Therefore, in Figure A4 we study the dynamics after a near-permanent shock to interst rates in the form of a inflation target shock; while in Figure A5 we look a temporary shock through the lens of the term permium shock.

On the one hand, impulse responses of output, inflation, debt and house prices are almost identical for near permanent shocks. If anything, the prepayment rates increase by twice as much in the Benchmark economy than in the ARM economy. However, it only translates into slightly larger increases in debt after 20Q (3.2% vs. 2.8%) as loan size increases by nearly the same amount in both economies. Moreover, the price to rent ratio increases by 1.5% after 20Q in both economies.

On the other hand, when shocks impose a temporary shift in mortgage rates the effect on debt and house prices is much stronger under fixed rate mortgages. Perhaps not surprisingly, borrowers rush to lock in lower rates before they go back to their initial levels, which results in a 3 times larger increase in the prepayment rates in the Baseline economy, which paired with an also higher loan size (1.6% vs. 1%) translates into an almost 2.5 times larger increase in debt at its peak (after 10Q). The house-price rent ratio also has substantially different dynamics dispite a smiliar initial response. In fact, the initial effect practically dies out after 1Q in the ARM economy while it takes much longer under fixed rate contracts.

The distinction between permanent versus transitory changes in mortgage rates is also relevant to understand redistributive effects under these two types of mortgage contracts. Unlike for aggregate dynamics, when shocks are temporary the short lived redistribution of consumption from borrowers to savers is identical across the two types of contracts, while for near permanent shocks this effect is more pronounced under adjustable rate mortgages.

D.5. Comparative Statics

D.5.1. Exogenous refinancing rates

To analyze the impact that different refinancing rates has on the dynamic responses of key macro aggregates, we solve the Benchmark economy with exogenous prepayment for various prepayment rates ranging from 0 to 1.

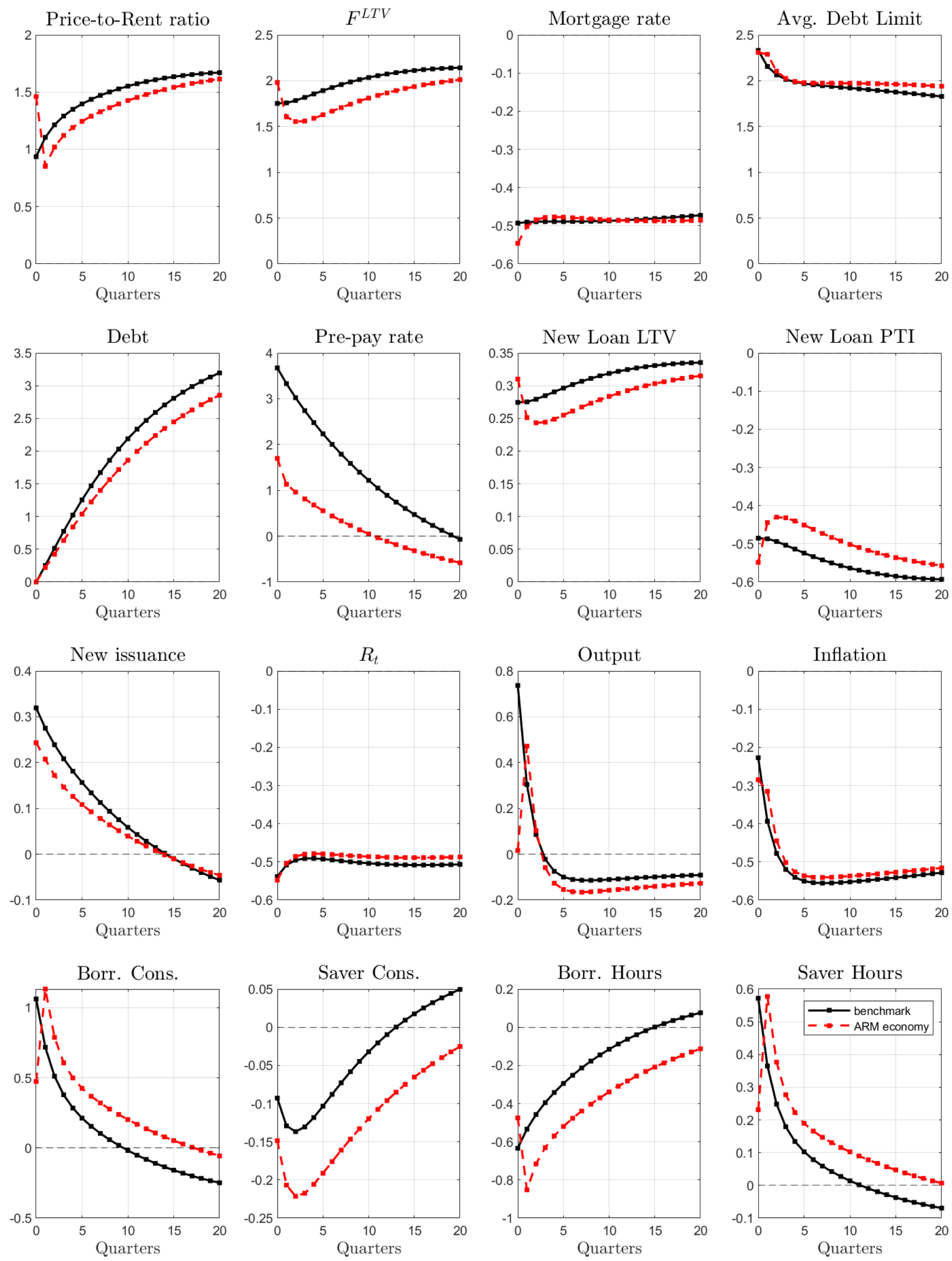

FIGURE A4. Response to –1% Inflation Target Shock.

Variable definitions are as follows. Price-Rent Ratio: $p_t^h/(u_t^h/u_t^c)$. Mortgage rate: $q_t^* - \nu$. Avg. Debt Limit $\bar{m}_t$. Debt: m_t. Prepay Rate: ρ_t. New issuance $\rho_t(m_t^* - (1-\nu)\pi_t^{-1}m_{t-1})$. New Loan LTV: $m_t^*/p_t^h h_{b,t}^*$. New Loan PTI: $(q_t^* + \alpha)m_t^*/w_t n_{b,t}$.

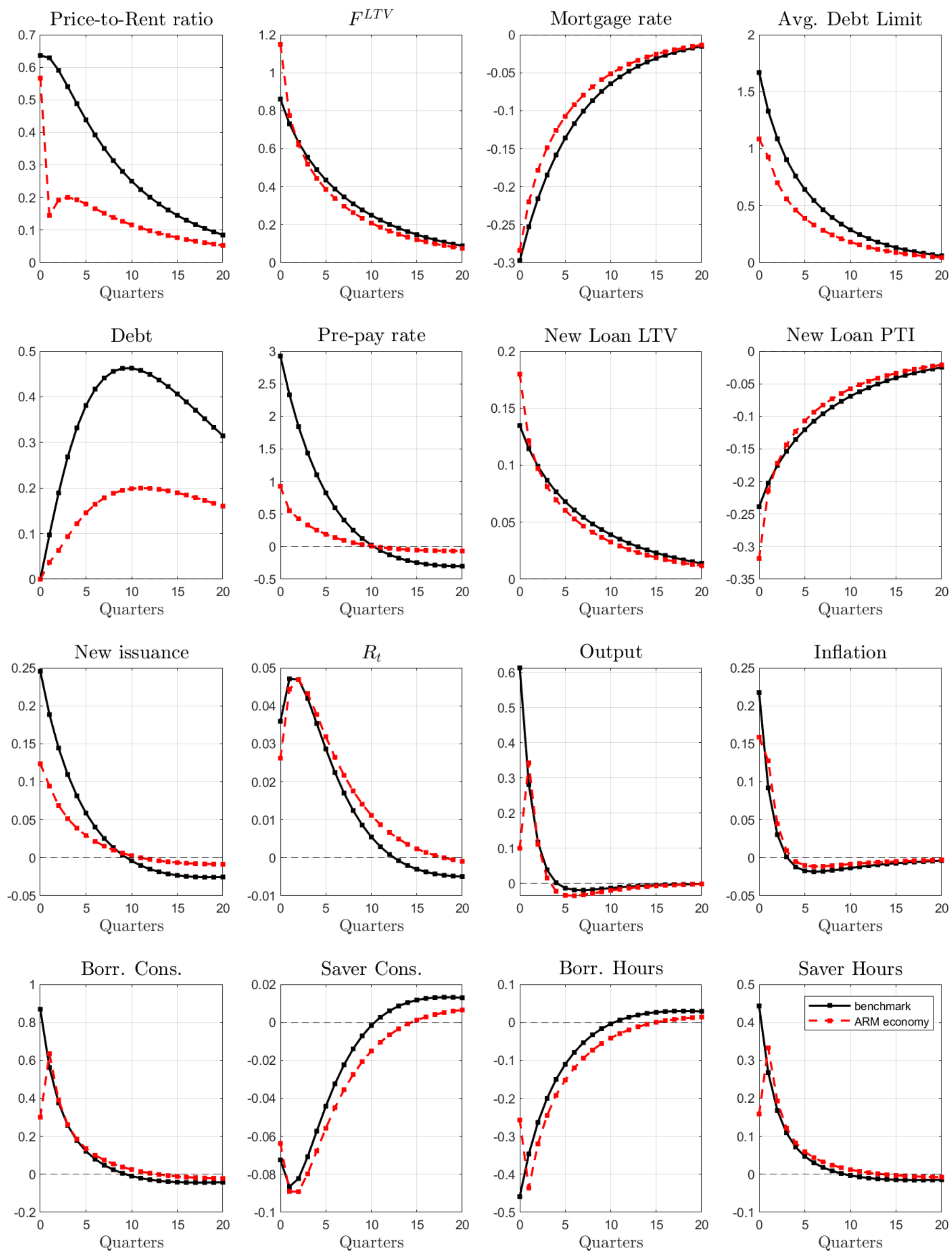

FIGURE A5. Response to –1% Term Premium Shock.

Variable definitions are as follows. Price-Rent Ratio: $p_t^h/(u_t^h/u_t^c)$. Mortgage rate: $q_t^* - \nu$. Avg. Debt Limit $\bar{m}_t$. Debt: m_t. Prepay Rate: ρ_t. New issuance $\rho_t(m_t^* - (1-\nu)\pi_t^{-1}m_{t-1})$. New Loan LTV: $m_t^*/p_t^h h_{b,t}^*$. New Loan PTI: $(q_t^* + \alpha)m_t^*/w_t n_{b,t}$.

Results are shown in Figure A6. This figure plots for each variable the response to a –1% infaltion target shock both upon impact (blue dots) and after 20Q (red dots) for various refiancing rates. The steady state level of the endogenous refinancing rate in the Benchmark economy is depicted as the solid black line in each of these subplots.

Debt. It is obvious that higher refinancing rates imply larger initial responses of new debt issuance. However, the long run impact on new issuance is slightly decreasing. The loan size is also increasing in the refinancing rate, but it is almost identical after 20Q. Since the effect on new issuance dies out quicker for high refinancing rates, and the loan size increases are similar across different refinancing rates, the stock of debt is very similar in the long run across the different economies.

Price-to-Rent ratio. The inital response of the house price to rent ratio is decreasing in the refinancing rate. The price to rent ratio can move for two reasons in the model. First, changes in collateral demand from new LTV-constrained borrowers which will lower housing demand and house prices. And second, movements in the marginal rate of substitution (MRS) between housing and consumption (user cost fromula for rental price). The initial impact on F^{LTV} is decreasing in the refinancing rate and it even becomes negative for very high values of $\bar{\rho}$. Moreover, the borrower consumption response is increasing in the refinancing rate, which should tranlate into higher MRS and therefore lower house price to rent ratios as the refinancing rate increases. In the long run, the response of the price to rent ratio is only slightly increasing in the refinancing rate as the fraction of LTV-constrained borrowers is also increasing but borrower consumption is not changing or slightly decreasing in the refinancing rate.

Output and Inflation. The initial response of both output and inflation is increasing in the refinancing rate, however, only in the short run as the response after 20Q is almost identical for different refinancing rates.

Need to understand why do mortgage rates respond defferently for different refinancing rates?

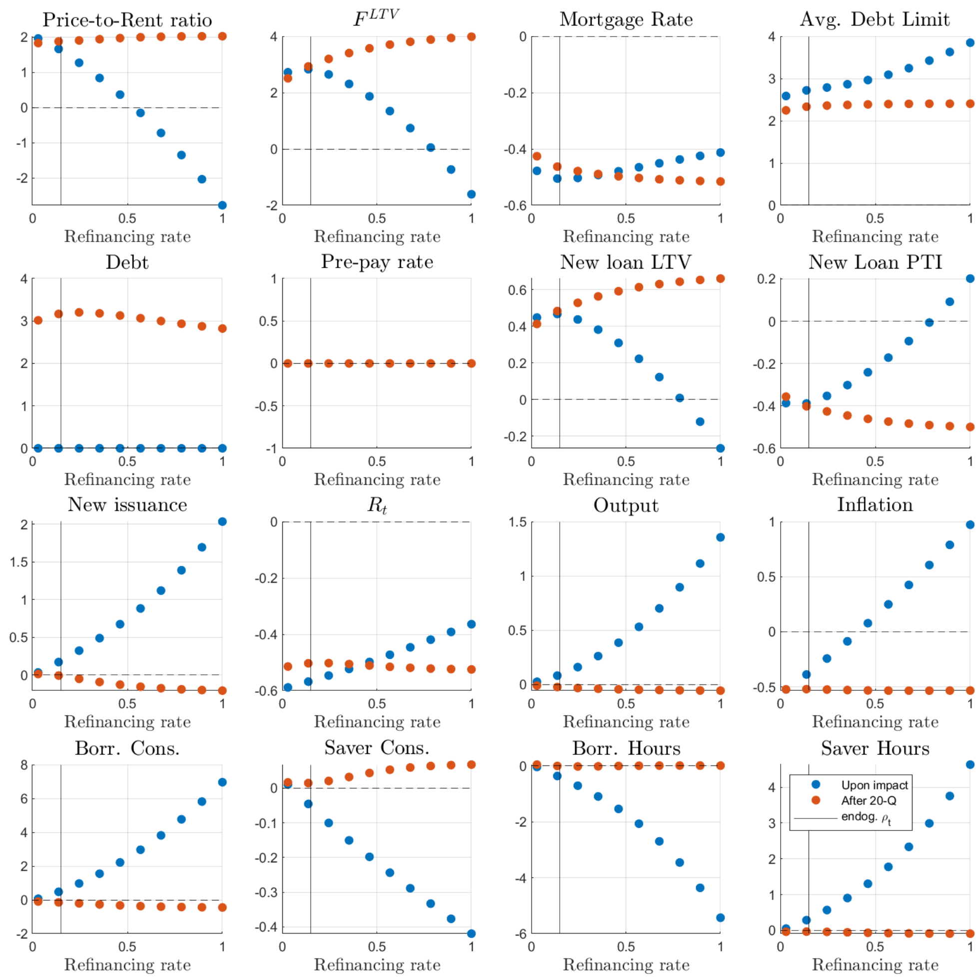

FIGURE A6. Q1 and Q20 Responses to –1% Inflation Target Shock.

Variable definitions are as follows. Price-Rent Ratio: $p_t^h/(u_t^h/u_t^c)$. Mortgage rate: $q_t^* - \nu$. Avg. Debt Limit $\bar{m}_t$. Debt: m_t. Prepay Rate: ρ_t. New issuance $\rho_t(m_t^* - (1-\nu)\pi_t^{-1}m_{t-1})$. New Loan LTV: $m_t^*/p_t^h h_{b,t}^*$. New Loan PTI: $(q_t^* + \alpha)m_t^*/w_t n_{b,t}$.